

# Backlash Vibration Suppression in Torsional System Based on The Fractional Order $Q$ -Filter of Disturbance Observer

Chengbin Ma

The Department of Electrical Engineering  
The University of Tokyo  
Email: ma@horilab.iis.u-tokyo.ac.jp

Yoichi Hori

Institute of Industrial Science  
The University of Tokyo  
Email: hori@iis.u-tokyo.ac.jp

**Abstract**—In this paper, a fractional order  $Q$ -filter  $\frac{1}{(Ts+1)^\alpha}$  is introduced to substitute the integer order  $Q$ -filter  $\frac{1}{(\tau s+1)^n}$  used in conventional disturbance observer for speed control of torsional system. The theoretical analysis and experimental results show that changing the  $Q$ -filter's order fractionally can give a more effective way to adjust control system's frequency and time responses than just tuning it among integer orders. The tradeoff between stability margin loss and the strength of vibration suppression is a common problem in torsional system control. By introducing Fractional Order Control (FOC) approach, control system can be designed more straightforwardly since control system's frequency responses can be altered between Integer Order Control (IOC) system's continuously, while less control parameters are needed to be decided. Design process and experimental results demonstrate that an "EASY & STRAIGHT-FORWARD DESIGN" can be achieved by introducing FOC control design concept. For implementation of the fractional order  $Q$ -filter, broken-line approximation method is applied. Even the realization issues for fractional order controllers are somewhat problematic. Experiment results show that the controllers can actually be realized quite acceptably.

## I. INTRODUCTION

The concept of Fractional Order Control (FOC) means controlled systems and/or controllers are described by fractional order differential equations. Expanding derivatives and integrals to fractional orders has a firm and long standing theoretical foundation. *Leibniz* mentioned this concept in a letter to *Hospital* over three hundred years ago (1695) and the earliest more or less systematic studies have been made in the beginning and middle of the 19th century by *Liouville*(1832), *Holmgren*(1864) and *Riemann*(1953) [1] [2]. As one of its applications in control engineering, FOC was introduced by *Tustin* for the position control of massive objects half a century ago, where actuator saturation requires sufficient phase margin around and below the critical point [3]. Some pioneering works were also done in 60's [4]. However the FOC concept was not widely incorporated into control engineering mainly due to the conceptually difficult idea of taking fractional order, the existence of so few physical applications and limited computational power available at that time [5].

In recent decades, researchers pointed out that fractional order differential equations could model various materials

more adequately than integer order ones and provide an excellent tool for describing dynamic processes [6]. It is a natural conclusion that fractional order models need fractional order controllers for more effective control of system's dynamics [7]. At the same time, letting control order be fractional can adjust control system's gain and phase characteristics simultaneously and continuously between Integer Order Control (IOC) systems'. This flexibility makes FOC a powerful tool in design of robust control system with less control parameters. Since the tradeoff of stability and other control demands always exists, this great flexibility obtained by introducing FOC makes it more straightforward to achieve a better tradeoff, which is one of central issues in control design. The advantages of FOC in modeling and control design motivated renewed interest in various applications of FOC [8] [9] [10] [11]. With the rapid development of computer performances, the modeling and realization of FOC systems also became possible and much easier than before.

Despite FOC's promising aspects in modeling and control design, FOC research is still at its primary stage, especially in motion control field. Parallel to the development of FOC theories, applying FOC to various control problems is also crucially important and should be one of top priority issues. The authors believe that FOC is a natural choice in control design and it's design process should be straightforward. There is no reason that the knowledge of extremely well-developed classical IOC theories is not made full use of in FOC applications.

