# Robust Design of Gain Matrix of Body Slip Angle Observer for Electric Vehicles and its Experimental Demonstration


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***Abstract*— Electric Vehicles (EVs) are inherently suitable for 2-Dimension control. To utilize EV's advantages, body slip angle $\beta$ and yaw rate $\gamma$ play an important role. However as sensors to measure $\beta$ are very expensive, we need to estimate $\beta$ from only variables to be measurable.**

**In this paper, an improved estimation method for body slip angle $\beta$ for EVs is proposed. This method is based on a linear observer from side acceleration $a_y$ and $\gamma$ sensors. We especially considered the design of gain matrix and we achieved succeeded in exact and robust estimation.**

**We performed experiments by UOT March$II$ (Fig. 1). This experimental vehicle was made for study of advanced control of EV to be driven by four in-wheel motors. Some experimental results are shown to verify the effectiveness of the proposed method.**


## I. Introduction

Electric Vehicles (EVs) are environment-friendly and expected to be a promising solution for solving today's energy problems. Thanks to the dramatic improvement of motors' and batteries' performances, EVs will become more popular in the near future. It is predicted that before pure EVs, Hybrid EVs (HEVs) will be widely used in the next 10 years.

However, it is not well recognized that EVs have other advantages over Internal Combustion engine Vehicles (ICVs) [1]. Those advantages can be summarized in three aspects.

First, motor's torque generation is fast and accurate. Electric motor's torque response is only several milliseconds, which is 10-100 times as fast as combustion engine's. This advantage can enable us to realize high performance control of EVs.

Second, motor torque can be known precisely. Therefore we can easily estimate driving and braking forces between tire and road surface in real-time. This advantage can be used to realize novel control based on road condition.

Third, in-wheel motors can be installed in EVs' each rear and front tires. We can control each torques of the four motors so that it is easier to control EVs' slip angle $\beta$ and yaw rate $\gamma$ than ICVs'. In order to make full use of EVs' advantages, it is essentially important to research on $\beta$ and $\gamma$ control and $\beta$ observer.

In order to observe $\beta$, many methods have been proposed. All these methods have some disadvantages. Their models are too complex and the observers used in those methods are not robust enough against disturbance and model error, or cannot exactly estimate $\beta$. For example, most approaches to design $\beta$ based only on $\gamma$ cannot give a correct estimation. To improve these disadvantages, we propose a novel method based on $\gamma$ and side acceleration $a_y$ and design the observer's gain matrix .

Fig. 1. UOT March$II$

We did experiments by using UOT March$II$. Experiments show the proposed observer can estimate $\beta$ accurately and the observer is robust against parameter variation. In order to verify our method's superiority, we also carried out an experiment using different gain matrix. The experimental result proved that our design of gain matrix was better.

## II. Modeling of EVs

We use two-wheel model [2] for two-dimensional movement of EVs as shown in Fig. 2. Generally, in order to describe vehicle's two dimension movement exactly, four-wheel model (see Fig. 3) is needed. However because four wheeled model is non-linear model, it cannot be used for linear observer design. Where P is the center of gravity, $l_f$ is the distance from P to the front wheel, $l_r$ is the distance from P to the rear wheel, $\alpha_f$ is the front wheel slip angle, $\alpha_r$ is the rear wheel slip angle and $\delta_f$ is actual steering angle at tire.

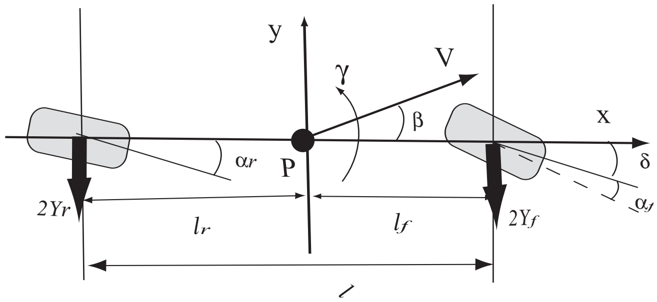



Fig. 2. Two-wheel model of vehicle motion

Usually, we express state equations with $\beta$, $\gamma$, and vehicle speed $v$. Motion equations are expressed in Equs. (1) $\sim$ (3).

