Paper

Backlash Vibration Suppression Control of Torsional System by Novel Fractional Order PID^k Controller

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This paper proposes a novel fractional order PID^k controller for torsional system's backlash vibration suppression control, in which the order k of D controller can not only be integer but also be any real number. Various methods have been proposed for two-inertia system's speed control, but the control systems designed by these methods may not be able to suppress the vibration caused by gear backlash. In order to improve control system's robustness against the backlash, several methods have been proposed. However their design processes are very complicated. Clear and straightforward design concept is required in practical applications. As a novel approach, in this paper the D controller's order is expanded to include any real number. Robust PID^k controller against backlash is designed by adjusting the order k directly. An approximation method based on Short Memory Principle is also introduced to realize the discrete D^k controller. Design process and experimental results demonstrate straightforward robust control design through the novel FOC approach, the PID^k control system's better robustness against backlash non-linearity and good approximation of the realization method.

Keywords: Fractional Order Control System, Torsional System, Backlash Vibration Control, Realization Method

1. Introduction

Gears are used widely in torsional system as an integral part of power transmission and position devices. Any faults in gear transmission will considerably affect the performance of such system. Especially, backlash non-linearity between the teeth is one of the most important faults in torsional system's speed control. It causes delay, vibration, speed inaccuracy and degrades overall control performance. In extreme cases, the severe backlash vibration can make it intractable and cause possible damage to the system.

Since in torsional system gear elasticity is much larger than shaft elasticity, control system is commonly designed based on the simplified two-inertia model, in which driving servomotor and gears are treated as unity inertia. The backlash non-linearity between gears is not expressed in the model. For two-inertia speed control, the history of control theory can be seen. Various design methods such as classical PID control, time derivative feedback, model following control, disturbance observer-based control, state feedback control and modern H∞ control have been proposed (1) (2) (3) (4). Among them, the PID control is the most widely used in real industrial applications.

Common weak point of the above methods is that the existence of backlash non-linearity is totally neglected. This weakness may make designed speed control systems unstable and give rise to backlash vibration. In order to be robust against backlash non-linearity, several methods have been proposed, but their design processes are very complicated. As an example, for PID control introducing a low-pass filter $K_d s/(T_d s+1)$ and redesigning the whole control system with three-inertia model can be a solution (5). Due to the necessity of solving high order equations, the design is not easy to carry out. Clear and straightforward design concept is required in practical applications.

In this paper a novel Fractional Order Control (FOC) approach of using fractional order PID^k controller is proposed for torsional system's backlash vibration suppression control. In PID^k controller D's order can be any real number, not necessarily be an integer. Tuning fractional order k can adjust control system's frequency response directly; therefore a straightforward design can be achieved for robust control against backlash non-linearity .

The concept of FOC means controlled systems and/or controllers are described by fractional order differential equations. Expanding derivatives and integrals to fractional orders had a firm and long standing theoretical foundation. *Leibniz* mentioned this concept in a letter to *Hospital* over three hundred years ago (1695) and the earliest more or less systematic studies seem to have been made in the beginning and middle of the 19th century by *Liouville*(1832), *Holmgren*(1864) and *Riemann*(1953) (6). As its application in control engineering, FOC was introduced by *Tustin* for the position control of massive objects half a century ago, where actuator saturation requires sufficient phase margin around and below the critical point (7). Some other pioneering works were also carried out around 60's (8). However the FOC concept was not widely incorporated

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into control engineering mainly due to the conceptually difficult idea of taking fractional order, the existence of so few physical applications and the limited computational power available at that time [9].

In the last few decades, researchers pointed out that fractional order differential equations could model various materials more adequately than integer order ones and provide an excellent tool for describing dynamic processes [6] [10] [11]. The fractional order models need fractional order controllers for more effective control of the dynamic systems. This necessity motivated renewed interest in various applications of FOC [12] [13]. With the rapid development of computer performances, modeling and realization of the FOC systems also became possible and much easier than before. But in motion control the FOC research is still in a primitive stage. This paper represents one of the first attempts towards applying FOC to motion control.

