離散型非整数次制御器による軸ねじれ系の振動抑制制御効果の実験的評価

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Experimental evaluation of torsional system's vibration suppression control performance by discrete fractional order controller

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Abstract : This article proposes a novel discrete fractional order PIDk controller for speed control of three-inertia torsional system. The experiments are also carried out to verify the theoretical robustness of the proposed PIDk control realized by the Short Memory Principle compared to the classical integer order PID control. The experimental results show the fractional order control system 's superior robustness performance against backlash nonlinearity and thus the backlash vibration can be suppressed. Applying fractional order control concept to motion control is still in a research stage, but it 's superior robustness against nonlinearities and other uncertainties highlights the promising aspects.

キーワード：非整数次制御、離散的実現法、 軸ねじれ系、振動制御
(Keywords: fractional order control, discrete realization method, torsional system, vibration suppression)

1. Introduction

The concept of expending derivatives and integrals to fractional (non-integer) order is by no means new. In fact,*Leibniz* mentioned this concept in a letter to *L'Hospital* over three hundred years ago (1695) and the earliest more or less systematic studies seem to have been made in the beginning and middle of the 19th century by *Liouville* (1832), *Holograms* (1864) and *Riemann* (1953)[(1)]. However, factional order control concept, when the controlled systems or controllers are described by fractional order differential equations, was not widely incorporated into control engineering mainly due to the conceptually difficult idea of taking fractional order and so few physical applications at that time[(2)].

In last few decades, researchers pointed out that fractional order differential equations could model various real materials more adequately than integer order ones and provide an excellent tool for the description of dynamical processes[(1)(3)(4)]. These fractional order models need the corresponding fractional order controllers be proposed and evoked the interest to various applications of fractional order control. The significance of fractional order control is that it is a generalization and " interpolation " of classical integral order control theory, which could lead to more adequate modeling and more robust control against various uncertainties.

However, most of these works were originated and concentrated on control of chemical processes while in motion control, the research is still in a primitive stage. In fractional order control, phase and gain could be adjusted continuously to give the systems more margin against uncertainties such as nonlinearities than their integer order counterpart. This theoretical superiority should highlight the promising aspects of fractional order control in motion control.

The paper is organized as follows: in section 2, basic mathematical aspects of fractional order control are mentioned to show the fractional order control is actually a natural generalization of classical integer control theory. The robustness of fractional $1/s^k$ system is also discussed; in section 3, fractional order PID^k controllers are proposed to the three-inertia torsional system 's speed control with backlash nonlinearity. The Short Memory Principle is introduced to realize designed fractional order D^k controller on digital computers; in section 4, the designed fractional order PID^k realized by the Short Memory Principle with different order k and memory length are evaluated by the experiment of torsion system 's speed control. The experimental results show that the discrete fractional order PID^k control systems display superior robustness against gear backlash nonlinearity and suppress the vibration caused by the backlash. Finally, in section 5, preliminary conclusions are drawn.

2. Theoretical Aspects

⟨2•1⟩ Mathematical definitions The mathematical definition of fractional calculus has been the subject of several different approaches. The most frequently encountered definition is called Riemann-Liouville definition (1) (2):

$$ {}_aD_t^k = \frac{1}{\Gamma(\gamma-k)} \frac{d^\gamma}{dt^\gamma} \int_a^t \frac{1}{(t-\xi)^{k-\gamma+1}} f(\xi) d\xi \qquad (1)$$

Where Gamma function

$$\Gamma(x) \equiv \int_0^\infty y^{x-1} e^{-y} dy \quad \cdots\cdots\cdots\cdots \quad (2)$$

a and t are limits and k, $(k \in R)$ the order of the operation. γ is an integer that satisfies $\gamma - 1 < k < \gamma$. The other approach for definition is $Grunwald - Letnikov$ definition (1) (3):

$$ {}_aD_t^k = \lim_{\substack{h\to 0 \\ nh=t-a}} h^{-k} \sum_{r=0}^{n} (-1)^k f(t-rh) \qquad (3)$$

Where the binomial coefficients

$$\begin{pmatrix} k \\ r \end{pmatrix} = \frac{k(k-1)\ldots(k-r+1)}{r!} \qquad (4)$$

