Development of the Visible Light Tomography Diagnostics for UTST Spherical Tokamak Plasmas

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Abstract — The University of Tokyo Spherical Tokamak (UTST) device has successfully demonstrated the merging start-up of spherical tokamak (ST) plasma by using the double-null merging method solely by external poloidal field coils. It was designed to form an ultra-high beta plasma by means of high power heating effect of magnetic reconnection. Since the most dangerous mode for the high beta ST is the ballooning mode, it is needed to measure the shape of the toroidal cross-section of plasma. The visible light computed tomography (CT) is one of the powerful diagnostics to study the MHD activities of fusion plasma. This paper describes a newly developed visible light CT measurement system with a tomographic reconstruction algorithm for an annular cross-section of plasma using modified Fourier-Bessel expansions method. Spatial distribution of HeII (656.0nm) light was reconstructed with this algorithm in the case of center solenoid coil (CS) and also in the case of poloidal field coils (PF) discharge, and 2D profile of toroidal mode n=1 of plasma emissivity has been observed successfully.

Keywords — spherical tokamak, high beta, visible light tomography, ballooning instability

I. INTRODUCTION

A spherical tokamak (ST) is a low-aspect-ratio tokamak that supports a compact and economical fusion reactor. The University of Tokyo Spherical Tokamak (UTST) device was designed to form an ultra-high beta plasma by means of high power heating effect of magnetic reconnection [1]. Since the most dangerous mode for the high ST is the ballooning mode, it is needed to measure the shape of the toroidal cross-section of plasma. The visible light computed tomography (CT) is one of the powerful diagnostics to study the MHD activities of plasma.

In UTST device the whole toroidal plane is annular cross-section because there is a center stack penetrating the plasma. So, this paper describes a newly developed visible light CT measurement system with a tomographic reconstruction algorithm for an annular toroidal cross-section of plasma ($z=0$) using modified Fourier-Bessel expansions method [2]. The visible light emissivity, $g(r, \theta)$ was expanded into the Fourier series in the azimuthal direction and a combination of Bessel and Neumann functions in radial direction, for the purpose of satisfying the annular boundary condition.

In order to demonstrate the validity of the modified Fourier-Bessel expansions method in the new measurement. We have performed a model calculation for various emissivity profile of plasma using the 24-channel projection signals. Experimental data of 24-channel HeII (656.0nm) light signals were achieved by the 24-channel photomultiplier tubes (PMTs) with optical fibers attached to a toroidal cross-section of the UTST device, and the spatial distribution of the HeII (656.0nm) light was reconstructed by the proposed algorithm of the modified Fourier-Bessel expansions.

II. THE UTST PLASMA MERGING METHOD

In the UTST device, the ST plasma was generated using double-null merging method (DNM) with the four PF coils located outside of the vacuum vessel. Fig. 1 show the schematic view of the DNM method [3].

At the step 1, two magnetic null points are generated at the upper and lower regions inside the vacuum vessel. At the step 2, two STs are generated at the null points. Step 3) Two STs are pushed to midplane. Step 4) A single high beta ST are generated.

Fig. 1. Schematic view of DNM method in UTST. Step 1) Two null points are generated by outer PF coils. Step 2) Two STs are generated at the null points. Step 3) Two STs are pushed to midplane. Step 4) A single high beta ST are generated.
II. CT MEASUREMENT SYSTEM

A. Purpose and Content of the Research

In this study, to observe the instability of the ultra-high beta plasma, a visible light tomography system measuring the shape of the annular cross-section of plasma has been built. Fig. 2 shows the schematic view of visible light CT measurement system. This measurement system mainly consists of the following two parts.

![Fig. 2. The schematic view of visible light CT measurement system.](image)

(1) Hardware Part

As shown in Fig. 2, the hardware of the measurement system mainly consists of pinholes, flexible optical fibers, monochromators, PMTs, digitizers and a computer. The view chords of detection are determined by the pinhole and the fiber locations.

The cross-section of the UTST device is also shown in Fig. 2. The circle in the center is the center stack, the graylines show the view chords, R is the distance between the center point and view chord.

![Fig. 3. Geometry of the coordinate system.](image)

(2) Software Part

On the inner and outer walls of the vacuum vessel, there is a boundary condition that the spatial distribution of light emissivity is zero. To satisfy these boundary conditions, the visible light emissivity was expanded into the Fourier series in the azimuthal direction and a combination of Bessel and Neumann functions in radial direction.

B. CT Image Reconstruction Algorithm

CT image reconstruction is a problem of determining the emission profile, \( g(r, \theta) \), from the projection signals, \( f(p, \phi) \). The coordinate system used in this method is shown in Fig. 3. The projection signal \( f(p, \phi) \) is represented as the line integral of \( g(r, \theta) \) along a sight chord whose length is \( L \),

\[
f(p, \phi) = \int_{L(p, \phi)} g(r, \theta) d\theta \quad (1)
\]

The plasma area which emits light is shown in gray color in Fig. 3, and outside of the annular cross-section \( g(r, \theta) \) has zero light emission. Emission profile \( g(r, \theta) \) was expanded into the Fourier series in the azimuthal direction and a combination of Bessel Neumann functions in radial direction as following equation.

\[
g(r, \theta) = \sum_{m=0}^{M_{\text{max}}} \sum_{l=0}^{L_{\text{max}}} \left[ a_{m}^{(c)} \cos m\theta + a_{m}^{(s)} \sin m\theta \right] g_m^l(r) \quad (2)
\]