The paper is organized as follows: in section 2, the experimental setup and modeling of the torsional system are introduced; in section 3, integer order PID control's robustness problem against backlash non-linearity is analyzed using Bode plot; in section 4, a novel PID^k controller is proposed to improve the robustness of the system. Realization method for the discrete D^k controller is also introduced; in Section 5, experiments are carried out to verify the effectiveness of the proposed PID^k controller for backlash vibration suppression control. Finally, in section 6, conclusions are drawn.

2. Experimental Torsional System

2.1 Three-inertia model The experimental setup of torsional system is depicted in Fig. 1. A torsional shaft connects two flywheels while driving force is transmitted from driving servomotor to the shaft by gears with gear ratio 1:2. Some system parameters are changeable, such as gear inertia, load inertia, shaft's elastic coefficient and gears' backlash angle. The encoders and tacho-generators are used as position and rotation speed sensors.

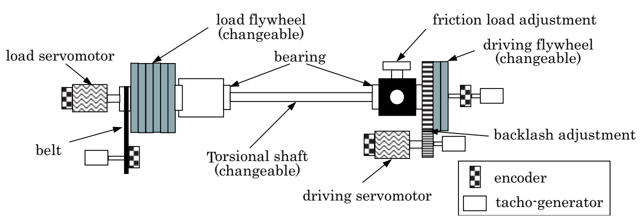


Fig. 1. Experimental setup of torsional system

The simplest model of the torsional system with gear backlash is the three-inertia model depicted in Fig. 2 and Fig. 3, where J_m, J_g and J_l are driving motor, gear (driving flywheel) and load's inertias, K_s shaft elastic coefficient, ω_m and ω_l motor and load rotation speeds, T_m input torque and T_l disturbance torque. In the modeling, the gear backlash is simplified as a deadzone factor with backlash angle band [-δ,+δ] and elastic coefficient K_g. Frictions between components are neglected due to

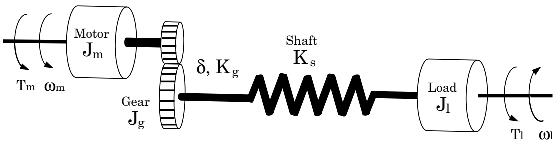


Fig. 2. Torsional system's three-inertia model

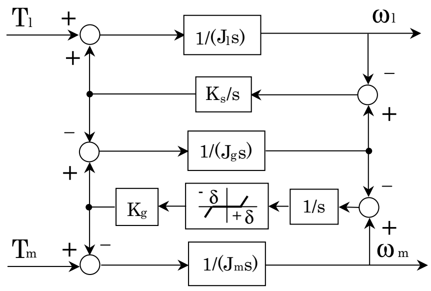


Fig. 3. Block diagram of the three-inertia model

their small values.

Parameters of the experimental torsional system are shown in Table. 1 with a backlash angle of $\pm 0.6deg$. The open-loop transfer function from T_m to ω_m is

$$\begin{aligned} P_{3m}(s) &= \{J_g J_l s^4 + [(K_s+K_g)J_l + K_s J_g]s^2 + K_g K_s\}/ \\ &\quad \{s\{J_m J_g J_l s^4 + [K_s(J_g+J_l)J_m + K_g(J_m \\ &\quad +J_g)J_l]s^2 + (J_m+J_g+J_l)K_s K_g\}\} \\ &= \frac{(s^2+\omega_{h1}^2)(s^2+\omega_{h2}^2)}{J_m s(s^2+\omega_{o1}^2)(s^2+\omega_{o2}^2)} \quad \cdots\cdots\cdots\cdots (1) \end{aligned}$$

where ω_{o1} and ω_{o2} are resonance frequencies while ω_{h1} and ω_{h2} are anti-resonance frequencies. ω_{o1} and ω_{h1} correspond to torsion vibration mode, while ω_{o2} and ω_{h2} correspond to gear backlash vibration mode (see Fig. 4).