$$\begin{aligned} ma_x &= F_{x\_fl} + F_{x\_fr} + F_{x\_rl} + F_{x\_rr} && (1)\\ ma_y &= F_{y\_fl} + F_{y\_fr} + F_{y\_rl} + F_{y\_rr} && (2)\\ I\dot{\gamma} &= l_f(F_{y\_fl} + F_{y\_fr}) - l_r(F_{x\_rl} + F_{x\_rr}) + N && (3)\\ N &= \frac{d}{2}(-F_{x\_fl} + F_{x\_fr} - F_{x\_rl} + F_{x\_rr}) \end{aligned}$$

where
$F_{x\_fr}$: front right wheel's force in x direction.
$F_{x\_fl}$: front left wheel's force in x direction.
$F_{x\_rr}$: rear right wheel's force in x direction.
$F_{x\_rl}$: rear left wheel's force in x direction.
$F_{y\_fr}$: front right wheel's force in y direction.
$F_{y\_fl}$: front left wheel's force in y direction.
$F_{y\_rr}$: rear right wheel's force in y direction.
$F_{y\_rl}$: rear left wheel's force in y direction.

$F_{y\_fl}$ and $F_{y\_fr}$ are given by Equ. (4) in linear area.

$$F_{y\_fl} = F_{y\_fr} = \alpha_f C_f, \quad F_{y\_rl} = F_{y\_rr} = \alpha_r C_r \qquad (4)$$

$C_f$ and $C_r$ are cornering power CP,

$$CP = \frac{\partial F_y}{\partial \alpha} \mid_{\alpha=0} \qquad (5)$$

Relationships between longitudinal acceleration $a_x$, side acceleration $a_y$, $\gamma$ and $\beta$ take the form of Equs. (6) and (7) with vehicle speed $v$.

$$\begin{aligned} a_x &= -v(\dot{\beta} + \gamma)\sin\beta + \dot{v}\cos\beta && (6)\\ a_y &= v(\dot{\beta} + \gamma)\cos\beta + \dot{v}\sin\beta && (7) \end{aligned}$$

Because cornering power is much larger than $\dot{v}$, $\dot{v}$ can be put to 0 in Equ. (7).

$$a_y = v(\dot{\beta} + \gamma)\cos\beta \qquad (8)$$

From Equs. (1) $\sim$ (4) we can get the state equation:

$$\dot{x} = Ax + B\delta_f \qquad (9)$$

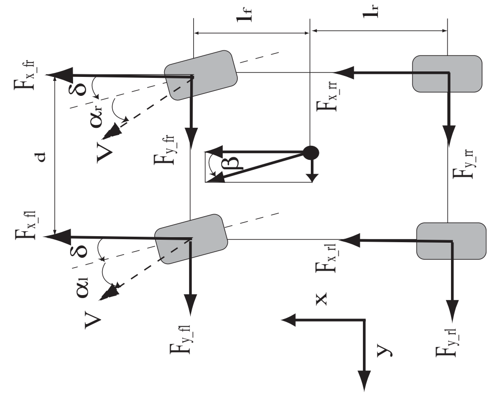



Fig. 3. Four-wheel model of vehicle motion

$$A = \begin{bmatrix} \frac{-2(C_f+C_r)}{mv} & \frac{-2(l_fC_f-l_rC_r)}{mv^2} - 1 \\ \frac{-2(l_fC_f-l_rC_r)}{I} & \frac{-2(l_f^2C_f+l_r^2C_r)}{Iv} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2C_f}{mv} \\ \frac{2l_fC_f}{I} \end{bmatrix}, \quad x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$$

## III. Design of proposed linear observer

Various methods for estimating $\beta$ were proposed previously. For example, direct integral method [3] estimates $\beta$ based on Equ. (10). In this method, estimated $\beta$ contains steady state error, therefore it can't estimate $\beta$ exactly. Nonlinear observers [4] [5] [6] [7] aim to design an accurate model based on actual vehicle's dynamics and to estimate $\beta$. These methods are suitable for simulation. But due to the complexity of the models, these methods are difficult for $\beta$ estimation in real-time.

$$v(\dot{\beta} + \gamma) = a_y \qquad (10)$$

The advantage of conventional linear observers is it's simple structure. However they are not robust enough against model error. Moreover it cannot estimate $\beta$ exactly in non-linear region.

