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離散的非整数次制御器のサンプリング周期スケーリング特性及び非整数次振動抑制制御実現法への応用

Sampling Time Scaling Property of Discrete Fractional Order Controller and Application to Realization of Fractional Order Vibration Suppression Control

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Abstract

This paper proposes an interpretation of discrete fractional order controllers as sampling time scaled classical discrete integer order controllers. The discrete fractional order control systems are proved to remaining being Linear Time-Invariant systems based on the proposed sampling time scaling property. Essence of fractional order control as control with scaled memory is proposed to explain the fractional order control systems' robustness against uncertainties. Finally, the sampling time scaling property is used to realize fractional order PID^k controllers which are designed to suppress torsional system's backlash vibration. The experimental results show good approximation of the proposed novel realization method and the PID^k control systems' superior robustness against backlash non-linearity. Applying fractional order control concept to motion control is still in a research stage, but it's superior robustness against non-linearities and other uncertainties highlights the promising aspects.

1 Introduction

The concept of expanding derivatives and integrals to fractional (non-integer) orders is by no means new. In fact, the concept had a firm and long standing theoretical foundation. *Leibniz* mentioned this concept in a letter to *Hospital* over three hundred years ago (1695) and the earliest more or less systematic studies seem to have been made in the beginning and middle of the 19th century by *Liouville*(1832), *Holmgren*(1864) and *Riemann*(1953) [1]. However, the concept of Fractional Order Control (FOC), in which the controlled systems and/or controllers are described by fractional order differential equations, was not widely incorporated into control engineering mainly due to the conceptually difficult idea of taking fractional order and the existence of so few physical applications at that time [2].

In last few decades, researchers pointed out that fractional order differential equations could model various real materials more adequately than integer order ones and provide an excellent tool for the description of dynamical processes [1, 3, 4]. Those fractional order models need the corresponding fractional order controllers to be proposed and evoked the interest to various applications of FOC [5–8]. The significance of FOC is that it is a generalization and "interpolation" of classical integral order control theory, which could lead to more adequate modeling of dynamic processes and more direct design of robust control systems against uncertainties.

It is well known that integer order derivatives and integrals have clear physical and geometric interpretations, such as slope or velocity for derivatives and area or distance for integrals generally. These clear and easily understandable interpretations simplified their applications to various problems in different fields, including control theory that is extremely well developed based on integer order differential equations. On the contrary, for fractional order derivatives and integrals, it was not so. The notorious lack of clear geometric interpretations made fractional order derivatives and integrals conceptually difficult and greatly obstructed their real applications. *Podlubny* proposed a simple geometric interpretation of fractional integrals as "changing shadows on the wall" and some pictures describing this changing process were given [9]. However, since most modern controllers are realized by digital computers, clear interpretation of fractional order controllers' roles in discrete domain is much more concerned and with practical importance.

The paper is organized as follows: in section II, basic mathematical aspects are mentioned; in section III, an interpretation for discrete fractional order controllers based on sampling time scaling property is proposed and basic concepts of discrete FOC systems are investigated by using the proposed property; in section IV, robustness of FOC systems is analyzed through their absolute and relative stabilities; in section V, experiments of applying discrete PID^k controller to backlash vibration suppression control are introduced; in section VI, preliminary conclusions are drawn; finally, in section VII, future works are mentioned.

2 Mathematical Aspects

2.1 Mathematical Definitions

The mathematical definition of fractional derivatives and integrals has been the subject of several different approaches [1] [3]. The most frequently encountered definition is called Riemann-Liouville definition, in which the fractional order integrals are defined as

$$ {}_{t_0}D_t^{-\alpha} = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\xi)^{\alpha-1} f(\xi) d(\xi) \tag{1} $$

while the definition of fractional order derivatives is

$$ {}_{t_0}D_t^{\alpha} = \frac{d^\gamma}{dt^\gamma} \left[{}_{t_0}D_t^{-(\gamma-\alpha)} \right] \tag{2} $$

where

$$ \Gamma(x) \equiv \int_0^\infty y^{x-1} e^{-y} dy \tag{3} $$

is the Gamma function, a and t are limits and α ($\alpha > 0$ and $\alpha \in R$) is the order of the operation. γ is an integer that satisfies $\gamma - 1 < \alpha < \gamma$.

The other approach is Grünwald-Letnikov definition:

$$_{t_0}D_t^\alpha = \lim_{\substack{h\to 0\\ nh=t-t_0}} h^{-\alpha}\sum_{r=0}^{n}(-1)^\alpha f(t-rh) \tag{4}$$

Where the binomial coefficients ($r > 0$)

$$\left(\begin{array}{c}\alpha\\0\end{array}\right)=1,\ \left(\begin{array}{c}\alpha\\r\end{array}\right)=\frac{\alpha(\alpha-1)\dots(\alpha-r+1)}{r!} \tag{5}$$

2.2 Laplace and Fourier Transforms

The Laplace transform of the Riemann-Liouville fractional order derivative with order $\alpha > 0$ [1] [3] is

$$L\left\{_0D_t^\alpha\right\} = s^\alpha F(s) - \sum_{j=0}^{n-1} s^j\left[_0D_t^{\alpha-j-1}f(0)\right] \tag{6}$$

where $(n-1) \le \alpha < n$. If

$$_0D_t^{\alpha-j-1}f(0) = 0,\quad j = 0,1,2,\dots,n-1 \tag{7}$$

then

$$L\left\{_0D_t^\alpha f(0)\right\} = s^\alpha F(s) \tag{8}$$

Namely, the Laplace transform of fractional order derivative is fractional order Laplace calculator s. Obviously, the Fourier transform of fractional derivative can be obtained by substituting s with $j\omega$ in its Laplace transform just like the classical integer order derivative's.