Based on these basic considerations, in this paper, the authors introduce a fractional order version of  $Q$ -filter  $\frac{1}{(Ts+1)^\alpha}$  to substitute the integer order  $Q$ -filter used in conventional disturbance observer for vibration suppression control of torsional system. Introducing fractional order  $Q$ -filter  $\frac{1}{(Ts+1)^\alpha}$  can achieve a clear-cut and effective adjustment of tradeoff between stability margin loss and the strength of vibration suppression. The necessity of this tradeoff adjustment is common and natural in oscillatory system's control [11] [12]. By changing fractional order  $\alpha$ , the torsional control system's frequency responses will be altered continuously. This advantage makes it easier to achieve a good tradeoff with improved vibration suppression.

sion performance and enough stability margin. The idea of taking fractional order  $Q$ -filter was proposed in Ref. [12]. However, only theoretical aspects were mentioned. In this paper, besides theoretical analysis, experiments are also carried out to demonstrate the fractional order  $Q$ -filter's advantages in real application.

## II. THE TESTING BENCH

The testing bench of torsional system is depicted in Fig. 1, which is a typical oscillatory system. Two flywheels are connected with a long torsional shaft; while driving force is transmitted from driving servomotor to the shaft by gears with gear ratio 1:2. Some system parameters are adjustable, such as gear inertia, load inertia, shaft's elastic coefficient and gears' backlash angle. The encoders and tachogenerators are used as position and rotation speed sensors.

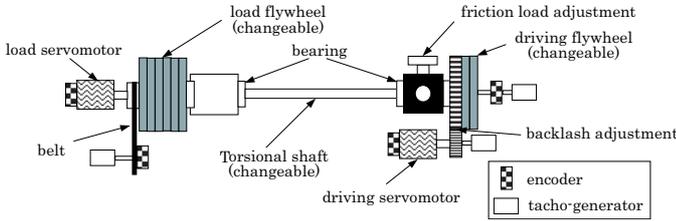


Fig. 1. Experimental setup of torsional system

The simplest model of the testing bench with gear backlash is three-inertia model, as depicted in Fig. 2 and Fig. 3, where  $J_m$ ,  $J_g$  and  $J_l$  are driving motor, gear (driving flywheel) and load's inertias,  $K_s$  shaft elastic coefficient,  $\omega_m$  and  $\omega_l$  motor and load rotation speeds,  $T_m$  input torque and  $T_l$  disturbance torque. In this modeling, the gear backlash non-linearity is simplified as a deadzone factor with backlash angle band  $[-\delta, +\delta]$  and elastic coefficient  $K_g$ . Frictions between components are neglected due to their small values.

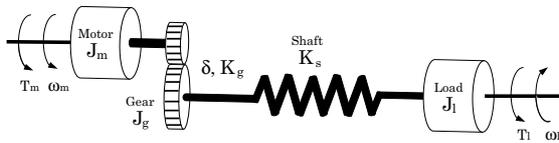


Fig. 2. Torsional system's three-inertia model

Parameters of the experimental torsional system are shown in Table. I with a backlash angle of  $\pm 0.6deg$ . Open-loop transfer function from  $T_m$  to  $\omega_m$  is in the following form:

$$P_{3m}(s) = \frac{(s^2 + \omega_{h1}^2)(s^2 + \omega_{h2}^2)}{J_m s(s^2 + \omega_{o1}^2)(s^2 + \omega_{o2}^2)} \quad (1)$$

where  $\omega_{o1}$  and  $\omega_{o2}$  are resonance frequencies,  $\omega_{h1}$  and  $\omega_{h2}$  are anti-resonance frequencies.  $\omega_{o1}$  and  $\omega_{h1}$  correspond to torsion vibration mode; while  $\omega_{o2}$  and  $\omega_{h2}$  correspond to gear backlash vibration mode (see Fig. 4).

