Design of Fractional Order PI$^{\alpha}$D Controller for Robust Two-inertia Speed Control to Torque Saturation and Load Inertia Variation

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I. INTRODUCTION

The concept of expending calculus to non-integer order is by no means new. In fact, the concept has a firm and long standing theoretical foundation. *Leibniz* mentioned it in a letter to *L'Hospital* three hundreds years ago (1695) [1]. The earliest systematic studies seem to have been made in the beginning and middle of the 19th century by *Liouville*, *Riemann*, and *Holmgren* [1][2].

However, the concept was not widely incorporated into control engineering until in last few decades, researchers pointed out that fractional order models could model various real materials more adequately than integer order ones and provide an excellent tool for the description of dynamical processes [1][2]. These fractional order models need the corresponding fractional order control be proposed and evoked the interest to it's various applications. The significance of fractional order control is that it is a generalization and "interpolation" of classical integral order control theory, which could lead to more adequate modeling and more robust control. However, most of these works were originated and concentrated on control of chemical processes while in motion control, the research is still in a primitive stage [2][3].

The article is organized as follows: in section II, mathematical aspects of fractional order control are mentioned; in section III, a integer order PID controller is designed for the speed control; in section IV, a frequency band PI$^{\alpha}$D controller is proposed and its broken-line realization method is also introduced; in Section V, Experimental results are presented to show the robustness of proposed fractional order PI$^{\alpha}$D controller. Finally, in section V, preliminary conclusions are drawn.

II. MATHEMATIC ASPECTS

A. *Mathematical definitions*

The mathematical definition of fractional calculus has been the subject of several different approaches [1][2]. The most frequently encountered definition is called Riemann-Liouville definition:

$$ {}_aD_t^k = \frac{1}{\Gamma(\gamma-k)}\frac{d^\gamma}{dt^\gamma}\int_a^t \frac{1}{(t-\xi)^{k-\gamma+1}} f(\xi)d\xi \qquad (1a)$$

Where Gamma function

$$\Gamma(x) \equiv \int_0^\infty y^{x-1}e^{-y}dy \qquad (1b)$$

a and t are limits and k, $(k \in R)$ the order of the operation. γ is an integer that satisfies $\gamma-1<k<\gamma$.

The other approach for definition is the Grunwald-Letnikov definition:

$$ {}_aD_t^k = \lim_{\substack{h\to 0\\ nh=t-a}} h^{-k}\sum_{r=0}^{n}(-1)^r\binom{k}{r}f(t-rh) \qquad (1c)$$

Where

$$\binom{k}{r} = \frac{k(k+1)...(k+r-1)}{r!} \qquad (1d)$$

B. Laplace and Fourier transforms

The Laplace transforms of the Riemann-Liouville fractional derivative of order k>0 is [2]

$$L\{{}_0D_t^k f(t)\} = s^k F(s) - \sum_{j=0}^{n-1} s^j [{}_0D_t^{k-j-1} f(0)] \quad \text{(2a)}$$

$$(n-1 \le k < n)$$

If

$$_0D_t^{k-j-1} f(0) = 0 \qquad j = 0,1,2,\ldots,n-1 \quad \text{(2b)}$$

then

$$L\{{}_0D_t^k f(t)\} = s^k F(s) \quad \text{(2c)}$$

Obviously, the Fourier transform of fractional order calculus could be obtained by setting $s=j\omega$ in its Laplace transform just like the classical integer order calculus'.

Fractional order calculus is also a generalization of classical integer order calculus in Laplace and Fourier transforms, which would mean that extremely well developed classical integer order control theory could still be fully used in fractional order control.

III. INTEGER ORDER PID CONTROL

A. Modeling of two-inertia system

The most simple model and block diagram of 2-inertia system are shown in Fig.1 and Fig.2.

