# A Novel Nonlinear Disturbance Predictor based on Reconstructed Attractor for Motion Control System

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## Abstract

*We propose a novel nonlinear disturbance predictor to be used for motion control system. As an example, we develop a nonlinear disturbance predictor to compensate for the interference force of multiple axes robot manipulator. The reconstructed attractor which is often used in chaos analysis is applied to express and to predict disturbance. Using this predictor, high performance servosystem is realized without the exact plant model of robot dynamics. It is effective when there are unknown dynamics such as friction force or even chaotic behavior difficult to be modeled. Some simulation results using the double pendulum system are shown to verify the effectiveness of the proposed method.*

**Key words:**
robot manipulator, reconstructed attractor, nonlinear prediction

## 1 Introduction

Robot manipulator control has been studied by many researchers. The interference force between different axes consists of the Coriolis effect and Centrifugal force and so on is usually treated as disturbance. Although many methods have been proposed to compensate for this disturbance, a new controller which can reject disturbance accurately is still necessary, for many robot manipulators work nearer around human [1].

In this paper, a nonlinear disturbance predictor to compensate for disturbance is proposed. To design the predictor, the reconstructed attractor which is often used in chaos analysis is applied. To be more specific,

1. Estimation of the interference force with a disturbance observer
2. Drawing of the reconstructed attractor based on the time series of the estimated interference force
3. Prediction of disturbance from the reconstructed attractor

By using this predictor, a high performance servosystem taking robot dynamics into account is realized without using the exact plant model. Usually the computed torque method or even adaptive control method need the exact plant model for high performance control [2]. In this proposed method, the reconstructed attractor can have the plant dynamics as the form of state space trajectory instead of exact knowledge of the plant model.

Fig.1 shows a configuration of the system including the nonlinear predictor. By using this predictor it can be realized to compensate disturbance without time delay.

## 2 Nonlinear Prediction Using Reconstructed Attractor

### 2.1 Attractor in the State Space

When a dynamic system is drawn in $d$ dimensional state space, the state vector expresses the instantaneous state of the physical value and the state vector's trajectory expresses the whole physical system behavior[3].

In general, a discrete time state equation is written as (1) using $d$ dimensional state vector $\boldsymbol{x}$. In (1), $\boldsymbol{f}$ is the nonlinear dynamic equation, $\boldsymbol{g}$ the output equation and $\boldsymbol{y}$ the vector of observable state variables. A physically stable phenomenon is drawn as stable trajectory in a state space. In particular, a stable trajectory which attracts

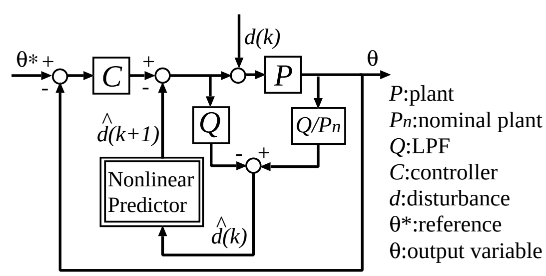



Figure 1. Configuration of the system including nonlinear predictor

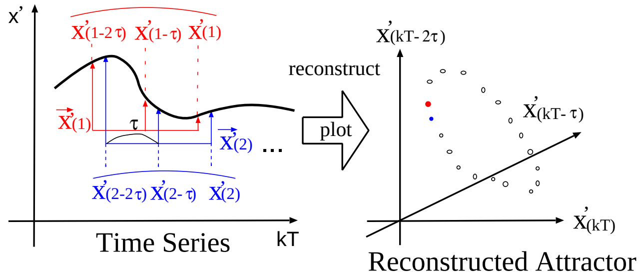



Figure 2. Concept of reconstructed attractor using time delay coordinates

neighboring trajectories is called an '**attractor**'[4].