In order to overcome these disadvantages, we propose a novel linear observer in this paper. Unlike conventional observers using only $\gamma$ as measurable signal, we utilize $a_y$ together with $\gamma$ to construct the linear observer [8], which can estimate $\beta$ in non-linear region.

To design the observer, it is necessary to restructure output equation by measurable parameters.

Parameters we can measure are

- longitudinal acceleration ($a_x$)
- side acceleration ($a_y$)
- yaw rate ($\gamma$)

Because $a_x$ cannot be expressed by linear equation, we use $\gamma$ and $a_y$ to restructure output equations. Using Equs. (8) and (9), $a_y$ can be restructured as:

$$a_y = v(a_{11}\beta + a_{12}\gamma + b_1\delta + \gamma) \qquad (11)$$

The output equation is:

$$y = Cx + D\delta_f \qquad (12)$$

$$C = \begin{bmatrix} 0 & 1 \\ va_{11} & v(a_{12}+1) \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ vb_1 \end{bmatrix}, \quad y = \begin{bmatrix} a_y \\ \gamma \end{bmatrix}$$

**Full order observer**

We use full order observer, which is defined by the following equations.

$$\dot{\hat{x}} = A\hat{x} + Bu - K(\hat{y} - y) \tag{13}$$

$$\hat{y} = C\hat{x} + Du \tag{14}$$

where the $K$ is observer matrix gain.

The estimation error $e = \hat{\beta} - \beta$ should satisfy the following error equation:

$$\dot{e} = (A - KC)e \tag{15}$$

By designing matrix gain $K$, we can change observer's characteristics. Proper selection of matrix gain $K$ is important for robust performance.

### A. Design of gain matrix for robustness

If the selected matrix gain is inadequate, the linear observer will have a poor robust performance against model error and sometimes cannot estimate $\beta$ exactly. To decide matrix gain, we must consider two important factors.

First, we must design the observer robust against model error. Since two-wheel model is used in our design, some model error exists more or less. Especially, cornering power $C_f$ and $C_r$ depend on road condition and loads on each tires. Therefore, their values are changing and cannot be measured.

Second, all eigenvalues of $A - KC$ must be located in stable region. $A - KC$ is the state transition matrix of Equ. (15). The positions of $A - KC$ eigenvalues will affect control system's time response performances, such as overshoot, rising time and settling time.

To make the observer robust, we referred [9]. By calculating Equs. (11) and (14), we can get $\dot{\hat{\beta}}$:

$$\dot{\hat{\beta}} = a_{11}\hat{\beta} + a_{12}\hat{\gamma} + b_{11}\delta_f - k_{11}(\hat{\gamma} - \gamma) - k_{12}(\hat{a_y} - a_y) \tag{16}$$

State equation of $\beta$ is given by Equ. (17), where any model error is not contained in this equation.

$$\dot{\beta} = a_{11}'\beta + a_{12}'\gamma + b_{11}'\delta_f \tag{17}$$

$a_{11}'$, $a_{12}'$ and $b_{11}'$ are the real values.

By Equs. (16) and (17), the state equation for $\dot{\hat{\beta}} - \dot{\beta}$ is expressed in following equation.

$$\begin{aligned} \dot{\hat{\beta}} - \dot{\beta} = {} & (\hat{\beta} - \beta)(a_{11} - k_{12}v) - (1 - k_{12}v)(a_{11}' - a_{11})\beta \\ & + (\hat{\gamma} - \gamma)[a_{12} - k_{12}v(a_{12} + 1) - k_{11}] \\ & - (1 - k_{12}v)(a_{12}' - a_{12})\gamma \\ & + (1 - k_{12}v)(b_1 - b_1')\delta \end{aligned} \tag{18}$$

