Backward Tumbling Control for Power-Assisted Wheelchair based on Phase Plane Analysis

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Abstract— **We propose a novel control strategy of backward tumbling for power-assisted wheelchair operated by moving the push rims. The power-assistance units of wheelchair amplify the torque applied to the push rims. Excessive amplified torque sometimes causes dangerous backward tumbling. This paper analyzes backward tumbling as the interaction between the position of center-of-mass on man-wheelchair system and its angular velocity on the phase-plane. However, center-of-mass(COM) of man-wheelchair system cannot be directly measured. Therefore, the proposed control method prevents dangerous backward tumbling by decreasing assistance-ratio based on both COM observer to estimate the position of the COM considering some uncertainties; and phase plane analysis using the estimated. We show the effectiveness of the proposed control method in some experiments.**

Keywords- **backward tumbling control, COM Observer, phase plane, power-assisted wheelchair**

I. Introduction

For years, people with mobility disabilities were generally using wheelchairs as the preferred prescriptions if the user possesses the capabilities to operate one. It encourages the user to remain physically active because it takes a huge effort to propel the large drive wheels. This is a good exercise, however, various driving environments such as ramps, lawn, and soft carpet often require too much force and endurance to propel manual wheelchairs. Therefore, in order to reduce physical fatigue of the users and improve wheelchair maneuverability, power-assisted wheelchair was developed as the existence between manual wheelchair and powered wheelchair of joystick operation type. In addition to reducing physical fatigue, power-assisted wheelchair gives some operators a kind of superiority against powered wheelchair of joystick-operation type from a viewpoint of active appearance.

The excessive amplified torque causes dangerous backward tumbling of the wheelchair. In this study, novel control method to prevent the dangerous backward tumbling of power-assisted wheelchair is proposed. The proposed strategy, which doesn't depend on mechanical instruments like safeguard bars, is composed of the idea of decreasing the power-assistance-ratio.

Backward tumbling phenomenon is caused by the interaction between the position of center-of-mass(COM) on man-wheelchair system and its angular velocity. So, the concept of phase-plane is applied to the analysis to develop the dangerous-index of backward tumbling. However, COM of man-wheelchair system cannot be measured directly. Therefore, first, COM observer to estimate the position of COM is proposed. Next, power-assistance-control method to prevent the dangerous backward tumbling is designed based on the phase-plane analysis. Finally, we perform some experiments to verify the estimation of COM while the front wheel is raising off the ground; and variable power-assistance-ratio control to prevent dangerous backward tumbling.

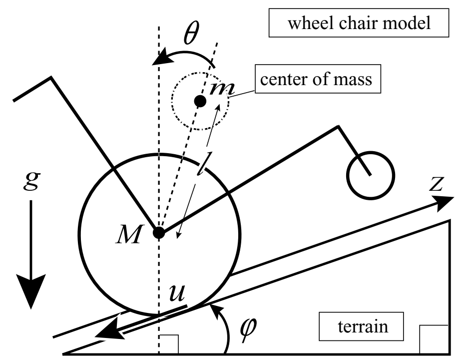


Fig. 1. Model of man and wheelchair system.

II. Fundamental description of power-assisted wheelchair

A. Motion equations using inverted pendulum

When the front wheel is raising off the ground, man-wheelchair system can be regarded as an approximate inverted pendulum model[1] shown by Fig.1. The operator's mass is assumed to be concentrated to COM with mass m linked to the rear wheel with distance l. And θ is the vertical angle term between the COM and the rear wheel axis. The model is controlled by force u. The motion equations of Fig.1 can be derived by Lagrange method and are denoted as

$$\begin{aligned} u &= (M+m)\ddot{z} + ml\cos\varphi(\ddot{\theta}\cos(\theta-\varphi) \\ &- \dot{\theta}^2\sin(\theta-\varphi)) + (M+m)g\sin\varphi \qquad (1) \\ 0 &= ml\ddot{z}\cos(\theta-\varphi)\cos\varphi + ml^2\ddot{\theta}\cos^2\varphi \\ &+ml^2\dot{\theta}^2\cos(\theta-\varphi)\sin(\theta-\varphi)(\frac{1}{\cos^2\varphi} - \cos^2\varphi) \\ &-mgl\sin(\theta-\varphi). \qquad (2) \end{aligned}$$

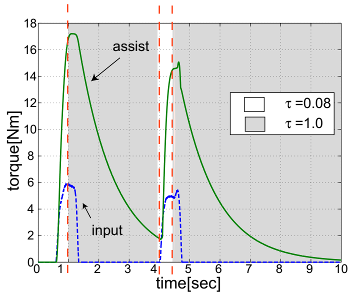


Fig. 2. Input torque and assistance torque versus time.