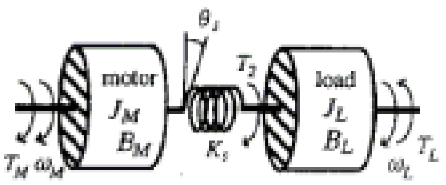

Fig. 1. Two-inertia system model

The open loop transfer function between T_M to ω_M is

$$G(s) = \frac{s^2 + \omega_h^2}{J_M s(s^2 + \omega_0^2)} \quad \text{(3a)}$$

where the resonance frequency ω_0 and the anti-resonance frequency ω_h are

$$\omega_0 = \sqrt{K_s\left(\frac{1}{J_M} + \frac{1}{J_L}\right)}, \quad \omega_h = \sqrt{\frac{K_s}{J_L}} \quad \text{(3b)}$$

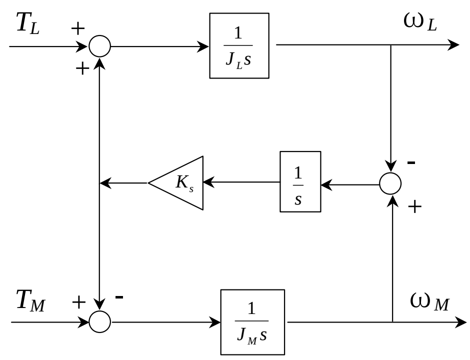

Fig. 2. Block diagram of 2-inertia system

Parameters of the experimental 2-inertia system are shown in Table.1.

Table 1. Parameters of the 2-inertia System

J_M (Kgm^2)	J_L (Kgm^2)	K_s (Nm/rad)
0.004	0.003	198.490

B.Design of PID controller

A set-point-I PID controller is introduced to control the 2-inertia system,

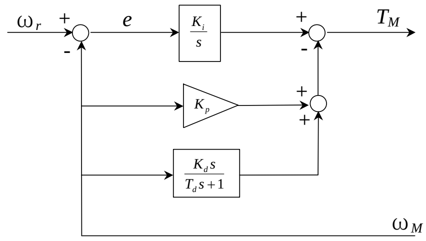

Fig. 3. Set-point-I PID controller

where

$$K_p = \frac{10\sqrt{2}}{11}\sqrt{J_L K_s}, \quad K_i = \frac{4}{11}K_s \quad \text{(4a)}$$

$$K_d = \frac{5}{11}J_L - J_M \quad \text{(4b)}$$

which is designed by Coefficient Diagram Method [4], a design method based on pole-placement. By equ(4a) and equ(4b),

K_p = 0.979, K_i = 72.178, K_d = -0.003, T_d = 0.02 (4c)

Time response by simulation shows the designed PID control system has a satisfactory performance (Fig.4). While in its frequency response, the enough phase margin is not kept in the neighborhood of the critical points, which could lower the robustness of the systems when nonlinearities such as saturation and parameter changing occur (Fig.5).