Because we can measure $\gamma$ by a gyro sensor, $\hat{\gamma}$ can be assumed to be equal to $\gamma$. The best condition for robustness in Equ. (18) is:

$$1 - k_{12}v = 0 \tag{19}$$

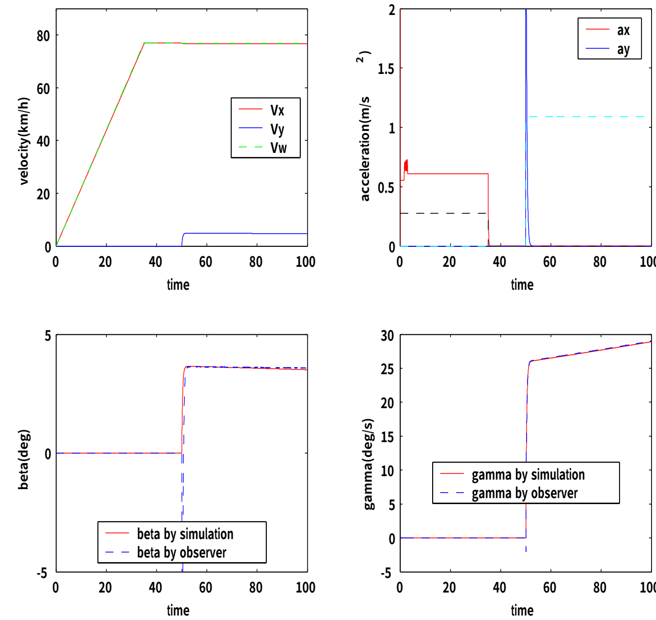



Fig. 4. Simulation result

By Equ. (19), we can get $k_{12}$:

$$k_{12} = \frac{1}{v} \tag{20}$$

Based on consideration of pole assignment and robustness against cornering power, $K$ is decided as:

$$K = \begin{bmatrix} \frac{\lambda_1\lambda_2}{C_f}\frac{(lf-lr)I}{2(lf^2+lr^2)+4lflr} - 1 & \frac{1}{v} \\ -\lambda_1 - \lambda_2 & \frac{m(lf^2+lr^2)}{(lf-lr)I} \end{bmatrix} \tag{21}$$

$\lambda_1$ and $\lambda_2$ are the assigned poles of the observer.

## IV. Simulation Results

### A. Simulation setup

Before experiments, we simulated the behavior of EV by using the four-wheel model. $\beta$ will be estimated by using the proposed observer. In simulation, all conditions can be set freely. Road type is supposed to be dry asphalt. Simulation time length is 100 [s]. In 0 $\sim$ 35 [s], front and rear tires torques are 556 [N] at each. Over 35 [s], torques become 0 [N]. Over 50 [s], $\delta_f$ is changed to be 5 [deg]. EV's parameters are supposed to be UOT March$II$'s parameters.

### B. Simulation result

Fig. 4 is the simulation results. It shows that the proposed observer can estimate $\beta$ exactly. Rising time for estimation is short enough and overshoot doesn't occur. When $\delta_f$ is changed, inverse response occurred. It happened because $\frac{1}{v}$ is included in the matrix gain $K$ and near-zero-divide problem appeared. This problem can be solved by limiting the value of $\beta$.

| PC to control | Pentium MMX 223[MHz] |
|---|---|
| | AMD K6-233[MHz] |
| OS | Slackware Linux 3.5 |
| | RTLinux rel. 9K |
| encoder pulse number | 3600[ppr] |
| acceleration sensor | ANALOG DEVICES ADXL202 |
| Yaw rate sensor | HITACHI OPTICAL FIBER GYROSCOPE HOFG-CLI(A) |
| Noncontact Optical sensor | CORREVIT S-400 |

TABLE I

SENSORS OF UOT MARCH$II$

| Experimental No. | $v$[km/h] | $\delta$[deg] | road type |
|---|---|---|---|
| 1 | 40 | 90 | dry |
| 2 | 40 | 180 | dry |
| 3 | EV was accelerated, and draws a circle with a radius of 26.5 meters in wet road | | |
| 4 | In Experiment No.1, another observer gain with model error was used | | |

TABLE II

EXPERIMENTAL CONDITIONS

## V. EXPERIMENTAL DEMONSTRATION BY UOT MARCH$II$

### A. Experiment setup

UOT March$II$ is our experimental EV built to prove EVs' advantages. We made this EV by ourselves, which is remodeling of Nissan March. The EV equips acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure $\beta$. Table. I explains specification.