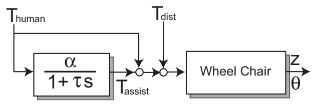


Fig. 3. Power-assisted controller.

where z is the position of moving direction, and the mass around the rear wheel is concentrated to the rear wheel axis with mass M. φ is the slope angle. Input force u is the sum of the forces given by the push rims and power-assistance units.

B. Fundamental method for power-assistance control

The power-assistance units amplify the manual inputs from the push rims with first order delay. The equation of power-assistance controller is

$$T_{assist} = \alpha \frac{1}{1+\tau s} T_{human} \tag{3}$$

where α is power-assistance-ratio, T_{assist} is the amplified torque from the push rim, T_{human} is the input torque from the push rim and τ is the time constant of first order delay. τ should be a suitable value realizing inertia for wheelchair. Therefore, τ at the beginning of propelling should be small value and that at the ending should be large as the following relations.

$$\tau = \begin{cases} \tau_{fast} & \frac{d}{dt} T_{human} > 0 \\ \tau_{slow} & \frac{d}{dt} T_{human} < 0 \end{cases}, \quad (\tau_{fast} < \tau_{slow}) \tag{4}$$

For example, our experiments adopt the following values respectively,

$$\tau_{fast} = 0.08[s], \quad \tau_{slow} = 1.0[s]. \tag{5}$$

And the behavior of this controller is shown in Fig.2.

Fig.3 shows the block diagram of power-assistance controller.

III. Analysis of backward tumbling using COM observer

Backward tumbling occurs as the interaction between the position of center-of-mass(COM) on man-wheelchair system and its angular velocity. But the position cannot be measured directly by using general sensors because operators' build types and sitting styles are widely differing. Conventional COM design methods for wheelchairs are done by static analysis based on anatomical viewpoint[2]. Then, we propose "COM Observer" to estimate dynamically the position of COM.

A. Derivation of COM observer

COM observer is derived from the linearized system such that it is consisted of the system (1),(2) around the unstable point of $(\theta, \dot{\theta}) = (0, 0)$. The linearized system around $(\theta, \dot{\theta}) = (0, 0)$ becomes the following equation.

$$\begin{pmatrix} \ddot{\theta} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0 & \frac{M+m}{Ml} g \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{1}{Ml} \\ 0 \end{pmatrix} u \tag{6}$$

Then, the output from wheelchair is defined as

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \theta \end{pmatrix}. \tag{7}$$

(7) is decided from the angular velocity of wheelchair's frame around the rear wheel axis. It is assumed that the operator's body and the frame of wheelchair rotate with the same velocity during backward tumbling.

COM observer is designed based on Gopinath method[3]. The estimated value is defined as $\hat{\theta}$. Substituting $\hat{\theta}$ for the first row of (6),

$$\dot{\hat{\dot{\theta}}} = \frac{(M+m)g}{Ml} \hat{\theta} - \frac{1}{Ml} u \tag{8}$$

is obtained. For a system of $\hat{\theta}$ with the feedback gain k of an error between $\ddot{\theta}$ and $\dot{\hat{\dot{\theta}}}$, the following equation is defined.

$$\dot{\hat{\theta}} = \dot{\theta} + k(\ddot{\theta} - \dot{\hat{\dot{\theta}}}) \tag{9}$$

In order to eliminate $\ddot{\theta}$, (9) is redefined as

$$\begin{aligned} \dot{\xi} &= \dot{\theta} - k\dot{\hat{\dot{\theta}}} - k\frac{M+m}{Ml} g \cdot k\dot{\theta} \\ &= [1 - k^2 \frac{(M+m)g}{Ml}]\dot{\theta} - k\frac{(M+m)g}{Ml}\xi + \frac{k}{Ml} u. \end{aligned} \tag{10}$$