⟨2•2⟩ Laplace and Fourier Transforms The Laplace transforms of the $Riemann - Liouville$ (1) (3) fractional derivative with order $k > 0$ is

$$L\left\{{}_0D_t^k\right\} = s^k F(s) - \sum_{j=0}^{n-1} s^j \left[{}_0D_t^{k-j-1} f(0)\right], (n-1 \le k < n) \qquad (5)$$

where $(n - 1 \le k < n)$. If

$$ {}_0D_t^{k-j-1} f(0) = 0 \quad j = 0, 1, 2, \ldots, n-1 \qquad (6)$$

then

$$L\left\{{}_0D_t^k f(0)\right\} = s^k F(s) \qquad (7)$$

Obviously, the Fourier transform of fractional order calculus could be obtained by setting $s = j\omega$ in its Laplace transform just like the classical integer order calculus'. Therefore, the frequency responses of fractional order $1/s^k$ systems can be plotted as Fig.**??**. Clearly, the fractional order systems' gain and phase change continuously between the classical integer order ones'.

Fractional order calculus is also a generalization of classical integer order calculus in Laplace and Fourier transforms, which would mean that extremely well developed classical integer order control techniques could still be fully referred in fractional order control.

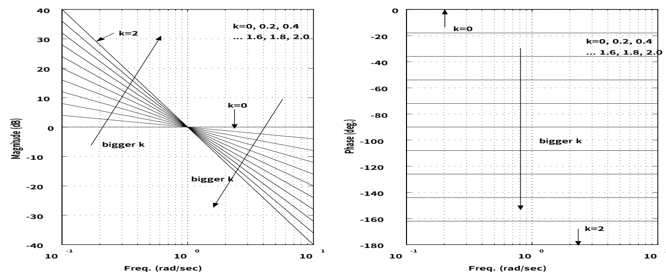

Fig. 0. Fractional order $\boldsymbol{1/s^k}$ system's Bode plots

⟨2•3⟩ Robustness to gain variation The close loop $1/s^k$ system 's characteristics equation with variable gain factor A is

$$1 + As^k = 0 \qquad (8)$$

The equation has two complex-conjugate dominate poles within $[-\pi, +\pi]$:

$$s_{1,2} = A^{-\frac{1}{k}} e^{\pm j\pi/k} \qquad (9)$$

The relative damping ratio ξ is

$$\xi = cos\left(\pi - \frac{\pi}{k}\right) = -cos\left(\frac{\pi}{k}\right) \qquad (10)$$

This result shows the relative damping ratio ξ is exclusively decided by order k and independent of the gain factor A. In frequency domain, the characteristic equation is

$$1 + AG(j\omega) = 0 \qquad (11)$$

Equ.(11) can be rewritten in the form:

$$G(j\omega) = -\frac{1}{A} \qquad (12)$$

The movement of $-1/A$ can be considered to be the locus of the critical point (Fig.**??**). when the gain variation occurs. For the integer order systems, this movement usually leads to less phase margin and low damping of overswings. But for fractional $1/s^k$ systems, phase margin and relative damping ratio can be kept constant in wide range of frequencies below and in the neighborhood of the critical point. This characteristic highlights the hopeful aspect of applying fraction order control to real engineering problems.

3. Fractional Order PID^k Speed Control

⟨3•1⟩ Modeling of the Torsional System The simplest model and block diagram of the torsional system with backlash nonlinearity between gears are the three-inertia model shown in Fig.3 and Fig.4, where J_m, J_g and J_l are driving motor, gear and load's inertias, K_s shaft elastic coefficient, ω_m and ω_l motor and load rotation speed, T_m the input torque and T_l the disturbance torque. In the modeling, the gear backlash nonlinearity is simplified as a deadzone factor with band [-δ,+δ] and elastic coefficient K_g.