Table 1. Parameters of the three-inertia system

J_m (Kgm^2)	J_g (Kgm^2)	J_l (Kgm^2)	K_g (Nm/rad)	K_s (Nm/rad)	δ (deg)
0.0007	0.0034	0.0029	3000	198.49	0.6

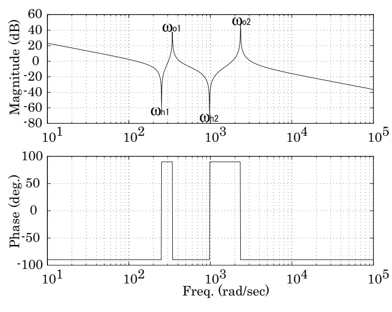


Fig. 4. Bode plot of the three-inertia model

2.2 Simplified two-inertia model Since the gear elastic coefficient K_g is much larger than the shaft elastic coefficient K_s ($K_g >> K_s$), for speed control design the two-inertia model is commonly used in which driving motor inertia J_m and gear inertia J_g are simplified to a single inertia $J_{mg}(= J_m + J_g)$ (see Fig. 5). The open-loop transfer function for the two-inertia model is

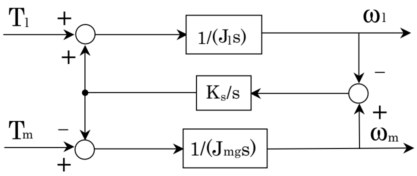


Fig. 5. Block diagram of the two-inertia model

$$P_{2m}(s) = \frac{s^2 + \omega_h^2}{J_{mg}s(s^2 + \omega_o^2)} \cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where ω_o is the resonance frequency and ω_h is the anti-resonance frequency corresponding to the torsion vibration mode, while the existence of backlash vibration mode is totally ignored in the simplified two-inertia model.

3. Design of PID Controller

In order to smooth the discontinuity of speed command ω_r by integral controller, a set-point-I PID controller is proposed for the torsional system's speed control (see Fig. 6).

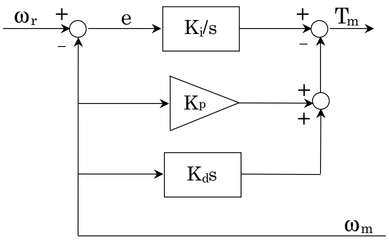


Fig. 6. Set-point-I PID controller

Based on the simplified two-inertia model, the PID controller's parameters are designed by the standard form ($\gamma_1 = 2.5, \gamma_2 = \gamma_3 = 2$) of Coefficient Diagram Method [(1) (15)]:

$$K_p = \frac{10\sqrt{2}}{11}\sqrt{J_l k_s}, K_i = \frac{4}{11}K_s, K_d = \frac{5}{11}J_l - J_{mg} \quad (3)$$

Equation (3) gives

$$K_p = 0.9754,\ K_i = 72.1782,\ K_d = -0.0028 \cdots (4)$$

where $K_d < 0$ means positive feedback of acceleration $\dot{\omega}_m$. Simulation results with the simplified two-inertia model show this integer order PID control system has a superior performance for suppressing torsion vibration (see Fig. 7).

For three-inertia plant $P_{3m}(s)$, the close-loop transfer function of integer order PID control system from ω_r to ω_m is

$$G_{close}(s) = \frac{C_I(s)P_{3m}(s)}{1 + C_I(s)P_{3m}(s) + C_{PD}(s)P_{3m}(s)} (5)$$

where $C_I(s)$ is I controller and $C_{PD}(s)$ is the parallel of P and D controllers in minor loop; therefore $G_{close}(s)$ is stable if and only if $G_l = C_I(s)P_{3m}(s) + C_{PD}(s)P_{3m}(s)$ has positive gain margin and phase margin. But as depicted in Fig. 8 the gain margin of $G_l(s)$ is negative. With the existence of gear backlash the designed integer order PID control system will easily be unstable and lead to backlash vibration.