As a result, the COM observer is given by

$$\hat{\theta} = \xi + k\dot{\theta}. \tag{11}$$

B. Determination strategy of observer gain considering some uncertainties

In general, observer gain k, which decides a speed when the error converges to zero; and when the estimated value converges to the true value, can be chosen as large as we want. However, practical modeling has some uncertainties as shown in Fig.4. Considering a general linearized dynamics with uncertainties D_A, D_B as

$$\begin{pmatrix} \ddot{\theta} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} + D_A \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \dot{\theta} \\ \theta \end{pmatrix} + \begin{pmatrix} B_1 + D_B \\ 0 \end{pmatrix} u, \tag{12}$$

we can decide an optimal value of k to minimize influences of D_A, D_B. We assume the output with observational error as follow.

$$y = (1 \;\; 0) \begin{pmatrix} \dot{\theta} \\ \theta \end{pmatrix} + \Delta\dot{\theta} \tag{13}$$

Rebuilding (11) by (13), we can obtain

$$\hat{\theta} = \frac{1 - k^2 A_{12}}{s + kA_{12}} \dot{\theta} + \frac{kB_1}{s + kA_{12}} u + \frac{1 - k^2 A_{12}}{s + kA_{12}} \Delta\dot{\theta}. \tag{14}$$

By (13), (9) becomes

$$\begin{aligned}\dot{\hat{\theta}} &= \dot{\theta} + \Delta\theta + k[A_{11}\dot{\theta} + (A_{12} + D_A)\theta \\ &\quad +(B_1 + D_B)u] - k(A_{12}\hat{\theta} + B_1 u). \qquad (15)\end{aligned}$$

Hence the error between the estimated and the true value is derived as the following equation from (15).

$$\begin{aligned}\dot{e} &= \dot{\hat{\theta}} - \dot{\theta} \qquad (16)\\ &= -kA_{12}e + \Delta\dot{\theta} + k(A_{11}\dot{\theta} + D_B u + D_A\theta)\end{aligned}$$

Considering $\theta \simeq \hat{\theta}$, the error system can be regarded as

$$\begin{aligned}\dot{e} &= -kA_{12}e + \Delta\dot{\theta} + k\{(A_{11} + \frac{(1-k^2A_{12})D_A}{s+kA_{12}})\dot{\theta} \\ &\quad +(D_B + \frac{kB_1D_A}{s+kA_{12}})u\} \\ &= -He + \Delta\dot{\theta} + k(S_A\dot{\theta} + S_B u) \qquad (17)\end{aligned}$$

where,

$$H = kA_{12} \qquad (18)$$

$$S_A = A_{11} + \frac{(1-k^2A_{12})D_A}{s+kA_{12}} \qquad (19)$$

$$S_B = D_B + \frac{kB_1D_A}{s+kA_{12}}. \qquad (20)$$

Because the error of (17) cannot be decreased in a condition of $k = 0$, k has to be as large as possible. Since man-wheelchair system is not generally affected by high-frequent disturbances, the numerator of H_2 norm by (19),(20) can be shown as

$$||S_A||_2^{num} = kA_{11}A_{12} + D_A(1-k^2A_{12}) \qquad (21)$$

$$||S_B||_2^{num} = k(A_{12}D_B + B_1D_A), \qquad (22)$$

which ignore influences of Laplacian operator s.

From calculation of $||S_A||_2^{num} = 0$, we can obtain two optimal observer gains as

$$k = \frac{A_{11}}{2D_A} \pm \frac{1}{2}\sqrt{(\frac{A_{11}}{D_A})^2 + \frac{4}{A_{12}}}. \qquad (23)$$

(22) tends to zero as decreasing k, hence the most optimal value suppressing influence of some uncertainties is decided from two solutions of (23).