3 Sampling Time Scaling Property

3.1 Geometric interpretation

By Riemann-Liouville definition, fractional order integral with order between 0 and 1 is

$$_0I_t^\alpha f(t) = \int_0^t f(\tau)dg_t(\tau),\ 0 < \alpha < 1 \tag{9}$$

where

$$g_t(\tau) = \frac{1}{\Gamma(1+\alpha)}[t^\alpha - (t-\tau)^\alpha] \tag{10}$$

Let $t \doteq nt_s$, where t_s is sampling time and n is the step currently under execution, then

$$g_{nt_s}(kt_s) = \frac{n^\alpha - (n-k)^\alpha}{\Gamma(1+\alpha)}t_s^\alpha,\ k = 1,...,n \tag{11}$$

Therefore, by sharing the same view of discrete integer order integration rules the "real" sampling time T of kth step is

$$\begin{aligned} T_n(k) &= \Delta g_{nt_s}(kt_s)\\ &= g_{nt_s}(kt_s) - g_{nt_s}[(k-1)t_s]\\ &= \frac{(n-k+1)^\alpha - (n-k)^\alpha}{\Gamma(1+\alpha)}{t_s}^\alpha \end{aligned} \tag{12}$$

Thus

$$\begin{aligned} T_n(n) &= \frac{1^\alpha - 0^\alpha}{\Gamma(1+\alpha)}{t_s}^\alpha\\ T_n(n-1) &= \frac{2^\alpha - 1^\alpha}{\Gamma(1+\alpha)}{t_s}^\alpha\\ &\dots\\ T_n(1) &= \frac{n^\alpha - (n-1)^\alpha}{\Gamma(1+\alpha)}{t_s}^\alpha \end{aligned} \tag{13}$$

Finally, based on the trapezoidal integration rule

$$_0I_{nt_s}^\alpha \approx \sum_{k=1}^{n}\frac{f(kt_s) + f[(k-1)t_s]}{2}T_n(k) \tag{14}$$

and

$$_0I_{nt_s}^\alpha = \lim_{\substack{t_s\to 0\\ t=nt_s}}\sum_{k=1}^{n}\frac{f(kt_s) + f[(k-1)t_s]}{2}T_n(k) \tag{15}$$

From Equ. (13), the interpretation of discrete fractional order integrals is the "deformation" of their integer order counterparts by the internal sampling time scaling (see Fig. 1). By using this interpretation, it is easily to understand that the past values are "forgotten" gradually in discrete fractional order integral due to their scaled tiny sampling time while in integer order ones all the values are "remembered" with the same weights.

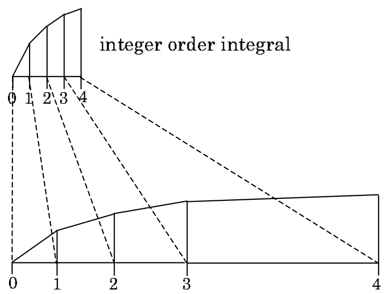

Fig.1: Fractional order integral's sampling time scaling

Similarly, discrete fractional order derivatives with order between 0 and 1 is

$$\begin{aligned} _0D_t^\alpha f(t) &= \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t\frac{f(\tau)}{(t-\tau)^\alpha}d\tau\\ &= \frac{d[\int_0^t f(\tau)dg_t(\tau)]}{dt},\quad 0 < \alpha < 1 \end{aligned} \tag{16}$$

where

$$g_t'(\tau) = \frac{1}{\Gamma(2-\alpha)}[t^{1-\alpha} - (t-\tau)^{1-\alpha}] \tag{17}$$

Thus

$$\begin{aligned} T_n'(n) &= \frac{1^{1-\alpha} - 0^{1-\alpha}}{\Gamma(2-\alpha)}{t_s}^{1-\alpha}\\ T_n'(n-1) &= \frac{2^{1-\alpha} - 1^{1-\alpha}}{\Gamma(2-\alpha)}{t_s}^{1-\alpha}\\ &\dots\\ T_n'(1) &= \frac{n^{1-\alpha} - (n-1)^{1-\alpha}}{\Gamma(2-\alpha)}{t_s}^{1-\alpha} \end{aligned} \tag{18}$$

Again based on the trapezoidal integration rule

$$\int_0^{nt_s} f(\tau)dg_t^{'}(\tau) \approx \sum_{k=1}^{n} \frac{f(kt_s)+f[(k-1)t_s]}{2} T_n^{'}(k) \qquad (19)$$

and

$$\int_0^{nt_s} f(\tau)dg_t^{'}(\tau) = \lim_{\substack{t_s\to 0\\ t=nt_s}} \sum_{k=1}^{n} \frac{f(kt_s)+f[(k-1)t_s]}{2} T_n^{'}(k) \qquad (20)$$

The interpretation of discrete fractional order derivatives is the derivatives of fractional $(1-\alpha)$ order integrals $\int_0^{nt_s} f(\tau)dg_t^{'}(\tau)$. Namely, it can be understood geometrically as the changing ratio of the "scaled integral area" (see Fig. 2) due to the sampling time scaling property.