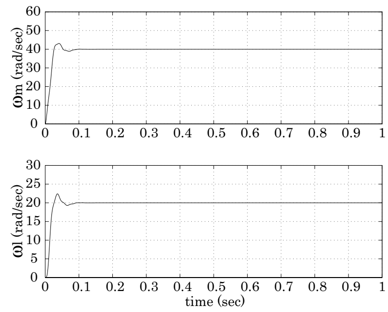


Fig. 7. Time responses of the integer order PID two-inertia system by simulation

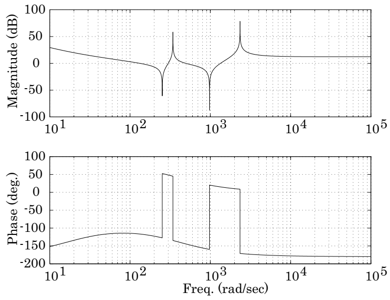


Fig. 8. Bode plot of $G_l(s)$ in PID control

4. Proposed PID^k Controller

4.1 Design of Fractional Order k In this paper, a novel fractional order PID^k controller is proposed to achieve a straightforward design of robust control system against gear backlash non-linearity. Instead of solving high order equations, by changing the D^k controller's fractional order k the frequency response of $G_l(s)$ can be directly adjusted (see Fig. 9). As depicted in Fig. 10, letting k be fractional order can improve PID^k control system's gain margin continuously. When $k < 0.84$ the PID^k control system will be stable; therefore with proper selected fractional order k the backlash vibration can be suppressed.

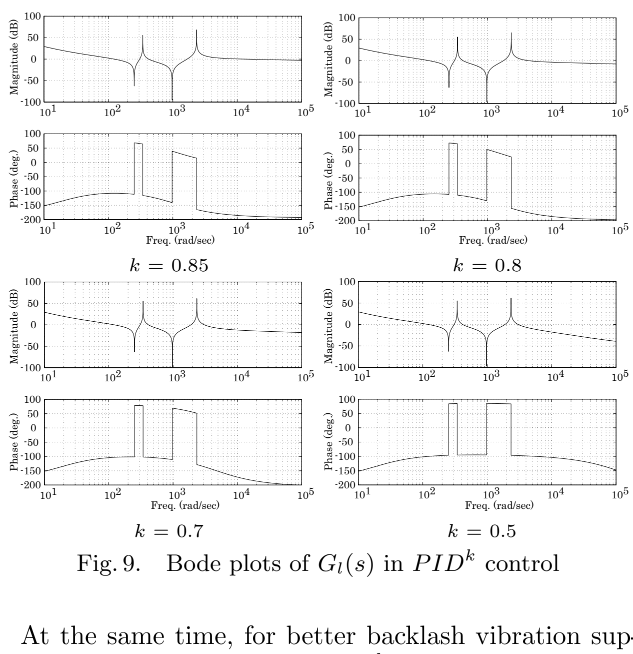


Fig. 9. Bode plots of $G_l(s)$ in PID^k control

At the same time, for better backlash vibration suppression performance higher D^k controller's order is

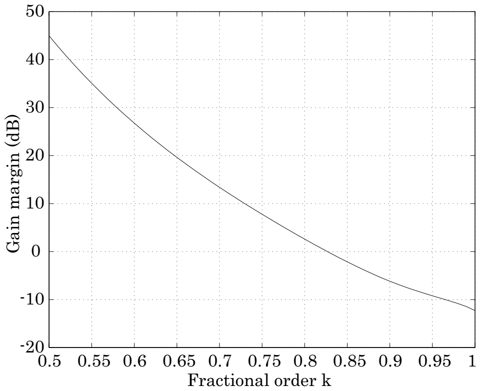


Fig. 10. Gain margin versus fractional order k

more preferable. As shown in open-loop gain plots of 0.85, 0.8, 0.7 and 0.5 order PID^k control systems (see Fig. 11), higher the D controller's order is taken lower the gain near gear backlash vibration mode is. Based on the tradeoff between robustness and vibration suppression performance, fractional order 0.7 is chosen as D^k controller's best order in this paper.