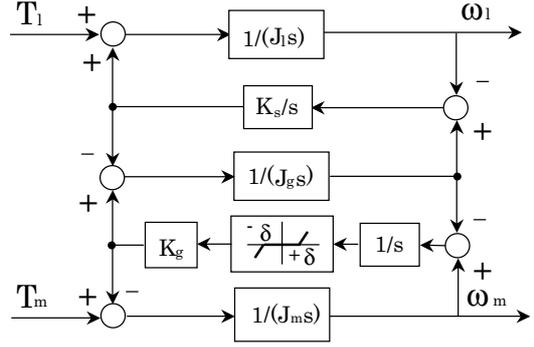


Fig. 3. Block diagram of the three-inertia model

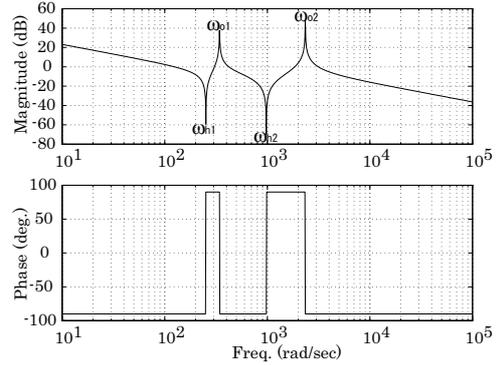


Fig. 4. Bode plot of the three-inertia model

TABLE I  
PARAMETERS OF THE THREE-INERTIA SYSTEM

$J_m$ ( $Kgm^2$ )	$J_g$ ( $Kgm^2$ )	$J_l$ ( $Kgm^2$ )	$K_g$ ( $Nm/rad$ )	$K_s$ ( $Nm/rad$ )	$\delta$ ( $deg$ )
0.0007	0.0034	0.0029	3000	198.49	0.6

## III. CONVENTIONAL DISTURBANCE OBSERVER

Disturbance observer can be applied in torsional system's speed control. As depicted in Fig. 5, the inverse plant model for disturbance observer is  $J_s$ , where  $J$  equals the sum of  $J_m$ ,  $J_g$  and  $J_l$ . In this simple inverse model, the three masses of driving motor, gear and load are considered to be connect with a rigid shaft and can be described as a single mass  $J$ . The  $Q$ -filter is a low-pass filter to restrict the effective bandwidth of the disturbance observer:

$$Q(s) = \frac{1}{(\tau s + 1)^n} \quad (2)$$

where  $\tau$  is the cutoff frequency and  $n$  is the relative degree of  $Q$ -filter.

The disturbance observer is applied to estimate disturbance torque  $\hat{T}_d$ , which is generated due to unmodeled dynamics in single inertia model  $J_s$ . Considering the frequency range of torsion vibration mode,  $\tau$  is taken as 0.005(=1/200). By choosing different relative degree  $n$ , the control system's frequency responses can be adjusted. As depicted in Fig. 6,  $n=1$  has the best vibration suppression performance. The three-inertia model is used as nominal model for the actual torsional system in Fig. 6.