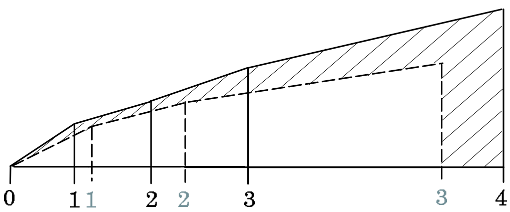

Fig.2: Changing of the "scaled integral area"

Clearly, when the orders are integers, the sampling time scaling effect disappears which means in discrete domain FOC is also a generalization and "interpolation" of the integer order control theory.

3.2 Causal LTI Systems

Some important concepts of discrete FOC systems become transparent when view in terms of the sampling time scaling property. Standard discrete control system is depicted in Fig. 3. For simplification the discrete controller F_d is fractional α order derivative or integral $(0 < \alpha < 1)$.

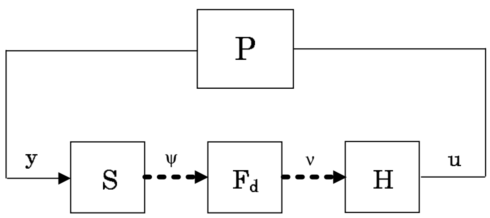

Fig.3: Block diagram of digital control system

Discrete α order derivative or integral's output at the kth step is a linear combination of all the past inputs $(\vec{v}(0) = 0)$:

$$\vec{\psi}(k) = [F]\vec{v}(l), \ (k, l = 1, 2, \ldots) \qquad (21)$$

The system matrix is

$$[F] = \begin{bmatrix} f(1,1) & 0 & 0 & \ldots \\ f(2,1) & f(2,2) & 0 & \ldots \\ f(3,1) & f(3,2) & f(3,3) & \ldots \\ \vdots & \vdots & \vdots & \end{bmatrix} \qquad (22)$$

where for α order integral

$$\begin{aligned} f(k,l) &= \frac{1}{2\Gamma(1+\alpha)} {t_s}^{\alpha}, \ l = k \\ f(k,l) &= \frac{(k-l+1)^{\alpha} - (k-l-1)^{\alpha}}{2\Gamma(1+\alpha)} {t_s}^{\alpha}, \ l < k \end{aligned} \qquad (23)$$

while for α order derivative the elements are a little complex:

$$\begin{aligned} f(k,l) &= \frac{1}{2\Gamma(2-\alpha)} {t_s}^{-\alpha}, \ l = k \\ f(k,l) &= \frac{2^{1-\alpha}-1}{2\Gamma(2-\alpha)} {t_s}^{-\alpha}, \ l = k-1 \\ f(k,l) &= \frac{(k-l+1)^{1-\alpha} - (k-l)^{1-\alpha}}{2\Gamma(2-\alpha)} {t_s}^{-\alpha} \\ &- \frac{(k-l-1)^{1-\alpha} - (k-l-2)^{1-\alpha}}{2\Gamma(2-\alpha)} {t_s}^{-\alpha}, \ l \leq k-2 \end{aligned} \qquad (24)$$

Based on Equ. (21) to Equ. (24), it is easy to prove that discrete FOC systems are causality that the outputs at step k depend only on the inputs up to step k ($[F]$ is lower triangular). Even with the sampling time scaling property, the FOC systems keep being Linear Time-Invariant since $[F]$ is constant along diagonals. Namely, if an input to FOC systems $\{\vec{v}(1), \vec{v}(2), \ldots\}$ produces the output $\{\vec{\psi}(1), \vec{\psi}(2), \ldots\}$, then the input $\{0, \vec{v}(1), \vec{v}(2), \ldots\}$ produces an output of the form $\{0, \vec{\psi}(1), \vec{\psi}(2), \ldots\}$ due to the causality and linear time-invariance of the discrete FOC systems. The Equ. (23) and Equ. (24) can also be used to realize fractional order controllers that will be discussed in detail in section. 5.4.

Although FOC is conceptually unfamiliar, it is in fact a natural generalization and expansion of integer order control theory. The FOC systems are also Causal LTI systems whose Laplace and Fourier transforms are similar to the integer order systems' but with fractional orders. These identities imply the future researches of FOC can still make good use of the extremely well-developed classical control theory for reference.

3.3 Control with Scaled Memory

The FOC systems have two time scales: feedback and sampling time scaler. As depicted in Fig. 4, a fractional order controller can be considered as the series of sampling time scaler and the classical integer order controller conceptually based on the proposed sampling time scaling property. Namely, the sampling time of input sequence $\{\vec{v}(1), \vec{v}(2), \ldots\}$ is pre-adjusted by the sampling time scaler before entering the integer order controller.