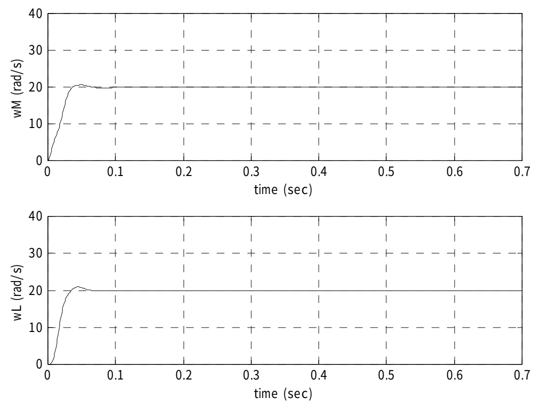

Fig. 4. Time response by simulation

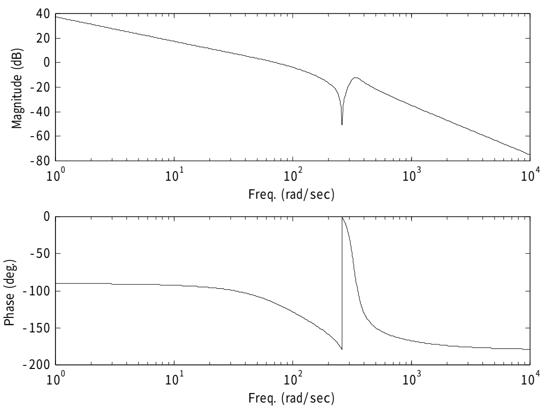

Fig. 5. Bode plot of designed integer order PID system

IV. FRACTIONAL ORDER PI$^{\alpha}$D CONTROL

A. Frequency band I$^{\alpha}$ controller

The most direct way to enhance the robustness of designed PID control system is to adjust I controller's order in order to giving more phase margin around the critical point. However, it is neither practicable nor desirable to try to make the order be fractional in all frequency range. Frequency-band fractional order controller is useful and required in real applications. Here, a frequency-band I$^{\alpha}$ controller is propose to substitute classical integer order I controller where the low band frequency ω_b = 10 *rad/sec* and high band frequency ω_h = 1000 *rad/sec* (equ.5).

$$\frac{1}{s}\left(\frac{\frac{s}{\omega_b}+1}{\frac{s}{\omega_h}+1}\right)^{1-\alpha} \qquad (5)$$

By changing the order of α, the phase margin of proposed fractional order PI$^{\alpha}$D control system can be adjusted directly and continuously (Fig.6).

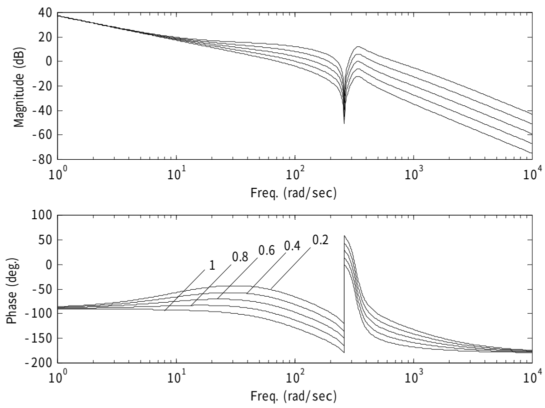

Fig. 6. Bode plot of fractional order PI$^{\alpha}$D system

B. Realization method

It is intuitive to approximate fractional order controllers by frequency-domain approach since the clear geometric interpretation in this domain. A broken-line approximation method is introduced to realize frequency-band fractional order controllers. Let

$$D(s)=\left(\frac{1+s/\omega_b}{1+s/\omega_h}\right)^k \approx D_N(s) \qquad (6a)$$

with

$$D_N(s)=\prod_{i=-N}^{N}\frac{1+s/\omega_i^{'}}{1+s/\omega_i} \qquad (6b)$$

From Fig.7, two recursive factors α and β are introduced where:

$$\alpha=\frac{\omega_i}{\omega_i^{'}}, \qquad \beta=\frac{\omega_{i+1}^{'}}{\omega_i} \qquad (7)$$

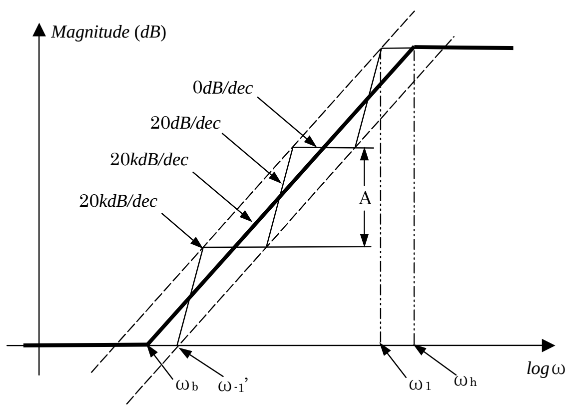

Fig 7. Broken-line approximation (N=1)