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{f}(\boldsymbol{x}(k)) \\ \boldsymbol{y}(k) &= \boldsymbol{g}(\boldsymbol{x}(k)) \end{aligned} \quad (1)$$

### 2.2 Reconstructed Attractor

To draw an attractor, all of $\boldsymbol{x}$'s elements must be observable. But it is impossible in most cases. In this case, it is often helpful to use 'delay coordinate embedding'. We have here a typical example, where the time series of a single scalar variable $x_l$ is the only information that we can measure. The 'delay coordinate vector' is reconstructed as in (2) (Fig.2).

$$\begin{aligned} \boldsymbol{x}^{'}(k) &= [x_l(kT), x_l(kT-\tau), x_l(kT-2\tau), \\ &\quad \ldots, x_l(kT-(q-2)\tau), x_l(kT-(q-1)\tau)] \end{aligned} \quad (2)$$

Embedding theorems guarantee that, if $q \geq 2d+1$, the vector $\boldsymbol{x}$ is generically a global one-to-one representation of the system state [5]. However such a representation may be achieved experimentally if $\tau$ is appropriately chosen even when $q < 2d+1$.

Using the 'delay coordinate vector' in this way, the reconstructed attractor is drawn in $q$ dimensional state space.

### 2.3 Nonlinear Prediction

The original dynamic system is transcribed to (3) by the 'delay coordinate vector'. It is possible to predict $\boldsymbol{x}^{'}(k+1)$ from $\boldsymbol{x}^{'}(k)$ by obtaining the map $\hat{\boldsymbol{f}}(\boldsymbol{x}^{'}(k))$.

$$\begin{aligned} \boldsymbol{x}^{'}(k+1) &= \hat{\boldsymbol{f}}(\boldsymbol{x}^{'}(k)) \\ y(k) &= (1, 0, \cdots, 0)\boldsymbol{x}^{'}(k) \end{aligned} \quad (3)$$

Although there are several methods to obtain the map $\hat{\boldsymbol{f}}(\boldsymbol{x}^{'})$, Jacobian method is applied in this paper. Jacobian matrix $\boldsymbol{A}_k$ is fixed by $N$ neighboring state vectors near $\boldsymbol{x}^{'}(k)$ on the reconstructed attractor as in Fig.3. $\boldsymbol{x}^{'}(k+1)$ is predicted by the approximate linear equation(4) which is constructed with a Jacobian matrix and the nearest state vector $\bar{\boldsymbol{x}}^{'}(k)$.

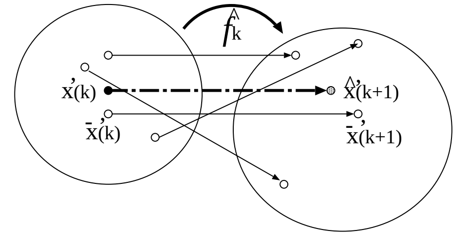



Figure 3. Prediction of the next step

$$\hat{\boldsymbol{x}}^{'}(k+1) = \boldsymbol{A}_k(\boldsymbol{x}^{'}(k) - \bar{\boldsymbol{x}}^{'}(k)) + \bar{\boldsymbol{x}}^{'}(k+1) \quad (4)$$

## 3 Nonlinear Disturbance Predictor For Manipulator Control

### 3.1 Dynamics of Double Pendulum

In this paper, a double pendulum shown in Fig.4 is considered. (5) is the dynamic equation of robot manipulator [6].

$$\boldsymbol{J}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{T}_{in} \quad (5)$$

Here, $\boldsymbol{J}(\boldsymbol{\theta})$ is the inertia matrix, $\boldsymbol{g}(\boldsymbol{\theta})$ is the gravitational force and $\boldsymbol{T}_{in}$ is the input torque. $\boldsymbol{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ represents the Coriolis effect and Centrifugal force to take the form of (6).

$$\boldsymbol{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) = \dot{\boldsymbol{J}}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} - \frac{1}{2}\frac{\partial}{\partial \boldsymbol{\theta}}(\dot{\boldsymbol{\theta}}^T \boldsymbol{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}) \quad (6)$$

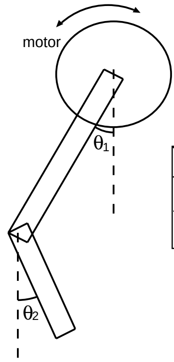
The interference force $\boldsymbol{T}_d$ is estimated by the disturbance observer. By using the disturbance observer each axis can be treated as an independent single pendulum. The interference force between different axes and the gravitational force can be treated as disturbance as given by the form of (8).