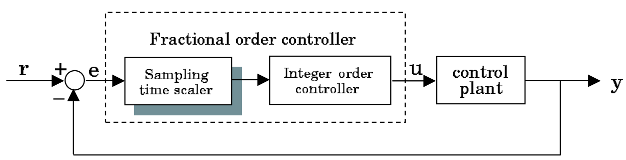

Fig.4: Sampling time scaler of FOC systems

Fractional order control can be regarded as some kind of control with "forgetting factors" $\lambda_n(k)$. For example, the "forgetting factors" of fractional order integral controllers equal $1/T_n(k)$ in Equ. (13), where large scaled sampling time of latest values means small "forgetting factor" and vice versa:

$$u(n) = \sum_{k=1}^{n} \frac{1}{\lambda_n(k)}[e(k) + e(k-1)] \tag{25}$$

It can be seen in Fig. 5 that in fractional order integral controllers the input values are memorized with scaled weights, while the integer order controllers give all the values with same weights. The rapid fading influences of the old values and dominance of the latest ones make fractional order controllers "adaptive"to present changes of the dynamic processes. This can also be a time domain explanation of FOC systems' robustness against uncertainties, while future researches are needed to make clear the theoretical importance of FOC as control with scaled memory.

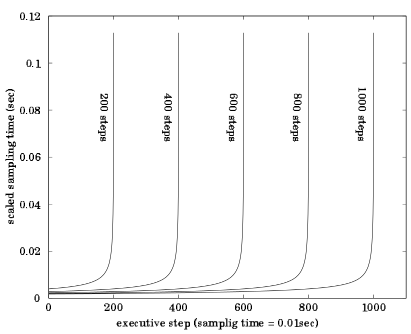

Fig.5: Discrete $I^{0.5}$ controller's scaled sampling time

4 Robust Control

4.1 Absolute Stability

The fractional α order control systems' characteristic equation is

$$(s^\alpha)^n + a_{n-1}(s^\alpha)^{n-1} + \ldots + a_1 s^\alpha + a_0 = 0 \tag{26}$$

where $0 < \alpha < 1$. Let $\sigma \doteq s^\alpha$, Equ. (26) can be rewritten as

$$\sigma^n + a_{n-1}\sigma^{n-1} + \ldots + a_1\sigma + a_0 = 0 \tag{27}$$

The requirement of the stability for Equ. (26) is the roots p_i of the characteristic equation in the principle sheet of the *Riemann* surface ($-\pi < arg(s) < \pi$) must be all located in the left-half s-plane, namely $-\frac{\pi}{2} < arg(p_i) < \frac{\pi}{2}$. Using the mapping $\sigma = s^\alpha$, the corresponding stability condition for Equ. (27) is that all its roots $p_i^{'}$ must be located in $-\frac{\pi}{2}\alpha < arg(p_i^{'}) < \frac{\pi}{2}\alpha$ of the σ-plane (see Fig. 6).

Compared with the characteristic equations of integer order control systems with same coefficients $\{a_{n-1}, \ldots, a_1, a_0\}$, the stability requirement of the FOC systems is looser. For uncertainties the coefficients of the characteristic equation change and consequently the root move about the complex plane. Looser stability requirement of FOC systems means better robustness performance against uncertainties.

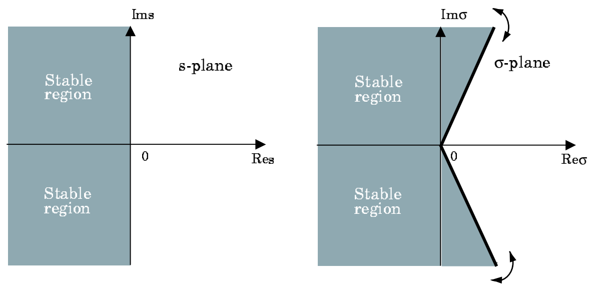

Fig.6: Larger stable roots' region for FOC systems

4.2 Relative stability

In frequency domain, the characteristic equation of unity-feedback system with open-loop transfer function $G(s) = 1/s^\alpha$ is

$$1 + AG(s) = 0 \tag{28}$$

where A is variable gain factor. Equ. (28) can be rewritten in the form:

$$G(j\omega) = -\frac{1}{A} \tag{29}$$

The movement of $-1/A$ can be considered to be the locus of the critical point as depicted in Fig. 7 when the gain variation occurs. For the integer order control systems, this movement usually leads to less phase margin since their open-loop frequency responses can only be adjusted between integer orders. But for fractional $1/s^\alpha$ systems, phase margin can be kept at any desired value and constant in wide range of frequencies below and in the neighborhood of the critical point. This characteristic highlights the hopeful aspect of applying fraction order controllers to real control problems.