Fig.3. Torsional system 's model

Fig. 4. Block diagram of the three-inertia system

The open loop transfer function between T_m to ω_m is

$$\begin{aligned} G(s) &= \{J_g J_l s^4 + [(K_s + K_g)J_l + K_s J_g]s^2 + K_g K_s\}/ \\ &\quad \{s\{J_m J_g J_l s^4 + [K_s(J_g + J_l)J_m + K_g(J_m J_l \\ &\quad + J_g + J_l)]s^2 + (J_m + J_g + J_l)K_s K_g\}\} \\ &= \frac{q_4 s_4 + q_2 s^2 + q_0}{s(p_4 s_4 + p_2 s^2 + p_0)} \\ &= \frac{(s^2 + \omega_{h1}^2)(s^2 + \omega_{h2}^2)}{J_m s(s^2 + \omega_{01}^2)(s^2 + \omega_{02}^2)} \quad \cdots\cdots\cdots\cdots\cdots\cdots \ (13) \end{aligned}$$

where ω_{01} and $\omega_{02}(\omega_{01} < \omega_{02})$ are the resonance frequencies while ω_{h1} and ω_{h2} $(\omega_{h1} < \omega_{h2})$ are the anti-resonance frequencies. As shown in the three-inertia system 's Bode plot (Fig.1), ω_{01} and ω_{h1} correspond to torsion vibration mode; ω_{02} and ω_{h2} correspond to gear backlash vibration mode. The frictions between the components are neglected due to their tiny values.

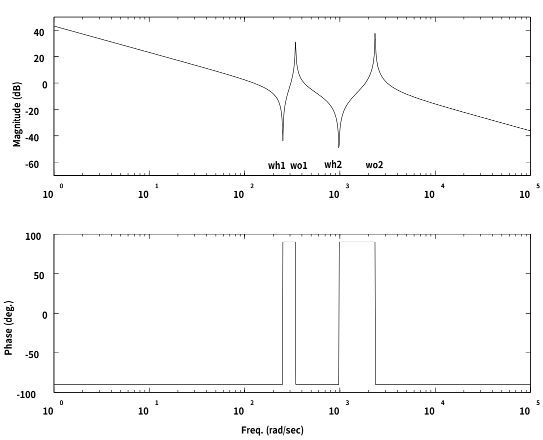

図 1 Bode plot of the three-inertia model

⟨3•2⟩ Fractional PID^k controller In order to smooth the discontinuity of speed command ω_r by the integral controller, a set-point-I PID k (Fig.6) controller is introduced to speed control of the torsional system.

Fig. 6. Set-point-I PID k controller Where,

$$K_p = \frac{10\sqrt{2}}{11}\sqrt{J_l k_s}, K_i = \frac{4}{11}K_s, K_d = \frac{5}{11}J_l - J_m \quad (14)$$

which is designed by Coefficient Diagram Method, a design method based on pole-placement of close loop characteristic equation [4]. The PID k controller's design is based on assuming D controller's fractional order k being integer order 1 firstly and simplifying the torsional system to two-inertia system where driving motor and gear are treated as unity inertia of $J_m + J_g$ and the backlash between gears is just neglected. Time responses by simulation with the simplified two-inertia model show the designed integer order PID control system has a satisfactory time-domain performance without backlash nonlinearity (Fig.2). However, when the designed inte-

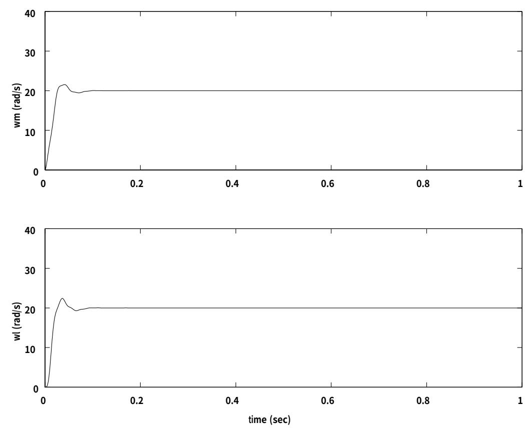

図 2 Time responses of the integer order PID two-inertia system by simulation

ger order PID controller is applied to the three-inertia system with gear backlash, the control system would be unstable and give rise to severe vibration due to the negative gain phase margin in PD and three-inertia plant's minor loop, as shown in Fig.3's k=1 case. The large phase delay of about -180 degree in high frequency range also indicates poor robustness again backlash nonlinearity and other uncertainties. In order to provide the minor loop enough stability margin, introducing a low pass D controller $K_d s/T_d s + 1$ to substitute pure D controller is the common method while the design process would become much more complex since the necessity of redesign the whole PID controller's parameter setting including the time constant T_d based on the sufficient condition of Multi-inertia system stability with PID control [6]. However, in this paper, a novel method of expanding D controller's order to fractional is proposed to adjust the minor loop's gain and phase margin