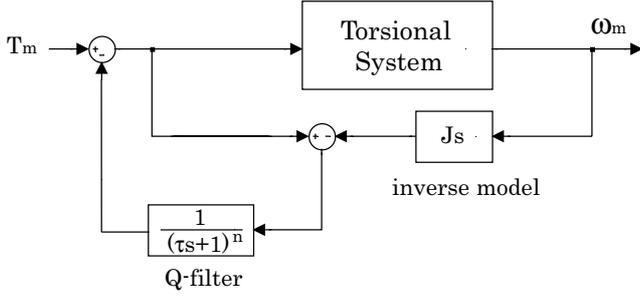


Fig. 5. Conventional disturbance observer

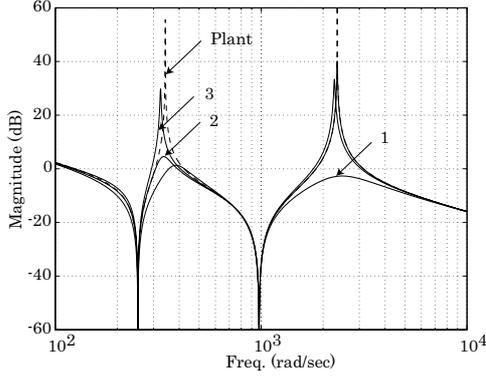


Fig. 6. Gain plot from  $T_m$  to  $\omega_m$  with different order  $n$

Simulation results show PI speed controller with disturbance observer ( $n=1$ ) can give a good control performance in nominal case, where  $K_i = 33.5384$ ,  $K_p = 1.6187$ , backlash angle  $\delta = 0.6deg$ . and maximum torque limitation is  $\pm 3.84Nm$  (see Fig. 7 and Fig. 8). The PI controller is design by using Coefficient Diagram Method (CDM) [11] [13] [14]. However, describing gear backlash using deadzone factor and elastic coefficient is far from adequate due to gear backlash's complex dynamics [11]. Whether this method can suppress backlash vibration effectively or not will be verified in experiments using the testing bench.

#### IV. NOVEL FRACTIONAL ORDER FILTER

Due to the negative feedback of the estimated signal in disturbance observer, larger  $n$  will give more phase margin, but also larger gain in control system's open-loop frequency responses, and vice versa. Namely, a tradeoff

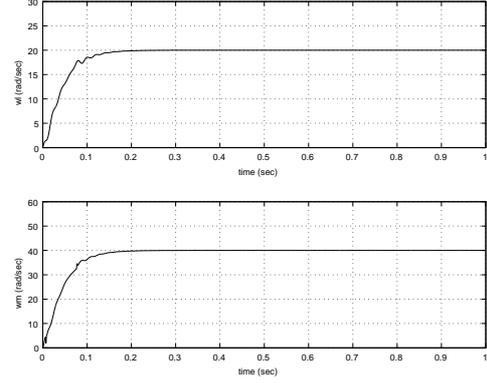


Fig. 8. Time responses in simulation ( $n=1$ )

between stability margin loss and vibration suppression exists in torsional system's speed control. The only tradeoff tuning knob is  $Q$ -filter's relative degree  $n$ .

For conventional disturbance observer, the possibility of achieving a better tradeoff is quite restricted since just integral order  $n$  can be chosen. As mentioned in above section, taking  $n$  as 1, the smallest value for  $n$ , gives the best vibration suppression performance for conventional disturbance observer. To further improve vibration suppression performance while keep enough phase margin, introducing  $Q$ -filter, whose order is between 0 and 1, is actually a natural choice (see Fig. 9).

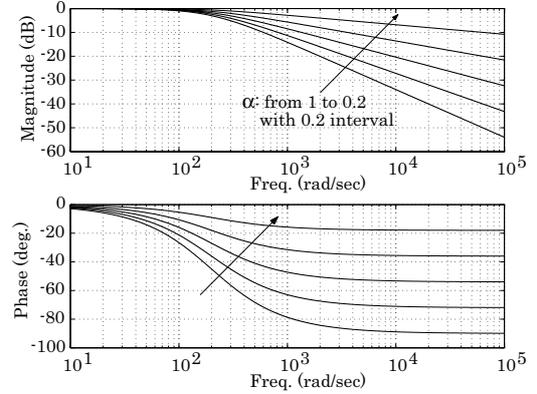


Fig. 9. Bode plots of fractional order  $Q$ -filter  $\frac{1}{(\tau s+1)^\alpha}$