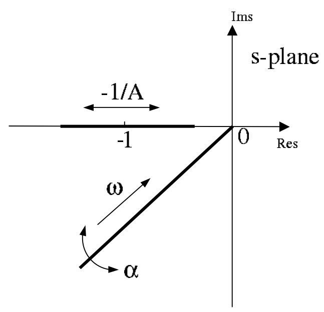

Fig.7: Any desired phase margin of $1/s^\alpha$ system

5 Application of Vibration Suppression

5.1 Experimental torsional system

The experimental setup of torsional system is depicted in Fig. 8. A torsional shaft connects two flywheels while driving force is transmitted from driving servomotor to the shaft by gears with gear ratio 1:2. Some system parameters are changeable, such as gear inertia, load inertia, shaft's elastic coefficient and gears' backlash angle. The encoders and tacho-generators are used as position and rotation speed sensors.

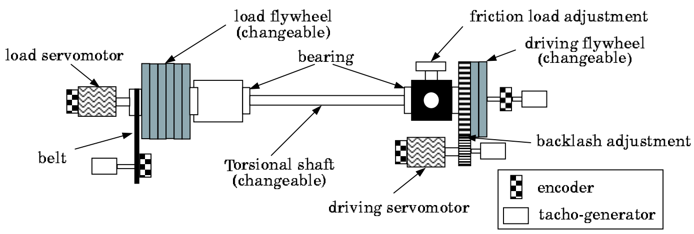

Fig.8: Experimental setup of torsional system

The experimental torsional system is controlled by a Pentium IV PC (see Fig. 9). Realtime operating system RTLinux^{TM} distributed by Finite State Machine Labs, Inc. is used to guarantee the timing correctness of all hard realtime tasks [10]. The control programs are written in RTLinux C threads which can be executed with strict timing requirement of control sampling time. The torque commands are calculated by the digital computer and sent to driving and disturbance servomotors through attached drivers. A 12-bit analogue input/output board with 4 output DA channels and 8 input AD channels is used to convert digital torque commands to analogue signals and analogue output voltage of tacho-generators to digital signals, while the pulse output signals of encoders are counted by a 4-channel 24-bit encoder pulse counter board.

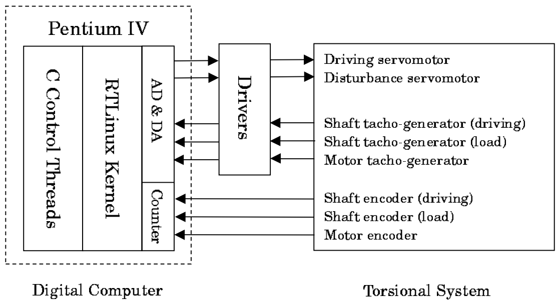

Fig.9: Digital control system of the experimental setup

5.2 Three-mass modeling

The simplest model and block diagram of the torsional system with backlash non-linearity between gears are the three-inertia model depicted in Fig. 10 and Fig. 11, where J_m, J_g and J_l are driving motor, gear and load's inertias, K_s shaft elastic coefficient, ω_m and ω_l motor and load rotation speed, T_m the input torque and T_l the disturbance torque. In the modeling, the gear backlash non-linearity is simplified as a deadzone factor with backlash angle band [-δ,+δ] and elastic coefficient K_g.

The open-loop transfer function between T_m to ω_m is

$$\begin{aligned} G(s) &= \{J_g J_l s^4 + [(K_s+K_g)J_l + K_s J_g]s^2 + K_g K_s\}/ \\ &\quad \{s\{J_m J_g J_l s^4 + [K_s(J_g+J_l)J_m + K_g(J_m J_l \\ &\quad +J_g+J_l)]s^2 + (J_m+J_g+J_l)K_s K_g\}\} \\ &= \frac{(s^2+\omega_{h1}^2)(s^2+\omega_{h2}^2)}{J_m s(s^2+\omega_{o1}^2)(s^2+\omega_{o2}^2)} \end{aligned} \quad (30)$$

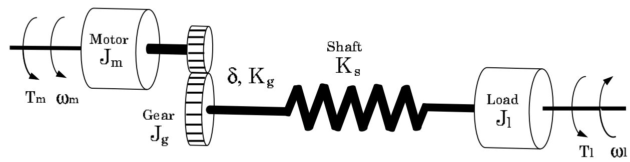

Fig.10: Torsional system's model

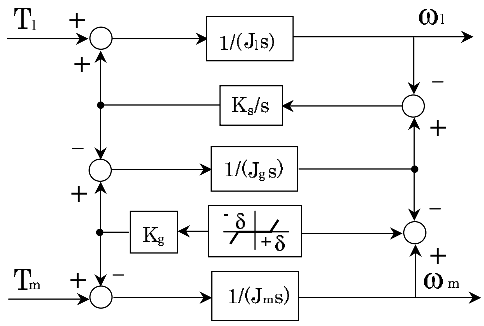

Fig.11: Block diagram of the three-mass model

where ω_{o1} and ω_{o2} are the resonance frequencies while ω_{h1} and ω_{h2} are the anti-resonance frequencies. ω_{o1} and ω_{h1} correspond to torsion vibration mode, while ω_{o2} and ω_{h2} correspond to gear backlash vibration mode (see Fig. 12). The frictions between the components are neglected due to their tiny values.