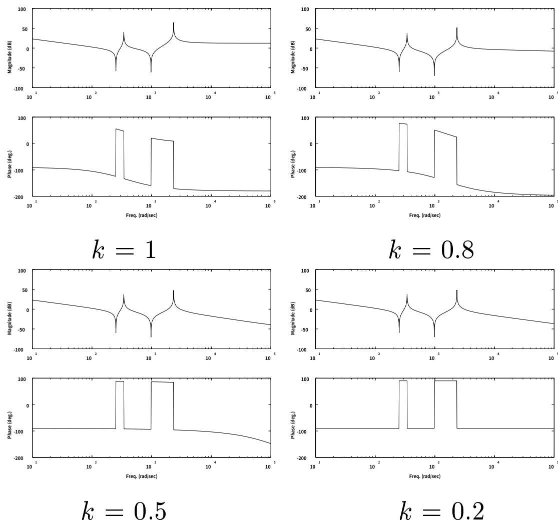

図 3　Bode plots of PD^k and plant's minor loop

directly and thus suppress the vibration caused by backlash nonlinearity, as shown in Fig.8.

〈3•3〉 Discrete Realization Method It is well known that the fractional order systems have an infinite dimension while the integer order systems finite dimensional. Proper approximation of designed fractional order controllers by finite difference equations is crucial for applying the fractional order control to real engineering problems. Especially, since most modern controllers are realized on digital computers, the discrete realization methods of fractional order controllers remain being more concerned.

Generally, there are currently three approaches to attain direct discretization of fractional order controllers: the definition approach Short Memory Principle [(3)], time-domain approach Lagrange interpolation method [(7)] and Tustin operator expansion approach [(8)]. Among them, the Short Memory Principle is being used most intensively in the simulation and realization of discrete fractional order systems by various literatures [(3)], (10)-(12) due to its easy programming and clear interpretation. The principle takes into account the behavior of $f(t)$ only in the "recent past", i.e. in the interval $[t-L, t]$, where L is the length of "memory":

$$ {}_aD_t^k f(t) \approx {}_{t-L}D_t^k f(t), (t > a + L) \tag{15}$$

The Short Memory Principle is based on the observation that the values of binomial coefficients near the "starting point" $t = a$ in the $Grunwald - Letnikov$ definition is small enough to be neglected or "forgotten" for large t. By using the principle, the discrete equivalent of the fractional order controller in discrete domain is given by

$$(\omega(z^{-1}))^{\pm k} = T^{\mp k} \sum_{j=0}^{[L/T]} c_j^{(\pm k)}, c_0^{(\pm k)} = 1 \tag{16}$$

where T is sampling time and the binomial coefficients are:

$$c_j^{(\pm k)} = (-1)^j \begin{pmatrix} \pm k \\ j \end{pmatrix} = \left(1 - \frac{1 + (\pm k)}{j}\right) c_{j-1}^{\pm k}, c_0^{\pm k} = 1 \tag{17}$$

Clearly, in order to have good approximation, small sampling time and long memory length are needed. In this paper, the Short Memory Principle is adopted to realize the discrete fractional k order D^k controller.

4. Experimental Results

In order to verify the backlash vibration suppression performance of fractional PID^k controllers and evaluate the Short Memory Principle discrete realization method in torsional system's speed control, experiments are carried out with sampling time T=0.001s, different D^k controller's order k and memory length L. Parameters of the experimental torsional three-inertia system are shown in Table.1. An encoder (8000pulse/rev) is used as the speed feedback sensor.