Since,

$$\omega'_{-N} = \beta^{1/2}\omega_b \text{ and } \omega_N = \beta^{-1/2}\omega_h \quad (8)$$

Therefore,

$$\alpha\beta = \left(\frac{\omega_h}{\omega_b}\right)^{1/(2N+1)} \quad (9)$$

with

$$\omega'_i = (\alpha\beta)^{i+N}\omega'_{-N}, \quad \omega_i = \alpha(\alpha\beta)^{i+N}\omega'_{-N} \quad (10)$$

Analysis of Fig. 7 gives

$$20k = \frac{A}{\log\alpha + \log\beta}, \quad and \quad 20 = \frac{A}{\log\alpha} \quad (11)$$

Therefore,

$$(\alpha\beta)^k = \alpha \quad (12)$$

thus, α and β can be expressed respectively by

$$\alpha = \left(\frac{\omega_h}{\omega_b}\right)^{k/(2N+1)} \text{ and } \beta = \left(\frac{\omega_h}{\omega_b}\right)^{(1-k)/(2N+1)} \quad (13)$$

Finally,

$$\omega'_i = \left(\frac{\omega_h}{\omega_b}\right)^{(i+N+1/2-k/2)/(2N+1)} \omega_b \quad (14)$$

and

$$\omega_i = \left(\frac{\omega_h}{\omega_b}\right)^{(i+N+1/2+k/2)/(2N+1)} \omega_b \quad (15)$$

Fig.8 shows the Bode plots of ideal frequency band $D(s)$ (k = 0.4, ω_b=10Hz and ω_h=1000Hz) and its 1st-order, 2nd-oder and 3rd-order approximations by above method. Even N = 2 could give satisfactory accuracy in frequency domain.

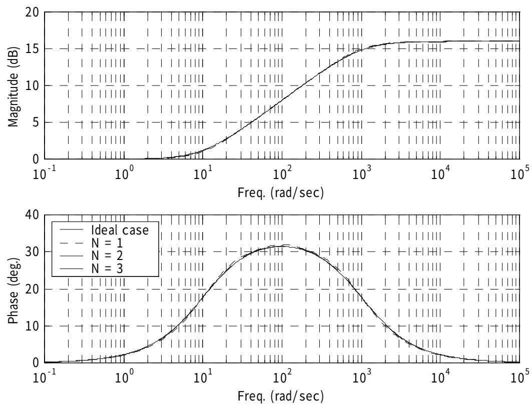

Fig. 8. Bode plots of ideal case and approximations

V. Experimental Results

Experiments of two-inertia speed control by integer order PID controller and frequency-band PI$^{\alpha}$D controller are carried out based on the parameters setting in Table.1 with maximum output torque limitation T_M = 3.84NM and an encoder (8000pulse/rev) as speed sensor. Here, α is taken as 0.6 to give proper phase margin around the critical point. The parameters K_i, K_p, K_d and T_d of PID and PI$^{0.6}$D controllers are kept as same as the settings in equ(4c).

For integral order PID controller, the step response is greatly changed when saturation occurs (Fig.9). While it can be seen the frequency-band PI$^{0.6}$D system showing a better robustness to saturation nonlinearity (Fig.10).