Inspired by this consideration, a straightforward method

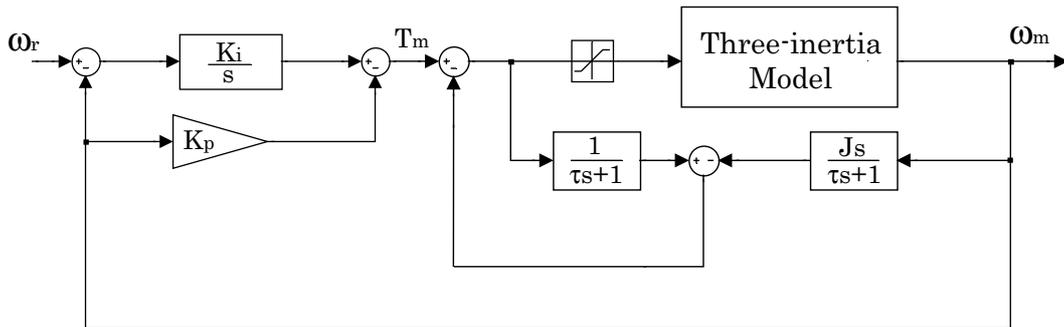


Fig. 7. Block diagram for simulation with three-inertia model

of substituting the integer order  $Q$ -filter  $\frac{1}{(\tau s+1)^n}$  with a novel fractional order one  $\frac{1}{(\tau s+1)^\alpha}$  is introduced, as depicted in Fig. 10.

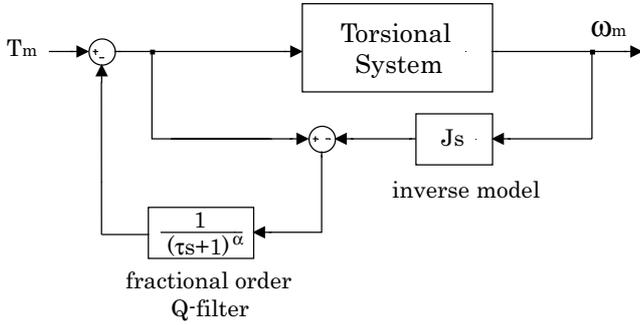


Fig. 10. Disturbance observer with fractional order  $Q$ -filter

It can be seen in Fig. 11 that letting the  $Q$ -filter's order be fractional can further enlarge the range of control system's frequency responses adjustment, while only one control parameter, the fractional order  $\alpha$ , is needed to be decided. These advantages of FOC approach provide much more flexibility in control design. A proper selected fractional order  $\alpha$  can easily give a better vibration suppression performance while keep enough stability margin, i.e. gain margin and phase margin.

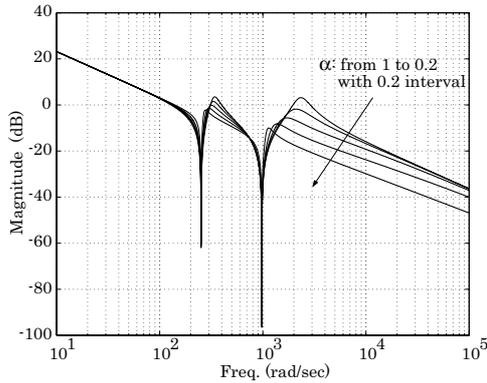


Fig. 11. Gain plot from  $T_m$  to  $\omega_m$  with different  $\alpha$

## V. REALIZATION METHOD

Design control system by FOC approach is straightforward. However, for realizing designed fractional order controller, it is not so. Because of fractional order systems' infinite dimension, proper approximation by finite difference equation is needed. Since FOC system's frequency response is actually exactly known, it is natural to approximate fractional order controllers by frequency domain approaches.

In this paper, a broken-line approximation method is introduced to approximate  $\frac{1}{(\tau s+1)^\alpha}$  in frequency range  $[\omega_b, \omega_h]$ , where  $\omega_b = \frac{1}{\tau}$ .  $\omega_h$  is taken as  $10^4$  to give an enough frequency range for the approximation. Let

$$\left(\frac{s}{\omega_b} + 1\right)^\alpha \approx \prod_{i=0}^{N-1} \frac{\frac{s}{\omega_i} + 1}{\frac{s}{\omega_i} + 1} \quad (3)$$