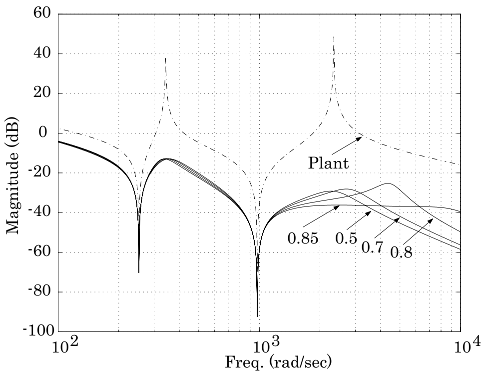


Fig. 11. Gain plots of the PID^k control systems and three-inertia plant

4.2 Realization of D^k controller

The mathematical definition of fractional derivatives and integrals has been a subject of several different approaches (6) (10). One of the most frequently encountered definitions is called Grünwald-Letnikov definition, in which the fractional α order derivatives are defined as

$$ {}_{t_0}D_t^{\alpha} = \lim_{\substack{h\to 0\\ nh=t-t_0}} h^{-\alpha}\sum_{r=0}^{n}(-1)^r \begin{pmatrix} \alpha \\ r \end{pmatrix} f(t-rh) \quad (6)$$

where h is time increment and binomial coefficients are

$$\begin{pmatrix} \alpha \\ 0 \end{pmatrix} = 1,\ \begin{pmatrix} \alpha \\ r \end{pmatrix} = \frac{\alpha(\alpha-1)\ldots(\alpha-r+1)}{r!} \quad (7)$$

It is clear that fractional order systems have an infinite dimension while integer order systems finite dimensional. Proper approximation by finite differential equation is needed to realize the designed PID^k controller. In this paper, Short Memory Principle is adopted to realize the discrete fractional order D^k controller. The principle is inspired by Grünwald-Letnikov definition. Based on the approximation of the time increment h through the sampling time T, the discrete equivalent of D^k controller is given by

$$Z\{D^k[f(t)]\} \approx \left[T^{-k}\sum_{j=1}^{\infty} c_j^k z^{-(j-1)}\right] X(z) \ \cdots \ (8)$$

where $X(z) = Z\{f(t)\}$ and the binomial coefficients are

$$c_1^k = 1,\ c_j^k = (-1)^{(j-1)} \begin{pmatrix} k \\ j-1 \end{pmatrix} \ \cdots\cdots\cdots\cdots \ (9)$$

The semi-log chart of Fig. 12 shows binomial coefficients value versus term order j in Equ. (8) when approximating $k = 0.5$ derivative. The observation of the chart gives that the values of binomial coefficients near "starting point" t_0 in Grünwald-Letnikov definition is small enough to be neglected or "forgotten" for large t.

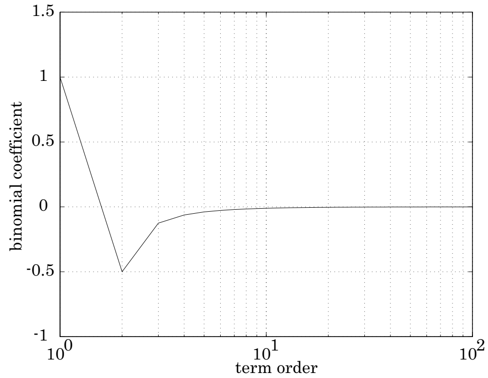


Fig. 12. Binomial coefficients value versus term order j when approximating $D^{0.5}$

Therefore the finite dimension approximation of D^k controller can be arrived by taking into account the behavior of $f(t)$ only in "recent past", i.e. in the interval $(t-L, t)$, where L is the length of "memory" and $[L/T]$ is the number of the recent sampled $f(t)$ values remembered by discrete D^k controller:

$$ {}_{t_0}D_t^k[f(t)] \approx {}_{t-L}D_t^k[f(t)],\ (t > t_0 + L) \ \cdots\cdots \ (10)$$

From Fig. 12, empirically memorizing 10 past values should have good approximation. Clearly, in order to have a better approximation, smaller sampling time and longer memory length are preferable. A detailed discussion on fractional order controller's realization methods will be held in succeeding papers.