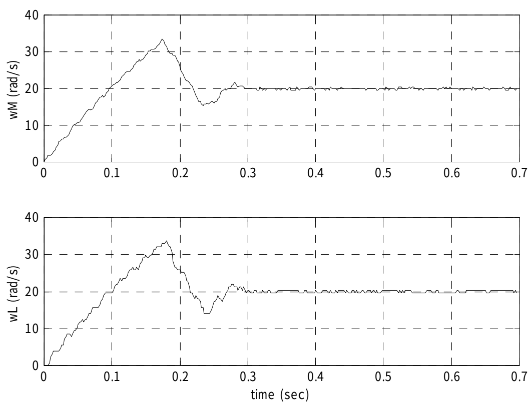

Fig. 9. Step response of PID system with saturation

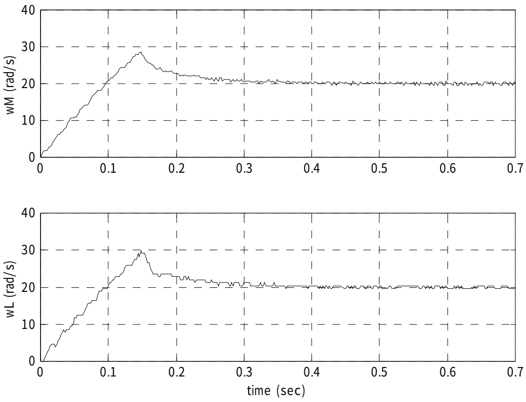

Fig. 10. Step response of $PI^{0.6}D$ system with saturation

Fig. 11 and Fig. 12 give the step responses of two control systems with different inertia on load side. Compared to the severe change of integer order PID control system's time responses with large overshoot and overswing, the frequency-band $PI^{0.6}D$ control system shows much better robustness to inertia variation with smaller and nearly constant overshoot.

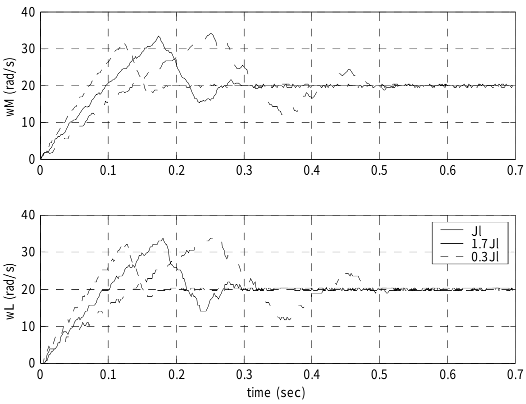

Fig. 11. Step responses of PID system with inertia variation

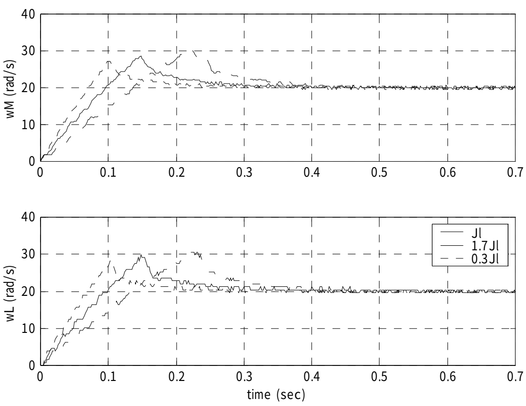

Fig. 12. Step responses of $PI^{0.6}D$ system with inertia variation

VI. PRELIMINARY CONCLUSIONS

In this paper, a frequency-band fractional order $PI^{\alpha}D$ controller is proposed for speed control of two-inertia system with torque saturation limitation and load side inertia variation. An intuitive broken-line approximate realization method of frequency-band controllers is also introduced which has a satisfactory accuracy in frequency domain. The experimental results show the robustness of proposed fractional control system. By changing fractional order, the robustness of the system can be enhanced directly which means the less complex design process and less tuning efforts in real industrial applications. Fractional order control to motion control is still in a research stage, but its superior robustness against parameter variation and nonlinearities highlight the promising aspects while future exploration of the applications to more complex cases is needed.

REFERENCES

1. Keith B. Oldham and Jerome Spanier, *The Fractional Calculus*, Academic Press, New York and London, 1974.
2. I. Podlubny, *Fractional Differential Equations*, Vol. 198, Mathematics in Science and Engineering, New York Tokyo, Academic Press, 1999
3. Ivo Petras and Blas M. Vinagre, *Practical application of digital fractional-order controller to temperature control*, Acta Montanistica Slovaca, Kosice , 2001
4. Yoichi Hori, *Control of 2-Inertia System only by a PID Controller*, T. IEE Japan, Vol. 115-D, No.1, 1995