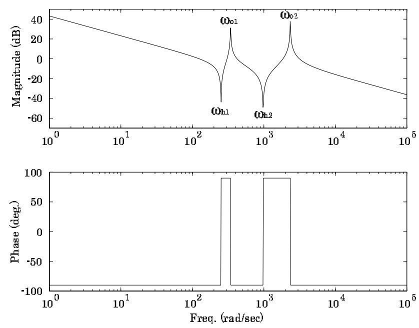

Fig.12: Bode plot of the three-inertia model

5.3 Fractional PID^k controller

In order to smooth the discontinuity of speed command ω_r by the integral controller, a set-point-I PID^k (see Fig. 13) controller is proposed for speed control of the torsional system where D controller's order can be any real number. Firstly, the integer order PID controller is designed by simplifying the torsional system to two-inertia system where driving servomotor and gears are treated as unity inertia of $J_m + J_g$ and the backlash non-linearity between gears is just neglected. The parameters of the PID controller are decided by Coefficient Diagram Method, a design method based on pole-placement of close-loop characteristic equa-

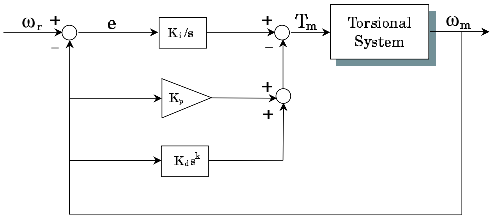

Fig.13: Set-point-I PID^k controller

tions [11] [12]:

$$K_p = \frac{10\sqrt{2}}{11}\sqrt{J_l k_s}, K_i = \frac{4}{11}K_s, K_d = \frac{5}{11}J_l - J_m \qquad (31)$$

Simulations with the simplified two-inertia model show the PID control system has a satisfactory performance (see Fig. 14).

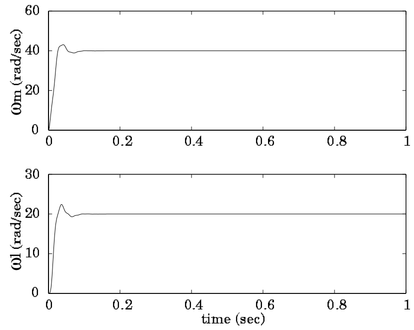

Fig.14: Time responses of the integer order PID two-inertia system by simulation

However, when the integer order PID controller is applied to the three-inertia system with gear backlash, the control system will be unstable and cause severe vibration due to the negative gain phase margin in PD and three-inertia plant's minor loop as depicted in Fig. 15's k=1 case.

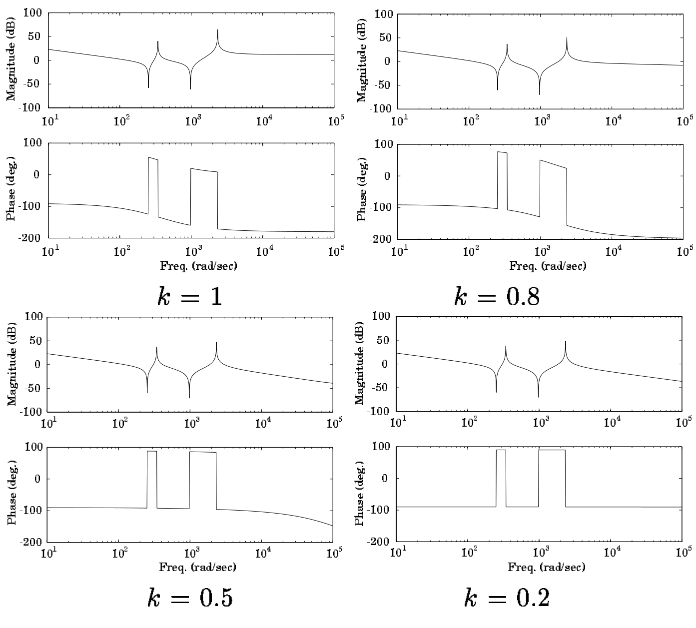

Fig.15: Bode plots of PD^k and plant's minor loop

In order to provide the minor loop with enough stability margin, introducing a low pass D controller $K_d s/(T_d s + 1)$ to substitute pure D controller is the common method while the design process will become much more complex since all the PID controller's parameters including the time constant T_d should be re-designed [13]. In this paper, a novel method of letting D controller's order to be fractional is proposed to adjust the minor loop's gain and phase margin directly and therefore suppress the vibration caused by backlash nonlinearity (see Fig. 15).

5.4 Realization method

It is well known that the fractional order systems have an infinite dimension while the integer order systems finite dimensional. Proper approximation by finite difference equation is needed to realize the PID^k controller. Generally, there are curretnly three approaches to attain direct discretization of fractional order controllers: the definition approach Short Memory Principle [3], time-domain approach Lagrange interpolation method [6] and Tustin operator expansion approach [14].