Based on Fig. 12, two recursive factors  $\zeta$  and  $\eta$  are

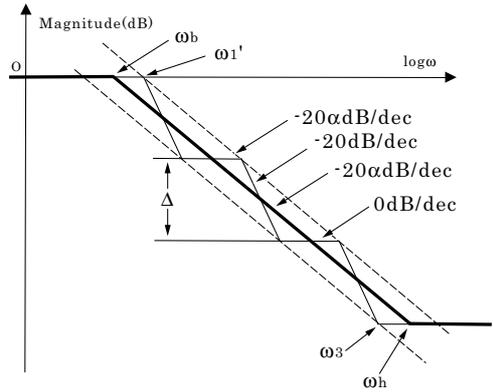


Fig. 12. An example of broken-line approximation ( $N = 3$ )

introduced to calculate  $\omega_i$  and  $\omega'_i$ :

$$\zeta = \frac{\omega'_i}{\omega_i}, \quad \eta = \frac{\omega_{i+1}}{\omega'_i} \quad (4)$$

Since

$$\omega_0 = \eta^{\frac{1}{2}} \omega_b, \quad \omega'_{N-1} = \eta^{-\frac{1}{2}} \omega_h \quad (5)$$

Therefore

$$\zeta \eta = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{1}{N}} \quad (6)$$

with

$$\omega_i = (\zeta \eta)^i \omega_0, \quad \omega'_i = \zeta (\zeta \eta)^i \omega_0 \quad (7)$$

The frequency-band fractional order controller has  $-20\alpha \text{ dB/dec}$  gain slope, while the integer order factors  $1 / (\frac{s}{\omega_i} + 1)$  have  $-20 \text{ dB/dec}$  slope. For the same magnitude change  $\Delta$ :

$$-20\alpha = \frac{\Delta}{\log \zeta + \log \eta}, \quad -20 = \frac{\Delta}{\log \zeta} \quad (8)$$

Thus

$$(\zeta \eta)^\alpha = \zeta \quad (9)$$

Therefore  $\zeta$  and  $\eta$  can be expressed respectively by

$$\zeta = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{\alpha}{N}}, \quad \eta = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{1-\alpha}{N}} \quad (10)$$

Finally

$$\omega_i = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+\frac{1}{2}-\frac{\alpha}{2}}{N}} \omega_b, \quad \omega'_i = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+\frac{1}{2}+\frac{\alpha}{2}}{N}} \omega_b \quad (11)$$

Figure. 13 shows the Bode plots of ideal frequency-band case ( $\alpha = 0.4$ ,  $\omega_b = 200 \text{ Hz}$ ,  $\omega_h = 10000 \text{ Hz}$ ) and its 1st-order, 2nd-order and 3rd-order approximations by broken-line approximation method. Even taking  $N = 2$  can give a satisfactory accuracy in frequency domain. Bilinear transformation is used to discrete the approximate controllers in this paper.

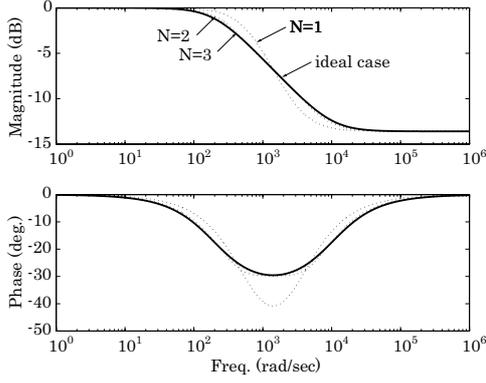


Fig. 13. Bode plots of ideal case, 1st, 2nd and 3rd-order approximations

## VI. COMPARATIVE EXPERIMENTS

As depicted in Fig. 14, the experimental torsional system is controlled by a PC with 1.6GHz Pentium IV CPU and 512M memory. Control programs are written in RTLinux C threads which can be executed with strict timing requirement of control sampling time. A 12-bit AD/DA multi-functional board is used whose conversion time per channel is  $10\mu sec$ .

Experiments are carried out with sampling time  $T=0.001sec$  and 2nd-order broken-line approximation for fractional order  $Q$ -filters. Two encoders ( $8000pulse/rev$ ) are used as rotation speed sensors with coarse quantization  $\pm 0.785rad/sec$ .