Where,

\[
S_m^l(r) = J_m(\lambda_m^{l+1} r) - \frac{J_m(\lambda_m^{l+1} r_{CS})}{N_m(\lambda_m^{l+1} r_{CS})} N_m(\lambda_m^{l+1} r) \quad (3)
\]

\[
J_m(r) \text{ is the } m\text{th Bessel function of the first kind,}
\]

\[
N_m(r) \text{ is the } m\text{th Neumann function. } \lambda_m^{l+1} \text{ is the (l+1)th zero of the following equation.}
\]

\[
J_m(\lambda \cdot r)N_m(\lambda \cdot r_{CS}) - N_m(\lambda \cdot r)J_m(\lambda \cdot r_{CS}) = 0 \quad (4)
\]

The projection signal, \( f(p, \phi) \), can be obtained using the line integral of Eq. (2) along the sight chord \( L(p, \phi) \),

\[
f(p, \phi) = \sum_{m=0}^{M_{\text{max}}} \sum_{l=0}^{L_{\text{max}}} \left[ a_{m}^{(c)} f_m^{(c)}(p, \phi) + a_{m}^{(s)} f_m^{(s)}(p, \phi) \right] \quad (5)
\]

Where,

\[
f_m^{(c,s)}(p, \phi) = \int_{L(p, \phi)} (\cos m\theta, \sin m\theta) g_m^l(r) ds \quad (6)
\]
After numerical integration of Eq. (5), Eq. (4) can be cast as a set of linear equations which can be written in matrix form,

\[ f = A \cdot g \]  

(7)

and it can be inverted to obtain the expansion coefficients, \( a_m^{(r,s)} \). This technique takes advantage of noise reduction through the least-squares fitting method.

C. Evaluation of the Algorithm for Observation System

(1) Assumed model

Assumed view chords of the model correspond with the view of experimental setup of the optical fibers. The plasma area is the gray colored region \( (r_{CS} \leq r \leq 1: \text{normalized}) \) as shown in Fig. 3. The visible emission from the plasma is collected by four sets of optical fiber arrays placed on the midplane of the device. Each array is composed of 6 optical fibers as shown in Fig. 1. To demonstrate the availability of this algorithm in the UTST CT measurement system, two source profiles were assumed, the first one is assumed to have the symmetric toroidal distribution of a hollow structure in radial direction, the second one is the distribution of the toroidal mode \( n=1 \). Fig. 4 (a) shows the 2D emissivity profile and the radial emissivity profile of the first source profile. Fig. 5 (a) shows the 2D emissivity profile and the radial emissivity profile of the second source profile at three different toroidal locations of \( \theta = 0, \pi, -\pi \) on the midplane.

(2) The Steps and Results of the Model Calculation

It is necessary to demonstrate the availability of the CT image reconstruction algorithm before putting it into the application on the experimental CT image reconstruction. The model calculation was carried out by the following four steps.

Step 1: Obtain the projection, \( f(p,\phi) \) of the assumed source profile along the view chord.

Step 2: Calculate the \( f_m^{(r,s)}(p,\phi) \) along the assumed view chord.

Step 3: Obtain the expansion coefficients, \( a_m^{(r,s)} \) by solving Eq. (7).

Step 4: put \( a_m^{(r,s)} \) into \( g(r,\theta) \) and reconstruct the CT image.

The results of the model calculation are shown in Fig. 4 (b) and Fig. 5 (b). Fig. 4 (b) shows the 2D emissivity profile and the radial emissivity profile of the first source profile reconstructed by the algorithm. Fig. 5 (b) shows...
the 2D emissivity profile and the radial emissivity profile of the second source profile reconstructed by the algorithm. Comparing the reconstructed results with the assumed emissivity profiles, it was found that the algorithm is available for the UTST CT image measurement system.

C. CT Image Reconstruction Results of Experimental Data

(1) Discharge Operation
The typical UTST plasma was generated by PF coils using DNM method with CS coils support (called PF+CS discharge). Fig. 6 shows the contours of the poloidal magnetic flux surface (lines) and toroidal current density (grayscale) in the case of PF+CS discharge.

![Fig. 6. Contours of the poloidal magnetic flux surface (lines) and toroidal current density (grayscale) in the case of PF+CS discharge.](image)

(2) Spatial Distribution of HeII (656.0nm) Light Intensity
In the case of PF+CS discharge, HeII (656.0nm) light intensity was measured and the spatial distribution of it was reconstructed by the algorithm of the modified Fourier-Bessel expansions technique. The reconstruction results were shown in Fig. 7. At 7.64ms and 7.66ms, the n=2 toroidal mode of plasma was observed, while n=1 toroidal mode of plasma was observed at 7.72ms and 7.74ms. It is demonstrated that this algorithm can be used to reconstruct the spatial distribution of UTST plasma emissivity profile, however, the ballooning instability has not been observed in the present PF+CS discharge.

![Fig. 7. The time evolution of 2D profile of HeII (656.0nm) light intensity in the case of PF+CS discharge.](image)

III. CONCLUSION

The toroidal midplane of the UTST device is annular cross-section because of the center stack. This paper introduced an algorithm based on the modified Fourier-Bessel expansions method which can satisfy the boundary condition of the emissivity $g(r, \theta) = 0$ in the inner an outer diameters of annular cross-section. Two source emissivity profile were assumed and reconstruction was demonstrated by the algorithm successfully. It is shown that the algorithm can be applied in CT image measurement system of the UTST device by reconstructing the spatial distribution of HeII (656.0nm) light.

IV. FUTURE WORK

Another set of fiber array with 5 channels will improve the accuracy of the measurement. With 30-channel fibers the toroidal mode n=2 will be measured more accurately. We are also planning to insert a carbon electrode and make it spark at the specific position in plasma to distinguish the magnetic surface on which the ballooning kink instability may occur. Hence, a local 2-dimensional CT image measurement system will be established by using the emissivity of the carbon lines from the carbon electrode.

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