Four patterns of experimental conditions were shown in Table. II. We changed vehicle velocity, driver's input steering wheel angle $\delta$ and road type to test robustness. When $\delta$ is large, $\beta$ is in non-linear region. In this region, we try to estimate $\beta$ by the proposed observer. Road types can affect cornering powers greatly. In dry road, J-turn is carried out (dry road is asphalt road). In wet road, EV was rotated in circle (wet road is rainy road).

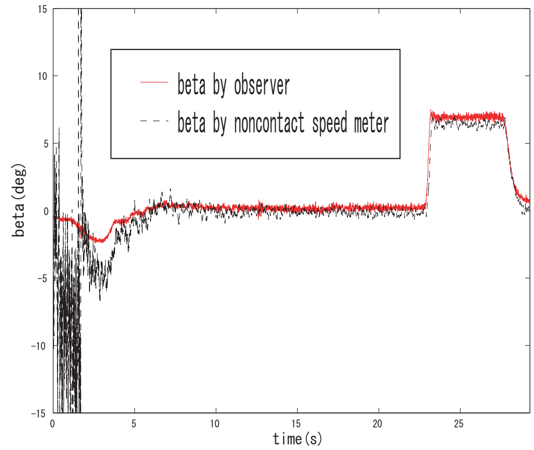
Experiment No.4's condition is equal to No.1's, but we add 30 percent error in $a_{11}$ intentionally. In experiment No.5, observer gain matrix $K$ is changed and compare it our to another linear observer with different matrix gain. The result can prove the proposed observer's robustness.

We recorded $\beta$, $\gamma$, $\delta$, $v$ and $a_y$ in hard disk drive by the sampling time of 1 [ms] and calculated by Matlab.

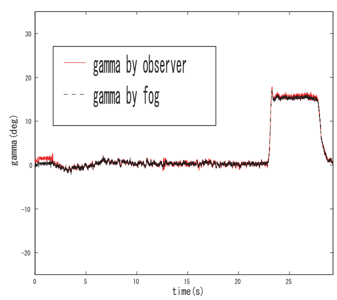
### B. Experimental results

Fig. 5 $\sim$ 8 show the results corresponding to No.1 $\sim$ 4 experiments under the difference conditions.

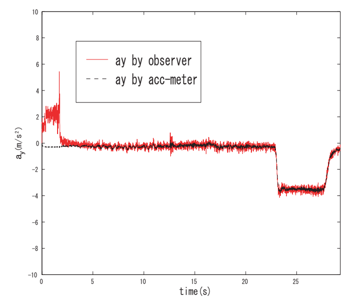
Fig. 5 shows No.1 experimental results. Fig. 5 (a) is measured and estimated $\beta$. Fig. 5 (b) and (c) are measured and output $a_y$ and $\gamma$. In Fig. 5 (a), EV is in linear region because steering angle $\delta$ is small. Fig. 5 shows us that the novel proposed observer can estimate $\beta$ well in linear region.




(a)Measured value and estimation of $\beta$




(b)Measured value and observer's output of $\gamma$




(c)Measured value and observer's output of $a_y$

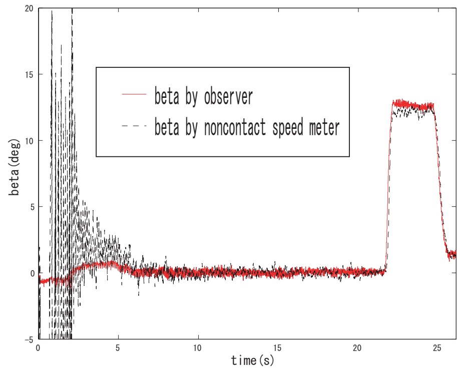
Fig. 5. No.1; Speed is 40km/h and steering angle $\delta$ is 90[deg]

Fig. 6 is No.2 experimental results. Because in experiment No.2, $\delta$ is lager than $\delta$ in No.1, $\beta$ becomes larger and EV enters non-linear region. Fig. 6 demonstrates that the novel observer can estimate $\beta$ even in non-linear region.