5. Experimental results

As depicted in Fig. 13, the experimental torsional system is controlled by a PC with 1.6GHz Pentium IV CPU and 512M memory. Realtime operating system RTLinux$^{\text{TM}}$ distributed by Finite State Machine Labs, Inc. is used to guarantee the timing correctness of all hard realtime tasks. The control programs are written in RTLinux C threads which can be executed with strict timing requirement of control sampling time. A 12-bit AD/DA multi-functional board is used whose conversion time per channel is $10\mu sec$.

Experiments on PID^k speed control are carried out with sampling time T=0.001sec and memory length of $L/T = 10$ (0.01sec). Two encoders (8000$pulse/rev$) are used as rotation speed sensors with coarse quantization $\pm 0.785 rad/sec$.

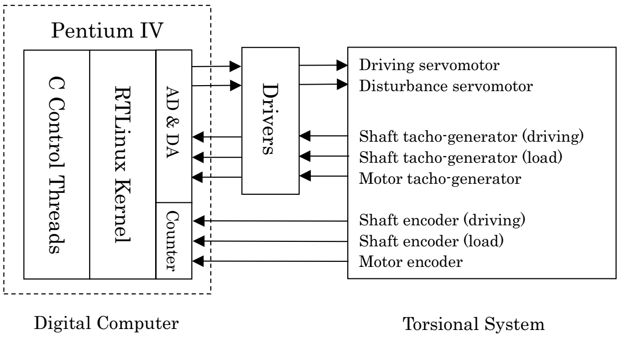
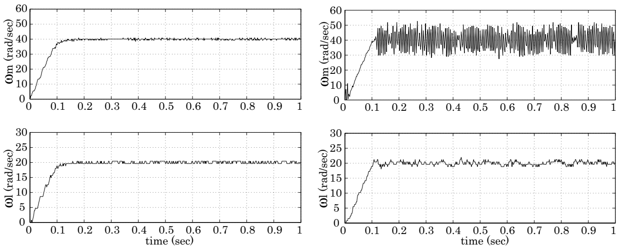


Fig. 13. Digital control system of the experimental setup

Since the driving servomotor's input torque command T_m has a limitation of maximum ± 3.84 Nm, K_i is reduced to 18.032 by trial-and-error to avoid large overshoot caused by the saturation. Firstly, integer order PID speed control experiment is carried out. As depicted in Fig. 14 the PID control system can achieve satisfactory response when the backlash angle is adjusted to zero degree ($\delta = 0$), while severe vibration occurs due to the existence of backlash non-linearity (see $\delta = 0.6$ case). This experimental result is consistent with the analysis in section 3.

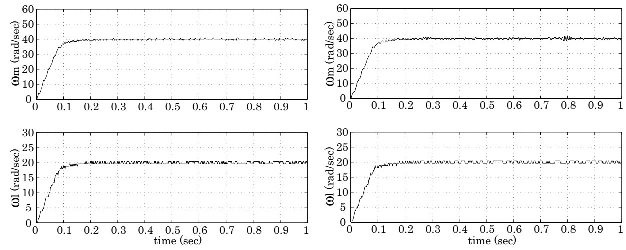


$\delta = 0deg.$ $\delta = 0.6deg.$

Fig. 14. Time responses of the integer order PID control

Figure 15 depicts the experimental results of fractional order PID^k control with 0.7 and 0.5 order D^k controllers. Severe backlash vibration in the integer order PID control case is effectively suppressed. The control system's stability and robustness against gear backlash non-linearity can be greatly improved by the FOC approach. $PID^{0.7}$ control system has a good robustness against backlash nonlinearity, while the error in the experimental response curves is for encoders' coarse quantization. The intermittent tiny vibrations in lower order 0.5 case are due to its relatively high gain near gear backlash vibration mode in open-loop frequency response.