表 1　Parameters of the three-inertia syste

J_m (Kgm^2)	J_g (Kgm^2)	J_l (Kgm^2)	K_g (Nm/rad)	K_s (Nm/rad)	δ (deg.)
0.0007	0.0034	0.0029	3000	198.49	0.5

Equ.(14) give

$$K_p = 0.979, K_i = 72.178, K_d = -0.003 \tag{18}$$

Since the torque input to driving motor from the experimental equipment's motor diver has a limitation of maximum 3.84NM, K_p is retuned to 18.032 by trial-and-error to avoid severe overshoot and overswing caused by the torque input saturation. Firstly, the speed control experiment is carried out by integer order PID control. As have been analyzed in section 3.2, the severe backlash vibration occurs due to the minor loop's negative stability margin (Fig.4).

Fig.5 to Fig.10 show the experimental results of fractional order PID^k control with 0.2 and 0.5 order D^k controllers and memory length 0.005sec and 0.1sec. By introducing fractional order controller, the control system's stability and robustness against backlash robustness are improved and thus the vibration could also

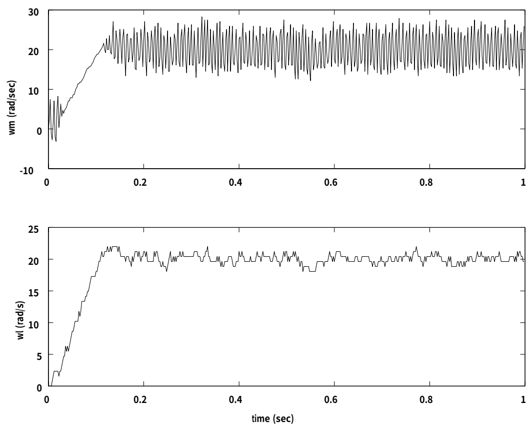

図 4　Time responses of the integer order PID control

be suppressed. The better approximation and performance could be achieved with longer memory length. Some transient tiny vibrations occur in the short memory length case $L/T = 5$, while for longer memory with $L/T = 100$, the tiny vibrations disappear and the time responses are more satisfactory. It 's interesting to find the time responses of the fractional order control systems also show the " interpolation " characteristic between their integer order counterparts that higher order 0.8 near integer order 1 leads to large overshoot and overswing, indicating a relative poor stability and robustness performance, while in lower order such as 0.2, there is nearly zero overshoot and the time responses indict the superior robustness against backlash nonlinearity. This "interpolation" characteristic is a key point to understand the superiority of fractional order control as providing much more flexibility in control design process.