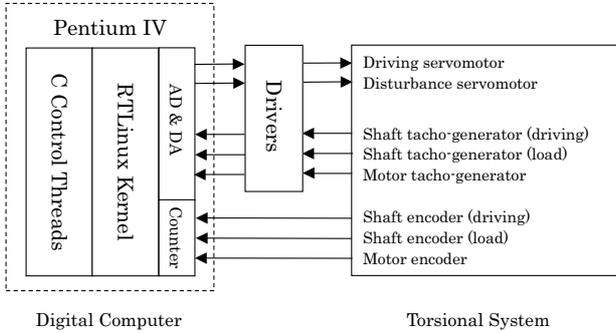


Fig. 14. Digital control system of experimental torsional system

Firstly, speed control experiment with integer order  $Q$ -filter is carried out. As depicted in Fig. 15, the control system can achieve satisfactory response when backlash angle is adjusted to zero degree ( $\delta=0$ ). With the existence of gear backlash non-linearity, persistent vibration occurs (see  $\delta=0.6$  case). Fig. 16 shows that compared with PI-only control, introducing disturbance observer can give better vibration suppression performance. However, this performance improvement is not enough to suppress effectively the vibration cause by gear backlash.

For higher order  $n$ , like  $n=2$  and  $n=3$ , the vibration suppression performance is actually deteriorated, while the control system still keeps stable (see Fig. 17). This experimental result verifies that a tradeoff between stability and the strength of vibration suppression exists and can be

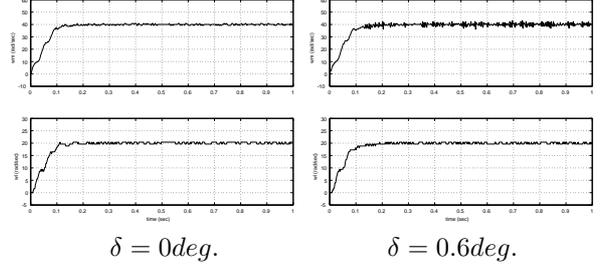


Fig. 15. Time responses with integer order  $Q$ -filter ( $n=1$ )

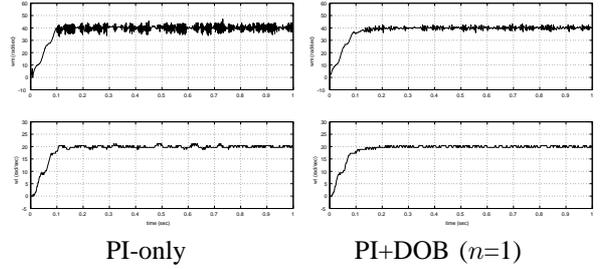


Fig. 16. Comparison of PI-only control and PI+DOB control

adjusted by different order  $n$  of the  $Q$ -filter.

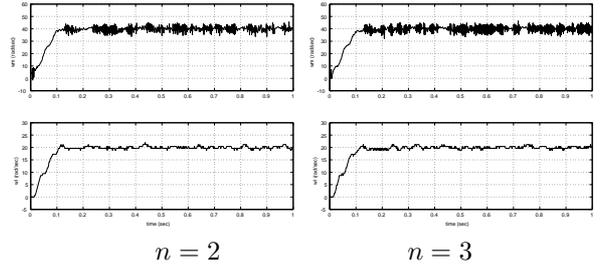


Fig. 17. Time responses with integer order  $Q$ -filter ( $n=2, 3$ )

Figure 18 depicts the experimental results with different  $\alpha$  for the fractional order  $Q$ -filter  $\frac{1}{(\tau s+1)^\alpha}$ . By taking  $\alpha$  as 0.8, the vibration caused by gear backlash is effectively suppressed and the best time response is achieved. Higher  $\alpha$ , for example 1.0, cannot suppression backlash vibration while the control system is still be stable. For small  $\alpha$  like 0.4, large phase lag of the fractional order  $Q$ -filter actually unstabilizes the control system. In the time response of  $\alpha = 0.6$  case, even backlash vibration is suppressed, the tiny overshoot and momentary vibration at the beginning reveal it's poor stability performance.