Finally, optimal observer gain k for man-wheelchair system is obtained as

$$k = \frac{-B}{2M_d} + \frac{1}{2}\sqrt{(\frac{B}{M_d})^2 + \frac{4Ml}{(M+m)g}}, \qquad (24)$$

where,

$$\begin{cases} A_{11} = -B \\ A_{12} = \frac{M+m}{Ml}g \\ B_1 = -\frac{1}{Ml} \\ D_A = M_d = \frac{M_{real}+m_{real}}{M_{real}l_{real}} - \frac{M+m}{Ml} \\ D_B = \frac{1}{M_{real}l_{real}} - \frac{1}{Ml}. \end{cases}$$

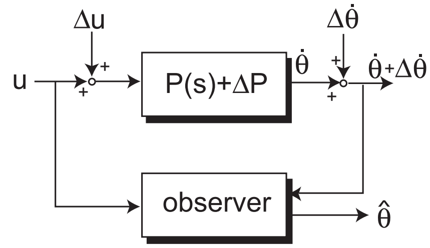
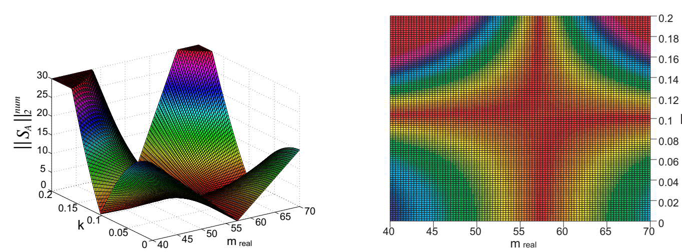


Fig. 4. COM observer with uncertainty.

C. Numerical analysis of observer gain

We derived (24) theoretically in above section. However, manual tuning of k is needed when we construct a controller for power-assisted wheelchair. Relation between the quantity of (21) and k is shown in Fig.5, where observer parameters are $[M = 25.0,\ m = 50.0,\ l = 0.3,\ g = 9.8,\ k = (0.0 \sim 0.2)]$; and man-wheelchair system's parameters are $[M_{real} = 25.0,\ l_{real} = 0.33,\ B = 0.05,\ m_{real} = (40.0 \sim 70.0)]$. Fig.5 denotes that $k \simeq 0.1$ minimizes $||S_A||_2^{num}$.

We can also obtain $k \simeq 0.1$ from varying l_{real}, hence it is found that tuning k around 0.1 is desirable to realize a robust estimation against uncertainties. Actually we adopted $k = 0.1085$ for the later experiments.



(a) (b)

Fig. 5. Evaluating result k versus (21) varying m_{real}.

D. Analysis of backward tumbling using phase plane

In this part, backward tumbling phenomenon is discussed with a phase plane for $\hat{\theta}$ and $\dot{\theta}$. As mentioned above, backward tumbling phenomenon is caused not only by the position of COM but also by its angular velocity. Fig.6 shows man-wheelchair phase plane, which divides the plane into three regions depending on the level of danger; A) proper safety zone ($\dot{\theta} < 0$ and below the negative slope asymptote), B) semi-safety zone ($\theta < 0$, $\dot{\theta} > 0$ and below the negative slope asymptote), and C) dangerous zone (above the negative slope asymptote).

Point P in Fig.6, which denotes man-wheelchair system maneuvering, is staying generally at the point $(\theta,\ \dot{\theta}) = (\theta_0, 0)$. But it will shift to C region through B region when backward tumbling occurs. Figs.7, 8 show the result of backward tumbling controlled by a subject without power-assistance control. There, of course, are some offset between gyro sensor(see Fig.9)'s angle and the estimated in

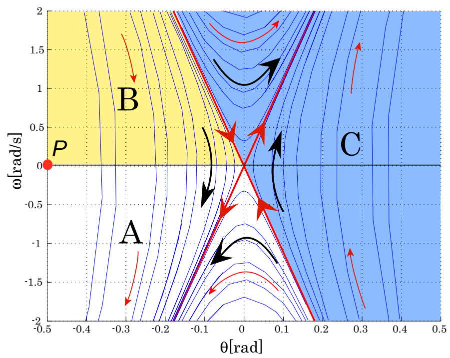


Fig. 6. Man-wheelchair phase plane.