$$diag\boldsymbol{J}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} = \boldsymbol{T}_{in} - \boldsymbol{T}_d \quad (7)$$

$$\boldsymbol{T}_d = (\boldsymbol{J}(\boldsymbol{\theta}) - diag\boldsymbol{J}(\boldsymbol{\theta}))\ddot{\boldsymbol{\theta}} + \boldsymbol{c}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \boldsymbol{g}(\boldsymbol{\theta}) \quad (8)$$

(8) is converted to the state equation (9) in the discrete time domain. From (9), disturbance to the first axis has some dynamics given by $\hat{\boldsymbol{f}}_{d,1}$.

$$T_{d,1}(k+1) = \hat{\boldsymbol{f}}_{d,1}(T_{d,1}(k)) \quad (9)$$




| | length | weight |
|---|---|---|
| arm1 | 10[cm] | 0.200[kg] |
| arm2 | 8[cm] | 0.100[kg] |

Figure 4. Model of the double pendulum and its parameters

If the prediction of $T_{d,1}(k+1)$ from $T_{d,1}(k)$ is realized, it is achieved to feedback the estimated disturbance without any time delay. But the map $\hat{\boldsymbol{f}}_{d,1}$ is too complicated to be formulated usually.

In practice, the 'delay coordinate vector' is constructed from the time series of the estimated disturbance as in (2). The reconstructed attractor is expected to be drawn even if the discrete state equation (9) might not be obtained.

### 3.2 Simulation Result

The sinusoidal curve reference angle is given to the first axis and its angle is controlled. The time series of the first axis $\theta_1$ and the second axis $\theta_2$ are shown in Figs.5 and 6. The interference force from the second axis to the first axis is shown in Fig.7.

The second axis is fluctuated by the interference force even if no torque is applied (Fig.6). In other words the first axis is interfered by the action-reaction effect from the second axis. This is not a problem if the interference force varies slowly enough to be suppressed by the position controller. But quick interference force is not suppressed by the existing feedback controller. The reconstructed attractor is drawn from the time series of disturbance (Fig.7) and future disturbance is predicted.

The reconstructed attractor is obtained with a dimension of 3, i.e., $q = 3$ in (2), and a delay time of 60[**ms**] (Fig.8). As the beautiful reconstructed attractor is drawn in Fig.8, we can see that the 3 dimensional delay coordinate vector has some dynamics and the prediction will be effective.

The error between the true disturbance and the estimated disturbance by the proposed method with one step ahead prediction is shown in Fig.9. In addition, the error between the true disturbance and the estimated disturbance by first order approximation (10) with one step ahead prediction is also shown.

$$\hat{x}^{'}(k+1) = \frac{d\hat{f}}{dx^{'}}(x^{'}(k) - x^{'}(k-1)) + x^{'}(k) \qquad (10)$$

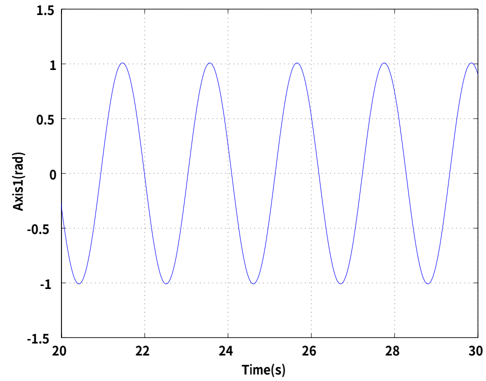



Figure 5. Time series of $\theta_1$ of the double pendulum

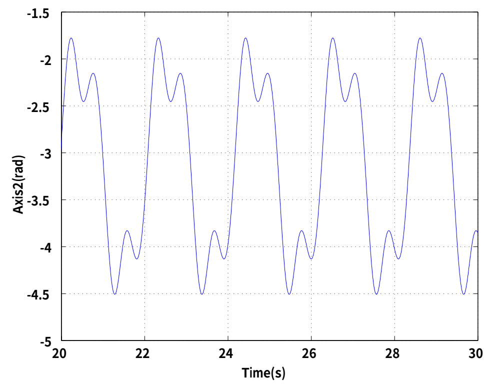



Figure 6. Time series of $\theta_2$ of the double pendulum

The error between the true disturbance and the estimated disturbance by the proposed method with twenty steps ahead prediction is shown in Fig.10, likewise, the error between the true disturbance and the estimated disturbance by first order approximation with twenty steps ahead prediction is also shown.