In this paper, the sampling time scaling property is used to realize the D^k controller due to its clear interpretation and easy programming. The method takes into account the behavior of $f(t)$ only in the "recent past", i.e. in the interval $[t - L, t]$, where L is the length of "memory":

$$_{t_0}D_t^k f(t) \approx_{t-L} D_t^k f(t), \ t > t_0 + L \qquad (32)$$

The realization method is based on the observation that the lengths of scaled sampling time near the "starting point" t_0 is small enough to be "forgotten" for large t (see Fig. 5). By using this observation and Equ. (24), the discrete equivalent of the D^k controller is given by

$$Z(s^k) \approx \frac{T^{-k}}{2\Gamma(2-\alpha)} \sum_{j=0}^{[L/T]} c_j z^{-j} \qquad (33)$$

where T is sampling time and the coefficients c_j are

$$\begin{aligned} c_0 &= 1 \\ c_1 &= 2^{1-\alpha} - 1 \\ c_j &= (i+1)^{1-\alpha} - i^{1-\alpha} \\ &- (i-1)^{1-\alpha} - (i-2)^{1-\alpha}, \ j \geq 2 \end{aligned} \qquad (34)$$

Clearly, in order to have better approximation, longer memory length are needed.

5.5 Experimental results

Experiments of torsional system's PID^k speed control are carried out with sampling time T=0.001sec, different D controller's order k and memory length L. Parameters of the experimental torsional three-inertia system are shown in Table. 1. Two encoders (8000$pulse/rev$) are used as the rotation speed sensors. Equation (31) gives

$$K_p = 0.979, \ K_i = 72.178, \ K_d = -0.003 \qquad (35)$$

where $K_d < 0$ means positive feedback of acceleration $\dot{\omega}_m$.

Since the driving servomotor's input torque command T_m has a limitation of maximum 3.84 NM, K_i is reduced to

Table 1: Parameters of the three-inertia system

J_m (Kgm^2)	J_g (Kgm^2)	J_l (Kgm^2)	K_g (Nm/rad)	K_s (Nm/rad)	δ $(deg.)$
0.0007	0.0034	0.0029	3000	198.49	0.5

18.032 by trial-and-error to avoid large over-shoot due to the saturation. Firstly, integer order PID speed control experiment is carried out. As depicted in Fig. 16 severe backlash vibration occurs due to the minor loop's negative gain margin, which is consistent with the analysis result in section 5.3.

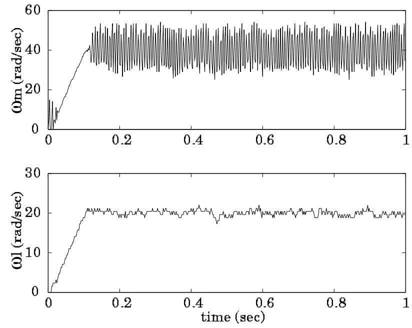

Fig.16: Time responses of the integer order PID control

Figures 17 and 18 depict the experimental results of fractional order PID^k control with 0.2, 0.4, 0.6, 0.8 order D^k controllers and memory length of $L/T = 5$ ($0.005sec$) and $L/T = 100$ ($0.1sec$). The control system's stability and robustness against backlash non-linearity are greatly improved and the severe backlash vibration in integer order PID control case is suppressed. It can be see in Fig. 17 and Fig. 18 better approximation and performances can be achieved with longer memory length, while even taking short memory length such as $L/T = 5$ can also give satisfactory performances. The intermittent tiny vibrations in lower order 0.6, 0.4 and 0.2 cases are due to their relative high gains near gear backlash vibration mode in open-loop frequency responses.

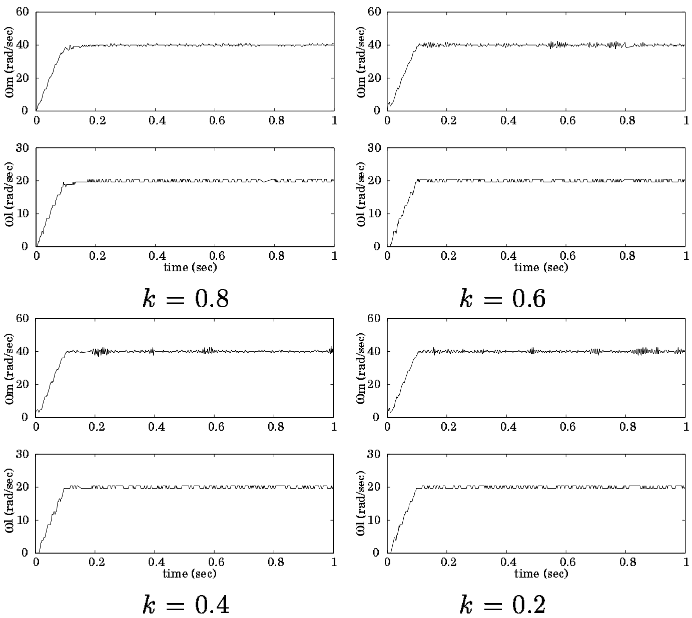

Fig.17: Time responses of PID^k control ($L/T = 5$)