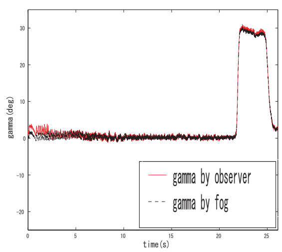
Fig. 7 shows No.3 experimental results. In No.3 the road type is different from the road type in No.1. Because cornering power is changed, while the novel observer's parameters are kept unchanged, No.3 experiment proves that the novel observer is robust against cornering power variation.

Fig. 8 is No.4 experimental result. No.4 experiment has the same experiment condition as No.1. The two linear observers were used in this experiment. One observer is the proposed observer, the other observer has different matrix gain (see Equ. (22) ). The novel observer can estimate $\beta$ exactly, but the other observer cannot. No.4 experiment shows us that our design of gain matrix can make the linear observer more robust.

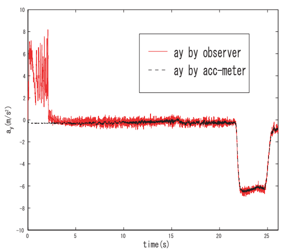
$$K = \begin{bmatrix} \frac{\lambda_1(a_{12}+1)}{a_{11}} - 1 & \frac{a_{11}-\lambda_1}{va_{11}} \\ a_{22} - \frac{a_{21}(a_{12}+1)}{a_{11}} - \lambda_2 & \frac{a_{21}}{va_{11}} \end{bmatrix} \quad (22)$$




(a)Measured value and estimation of $\beta$

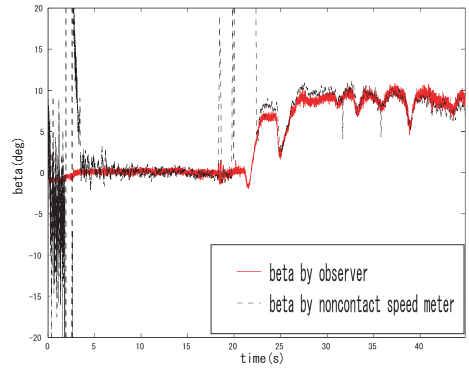



(b)Measured value and observer's output of $\gamma$

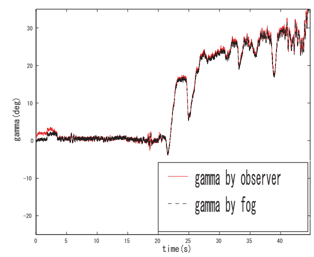



(c)Measured value and observer's output of $a_y$

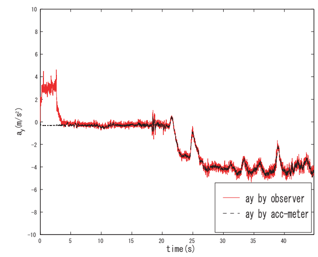
Fig. 6. No.2; Speed is 40km/h and steering angle $\delta$ is 180[deg]




(a)Measured value and estimation of $\beta$




(b)Measured value and observer's output of $\gamma$




(c)Measured value and observer's output of $a_y$

Fig. 7. No.3; EV was rotated in circle on the wet road

## VI. Conclusion

In this paper, we proposed an improved method for the estimation of body slip angle $\beta$ for EVs, based on linear observer using side acceleration and yaw rate $\gamma$ sensors. By this method, we can estimate $\beta$ robustly and accurately. Experiments by UOT March$II$ demonstrated that the proposed observer was robust and even if velocity, steering angle and road condition changed, the observer could still estimate $\beta$. EV's advantages are not only in environment-friendly properties but also its possibilities for safer and more sophisticated control. In the future, pure EVs will become more popular and contribute to solving energy problems. It is important to pursue the researches on making full use of pure EVs' advantages.

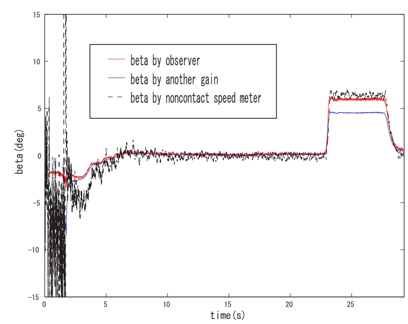



Fig. 8. No.4; Measured value and estimation of $\beta$ with two different gain matrixes; Speed is 40km/h and steering angle $\delta$ is 90[deg] when observer model has error

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