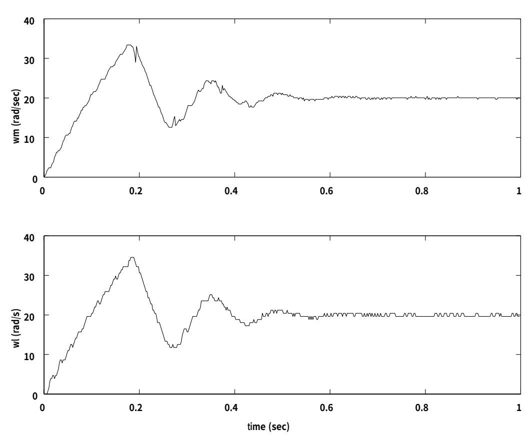

図 5　Time responses of fractional order $PID^{0.8}$ with $L/T = 5$

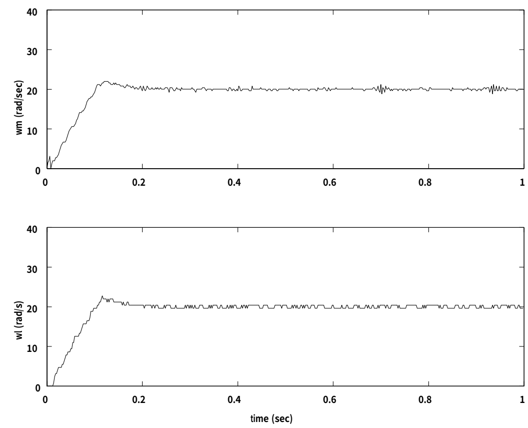

図 6　Time responses of fractional order $PID^{0.5}$ with $L/T = 5$

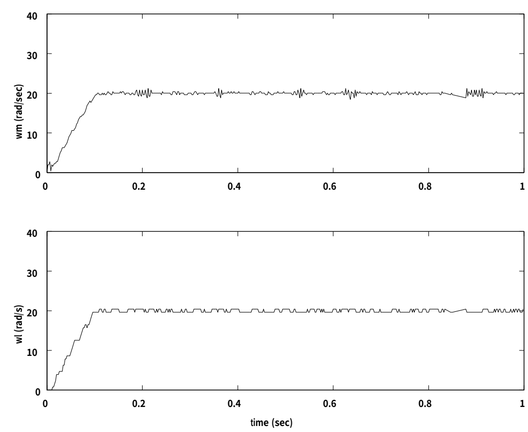

図 7　Time responses of fractional order $PID^{0.2}$ with $L/T = 5$

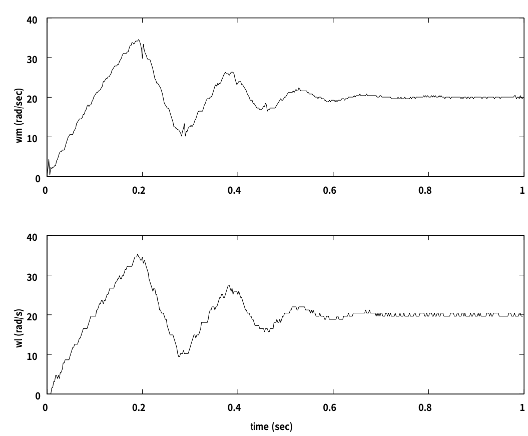

図 8　Time responses of fractional order $PID^{0.8}$ with $L/T = 100$

5. Preliminary Conclusions

In this paper, novel discrete fractional order PID^k controllers realized by the Short Memory Principle are proposed for speed control of three-inertia torsional sys-

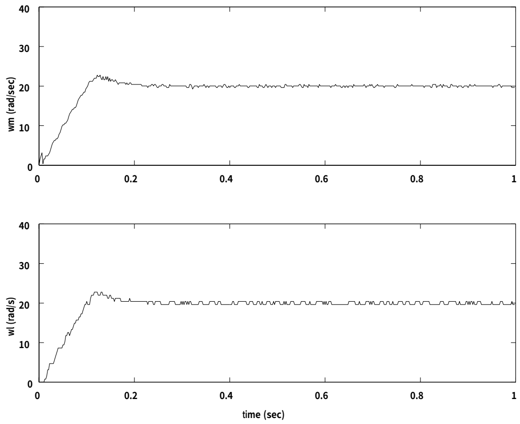

図 9　Time responses of fractional order $PID^{0.5}$ with $L/T = 100$

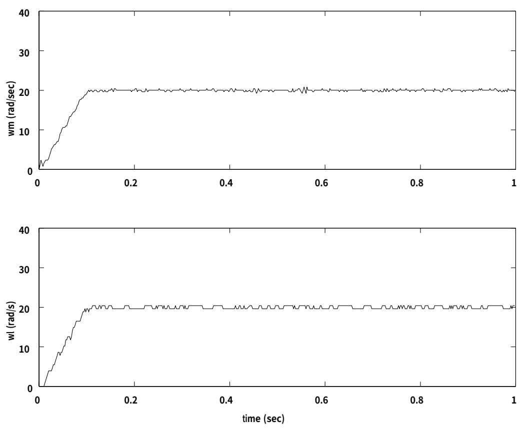

図 10　Time responses of fractional order $PID^{0.2}$ with $L/T = 100$

tem with gears backlash nonlinearity. The experimental results show the improved robustness and stability performances of proposed fractional PIDk order control system. By changing the fractional order, the system's robustness can be improved directly which means less complex design process and less tuning efforts in real industrial applications. Applying fractional order control concept to motion control is still in a research stage, but its superior robustness against nonlinearities and other uncertainties highlights the promising aspects while future exploration of more complex cases is needed.

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