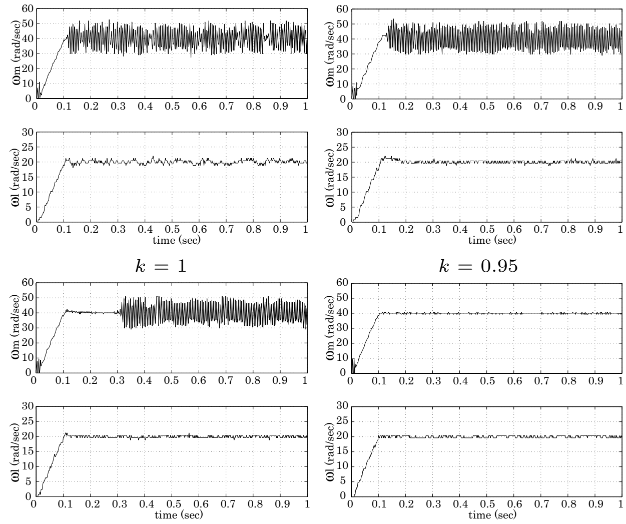
It is interesting to find the vibration suppression performance of fractional order PID^k control system shows somewhat "interpolation" characteristic. As depicted



$k = 0.7$ $k = 0.5$

Fig. 15. Time responses of PID^k control

in Fig. 16, PID^1 control has the most severe backlash vibration, while $PID^{0.85}$ is on the verge of instability. $PID^{0.95}$ and $PID^{0.9}$ have intermediate time responses. This experimental result is natural since these orders are continuous. The "interpolation" characteristic is one of main points to understand the superiority of FOC as providing more flexibility for designing robust control systems. At the same time, this experimental consistency with the logicality also verifies the good approximation of the realization method based on Short Memory Principle.



$k = 1$ $k = 0.95$

$k = 0.9$ $k = 0.85$

Fig. 16. Continuity of PID^k control's vibration suppression performance

6. Conclusions

In this paper, an integer order PID controller is firstly designed by CDM's standard form. Severe vibration occurs due to the gear backlash non-linearity. As mentioned in introduction part, there are several methods to suppress the backlash vibration of torsional systems. However fractional order PID^k control is an innovative proposal. In this paper, PID^k controller is designed in a manner that all the PID parameters are kept unchanged and only D controller's order is permitted to be adjusted fractionally. Experimental results show PID^k control system's improved robustness against backlash non-linearity compared to the PID control design based on CDM's standard form. A good approximation of the realization method is also verified in experiments. By changing D^k controller's order k, the control system's frequency response can be adjusted continuously. This flexibility can lead to straightforward design and better tradeoff between stability margin loss and the strength of backlash vibration suppression.

Even having a little higher hardware demand, cheaper design cost and better robustness of FOC demonstrated in this paper still highlight its promising aspects. Rapid development of computational power also makes fractional order controller's implementation not really problematic. On the other hand, applying FOC concept to motion control is still in a research stage. Future researches on FOC theory and its applications to more complex control problems are needed.

Finally the authors must point out the problem for integer order PID controller designed by using fixed gammas. Standard form was emphasized at CDM's early stage. In recent development, the selection of the gammas is recommended. Designers should choose proper gammas that can guarantee not only stability but also robustness. As discussed in Ref. (15), what is really needed for controller design of two-inertia resonant system is phase lag, not phase lead. For this reason, the value of D control becomes negative, which means phase lag. However positive feedback of D control will lead to poor robustness and should be avoided as much as possible in control design. Namely, the big value of negative K_d causes problem for the integer order PID control design in section 3. In fact a PI controller, where the assignment is $\gamma_1 = 2.5, \gamma_2 = 2$ and $K_d = 0$, should give a satisfactory performance in both stability and robustness. That is, in order to avoid positive feedback of D control, γ_3 can not be assigned and must be the value calculated under above assignment.

Therefore, care must be taken about this paper's purpose. It is not to claim PID^k controller as a good controller for three-inertia system, but to contribute to being a valuable experience for novel but still primitive FOC research. Especially this paper shows the possibility of better tradeoff between stability margin loss and the strength of backlash vibration suppression by FOC approach. This tradeoff is a common and natural problem in oscillatory systems' control [16]. For such kind of systems, a fractional order controller in the following form:

$$G_c(s) = \frac{K_p s + K_i}{(T_d s + 1)^k s} \quad \cdots\cdots (11)$$

is expected to be a general solution, where better tradeoff between stability margin and vibration suppression can be achieved by choosing proper fractional order k. Future research is required in this field.

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