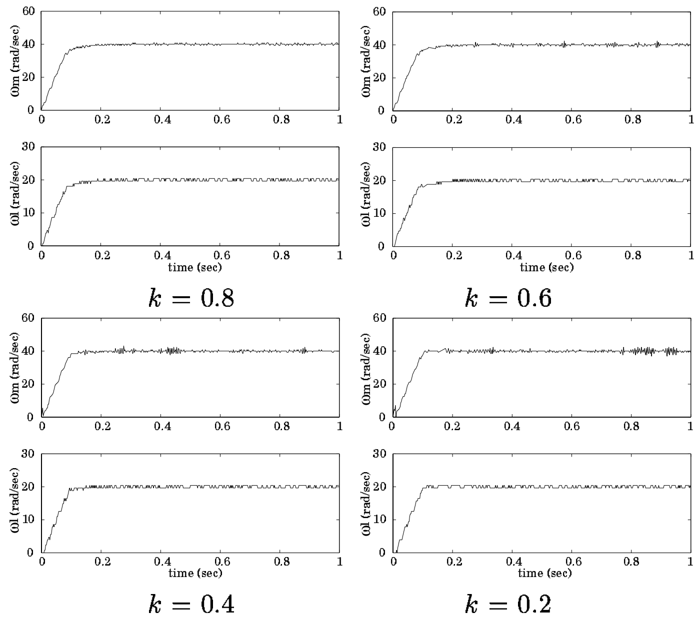

Fig.18: Time responses of PID^k control ($L/T = 100$)

It is interesting to find the time responses of the fractional order PID^k control systems also show the "interpolation" characteristic between their integer order counterparts. As depicted in Fig. 19, the time responses of $PID^{0.99}$ and $PID^{0.01}$ closely resemble PID^1 and PID^0's time responses, while this experimental reslut is natural since the orders are nearly same. The "interpolation" characteristic is one of main points to understand the superiority of FOC as providing more flexibility for designing robust control systems. At the same time, the experimental consistency with the logicality also verifies the good approximation of proposed novel realization method based on the sampling time scaling property.

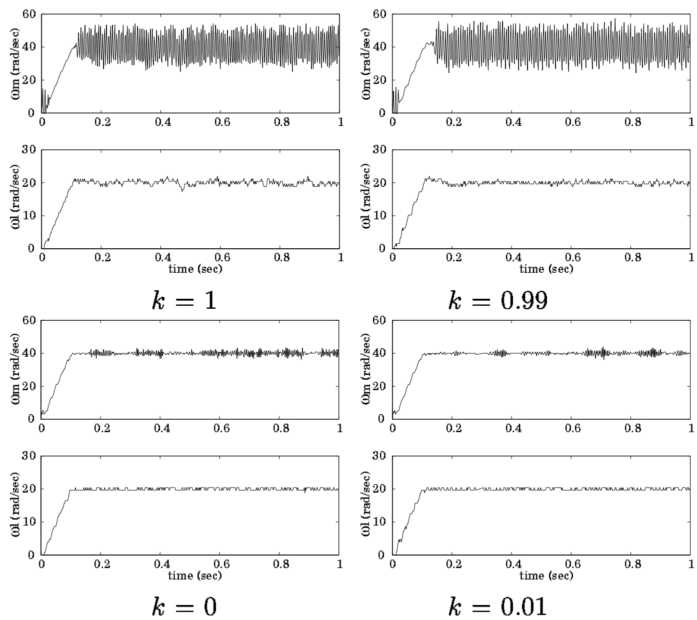

Fig.19: Continuity of PID^k control's time responses (the fractional order controllers are realized with $L/T = 100$)

6 Preliminary Conclusions

In this paper, the sampling time scaling property is proposed to give an interpretation of fractional order controllers in discrete domain. This interesting property might be one of main points to understand the essence of FOC systems in discrete domain as remaining being Causal Time-Invariant systems and control with scaled memory. The property is also used to realize fractional order PID^k controllers which are designed to suppress torsional system's backlash vibration. The experimental results show the improved robustness of PID^k control system and good approximation of the realization method. By changing D^k controller's order k, the control system's robustness against backlash non-linearity can be adjusted directly which means less design and tuning efforts in real industrial applications. Applying FOC concept to motion control is still in a research stage, but its superior robustness against nonlinearities and other uncertainties highlights the promising aspects.

7 Future Works

- *Theory*:
  1. Making clear the theoretical significance of interpreting FOC as control with scaled memory will be considered. Especially, the FOC systems have their own time scaler and infinite memeory. Theoretical explanation of these two important charaterstics should give insight to the FOC's essence.
  2. In adaptive control, there are also two time scales: a fast time scale for ordinary feedback and a slower one for updating the controller parameters actively to follow the process variations [15]. Some basic considerations and research methods of the adadaptive control may be referred in theoretical studies of FOC research.
  3. The current researches of FOC are mainly in continous domain. Discussing the FOC with latest digital control theory would be challenging and fruitful.
- *Modeling*: Fractional order model can model distributed-paramter systems more accurately [4]. Modeling aspects of the FOC theory are also important. The experimental torsional system should be a good object.
- *Application*: Applying FOC to other digital motion control such as robot actuators' vibration suppression and harddisk tracking control problems will be considered.
- *Realization*:
  1. The proposed novel realization method should be discussed in more detail and compared with the other realization methods to verify its superiority.
  2. FOC systems can only be realized by integer order approximations. Logically the realizable FOC systems are in fact integer order systems. The FOC can be looked as a "mine" of classical integer order control theory. Discussion of FOC approach directly from the viewpoint of integer order control theory would be interesting and profound.

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