The 2nd-order approximation of fractional order  $Q$ -filter  $\frac{1}{(\tau s+1)^\alpha}$  between frequency band  $[200, 10000]$  is in following form:

$$C_f(s) = \frac{0.043734(s+1163)(s+8223)}{(s+243.2)(s+1720)} \quad (12)$$

It can be seen that the realization of  $\frac{1}{(\tau s+1)^\alpha}$  is actually a quite simple 2-order controller. It's digital implementation only needs two arrays with length of 3.

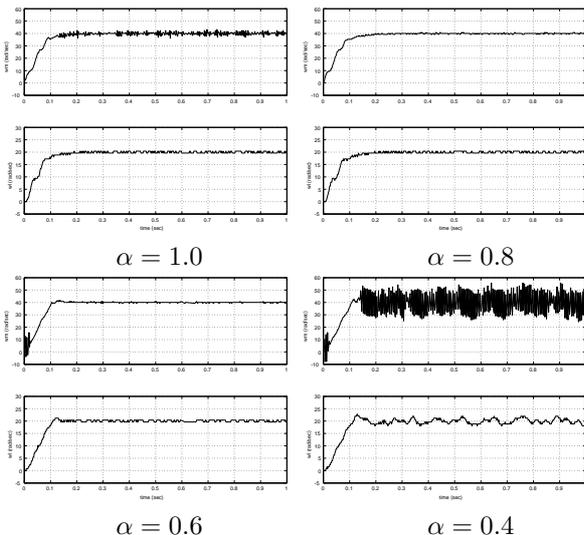


Fig. 18. Time responses with different fractional order  $\alpha$

## VII. CONCLUSIONS

In this paper, a fractional order  $Q$ -filter of disturbance observer,  $\frac{1}{(Ts+1)^\alpha}$ , was introduced to substitute the integer order  $Q$ -filter  $\frac{1}{(\tau s+1)^n}$  used in conventional disturbance observer for speed control of torsional system. The theoretical analysis and experimental results show that changing  $Q$ -filter's order fractionally can give a more effective way to adjust control system's frequency and time responses than just tuning it among integer orders.

The tradeoff between stability margin loss and the strength of vibration suppression is a common problem in torsional system control. By introducing FOC approach, control system's frequency responses can be adjusted straightforwardly and continuously between IOC system's, while less control parameters are needed to be decided. "EASY & STRAIGHTFORWARD DESIGN" can be achieved by expanding controller's order to being fractional. On the contrary to FOC control design, the implementation of fractional order controllers is not such straightforward. Some proper approximations are needed. However, as verified in experimental results, implementation issue is not problematic actually.

FOC should not be an independent concept of the well-developed IOC. Knowledge and design methods developed in IOC can still be made full use of in FOC research, as demonstrated in this paper. It is interesting to notice that even the theoretical analysis and design are based FOC approach, the implementation of fractional order controllers are certainly integer order controllers (see Equ. 12). Therefore, FOC should not be thought as a novel and conceptually difficult idea, but actually a natural and more effective control design tool. By FOC, control system's responses can be designed with much more flexibility. This enlarged flexibility will provide more possibility to find excellent solutions for various control problems. Some well-designed IOC system might be looked as a good approximation of FOC system. If this hypothesis can be established, it will

further verify FOC's advantages in control field.

Finally, introducing FOC approach in this paper is "can", not "must". Other IOC design methods can also solve the problem; while they maybe not as straightforward as the FOC approach. However, for example, if the long shaft is thin enough, fractional order modeling will be more effective due to the shaft's distributed-parameter characteristic. In this case, fractional order controller is a natural choice. Future research will be carried out on this issue.

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