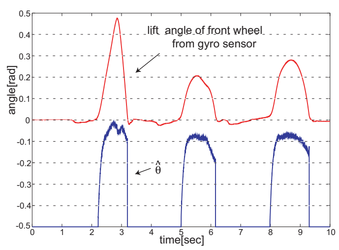


Fig. 7. Estimated angle $\hat{\theta}$ and Integral of gyro sensor's output.

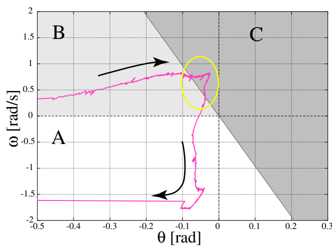


Fig. 8. Phase plane of Fig.7.

Fig.7. And it is shown in Fig.8 that the subject operated the wheelchair successfully to return from C region.

IV. Power-assistance-ratio control method based on phase-plane analysis

A. Design of power-assistance-ratio

Power-assistance-ratio control strategy for backward tumbling is proposed in this section. Power-assisted wheelchair should be controlled by the operator to be some extent from a viewpoint of secure feeling, hence we aim at realization of backward tumbling compatible controller with operations. The proposed power-assistance-ratio adjustment is shown as

$$\alpha = \alpha_{max} \exp(\beta \frac{\dot{\theta}}{\hat{\theta}}), \tag{25}$$

where, β is the decreasing constant which decides a speed of decreasing power-assistance-ratio. $\dot{\theta}/\hat{\theta}$ implies the slope from origin on the phase plane, namely the level of danger. α_{max} is the maximum power-assistance-ratio.

B. Experiments

Fig.9 shows the construction of the experimental apparatus. Experiments with $\beta = [0.5,\ 3.0]$ are performed that the subject propels rims so that backward tumbling would occur intentionally. The results are shown in Fig.10. Fig.10(a), in case of $\beta = 0.5$, denotes a dangerous incident that the operator can no longer get up himself, however, Fig.10(b), in case of $\beta = 3.0$, denotes a safe front-wheel raising to get over the step on the uneven grand. β can adjust the extent of backward tumbling control and should be

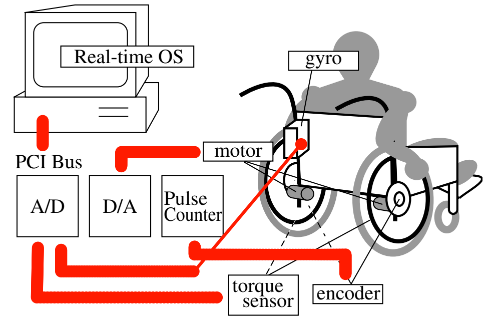
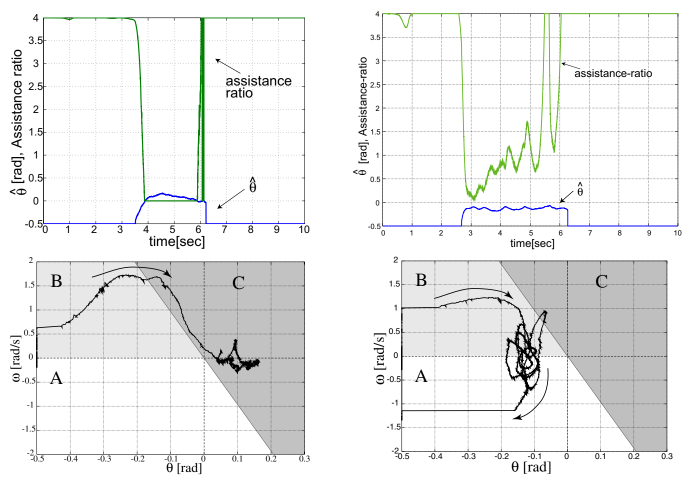


Fig. 9. Experimental set up.



(a) $\beta = 0.5$ (b) $\beta = 3$

Fig. 10. Experimental results.

chosen considering the operators' preferences and driving characteristics.

V. Conclusion

In this paper, as one of the studies aiming to realize a higher performance and safety driving for power-assisted wheelchair, novel backward tumbling control strategy with estimated COM has been proposed, furthermore, design method of COM observer gain k to suppress the influence from parameter errors has been presented. The effectiveness of the proposed strategy was verified by some practical experiments. This method can be applied to the operating on uphill.

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