In Figs.9 and 10, when the one step ahead disturbance is predicted, the proposed method and the existing method give similar results. But when the twenty steps ahead disturbance is predicted, the predicted by the proposed method is superior to that obtained by the existing method, because first order approximation use just a linear model. On the contrary, the nonlinear disturbance predictor involves the dynamics of disturbance.

Through these simulation results, it is proved that the proposed method is effective for the system where the sampling time is long and the system which disturbance varies quickly.

## 4 Conclusion

In this paper, the nonlinear disturbance predictor using the reconstructed attractor is proposed. In the simulation of a double pendulum model, the

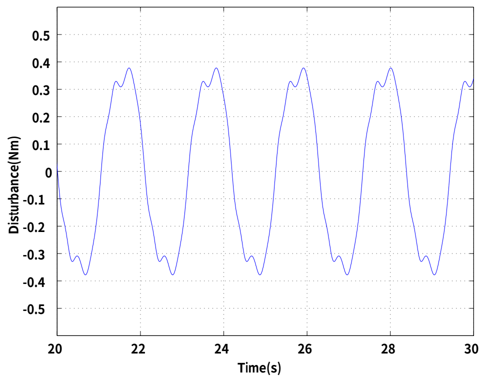



Figure 7. Disturbance input to the axis1 estimated by the disturbance observer

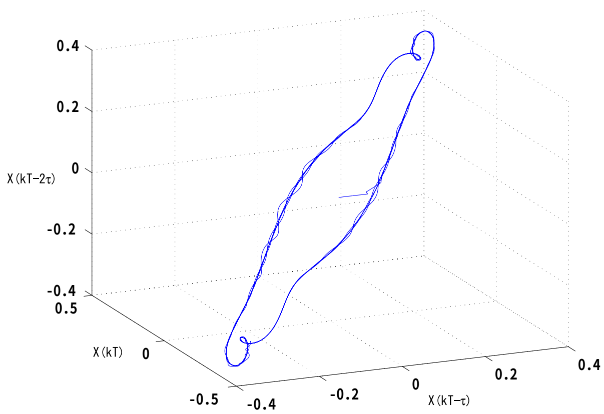



Figure 8. Three dimensional reconstructed attractor of disturbance at axis1

reconstructed attractor is drawn from estimated disturbance. We succeeded in predicting future disturbance with one step ahead and twenty steps ahead. Although only periodic disturbance is treated in this paper's simulation, this predictor is effective even if there are unknown dynamics such as friction force or even chaotic behavior which is difficult to be modeled. By feeding back the predicted disturbance, any disturbance can be compensated for almost completely without any time delay.

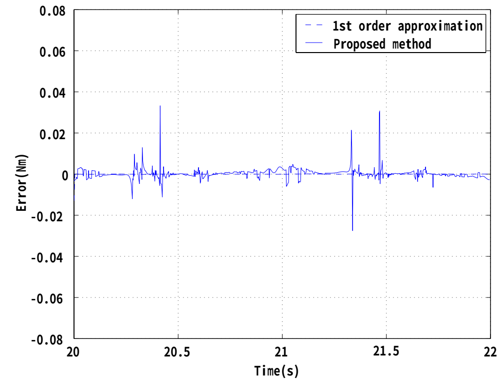



Figure 9. Error between the true disturbance and one sample time prediction by the proposed method with 1st order approximation

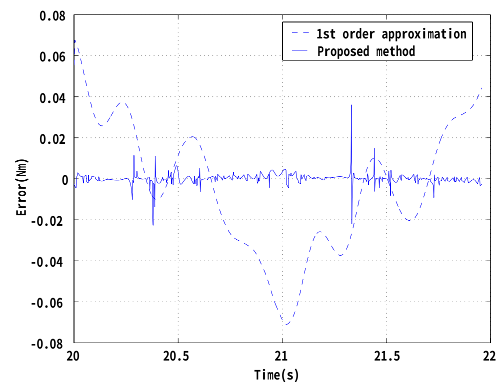



Figure 10. Error between true disturbance and twenty sample times prediction by the proposed method with 1st order approximation

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