Resolving Actuator Redundancy for 4WD Electric Vehicle by Sequential Quadratic Optimum Method

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Force and moment control distribution or allocation was a method that was used to design control system for over actuated electric vehicle. This kind of method was quite suitable for resolving the actuator redundancy when distributed the given force or moment command among available sets of actuators. In this paper, dynamic control distribution method, which was based on sequential quadratic optimum method, was discussed and used to control an electric vehicle which was equipped with four in wheel motors. The control objective was that the vehicle was controlled to track a desired yaw rate trajectory while sideslip angle was minimized. The upper control unit specified the required yaw moment by optimal feedback control method. And then the distribution controller allocated the yaw moment by determining appropriate slip ratio of individual wheel. According to the slip ratio command, the lower control unit controlled the in-wheel motors. Simulation results showed that the control strategy which used redundancy resolving algorithm quiet well enhanced the stability of yaw dynamics of EV.

Keywords: Electric Vehicle, Actuator Redundancy, Sequential Quadratic Optimum, Yaw Rate Control

1. Introduction

For reasons of environmental protection and energy conservation, electric vehicle who used electric motors as the basis of its operation was expected to replace the conventional internal combustion engine vehicles (ICV) in the near future [(1)].

On the other hand, since torque generation of electric motor is very quick and could be easily measured and accurately controlled, it was more convenient to improve the motion dynamics of EV through control way. Furthermore, some sophisticated configurations which made full advantagies of electric motors were developed for EV. For example, electric motors could be integrated into each wheel and be controlled independently [(2)]. Based on those merits, many control methods were broght out for EV to obtain well maneuverability, stability and high power efficiency.

However when the number of operating motors exceeded the number of states being controlled, the EV would be over actuated. For example, if a vehicle had four drive motors and its yaw rate was stabilized by direct yaw moment control method, different combinations of traction force generated by controlling motors could produce the same desired yaw moment. As Fig1. showed, an electric vehicle named "UOTII" has four-in-wheel motors, which might bring such problem. From this view point, there was a redundancy of drive motors.

Althouth redundancy made control strategies more complex, it also brought chances to choose an optimum solution which can minimize the control effort. Besides that, when an operating motors failure, the drive system can be reconfigurated due to the redundancy.

Fig. 1. "UOT II" with four in wheel motors

Redundancy resolution was generally used for the space vehicle control or marine vessel control. It have previously been used in ref [(3)] [(4)] for automotive vehicles control. Current methods for solving redundancy of actuator were static mapping methods based on least square method, which include redistributed pseudo-inverse, approximate quadratic programming method, the fixed-point method and so on. Although those methods were easily implemented and computationally efficient, they do not consider constraints of actuators'. And can not obtain the unique results every where in constrained domain [(5)]. Recently, dynamic resolving strategies based on sequential quadratic optimum method have been proposed. Those strategies try to find the exact optimal solutions in a finite number of steps.

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This paper was main focused on the algorithm of sequential programming method and its use for EV control. The paper consists of several parts: first part was the redundancy analysis and dynamic resolution. The second part was the application of dynamic redundancy resolution method for optimal control design for 4WD EV. And the last part was the simulation and conclution.

2. Redundancy analysis and dynamic optimal resolution

2.1 **Redundancy analysis** Considered the linear state-space model of a system

$$\begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix} = A_{n\times n} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + B_{n\times m} \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix} \cdots\cdots (1)$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_p \end{pmatrix} = C_{p\times n} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdots\cdots (2)$$

where $x \in R^n$ was the system state vector. $u \in R^m$ was the control input vector, and $n < m$. $y \in R^p$ was the system output to be controlled.

Assume that $Rank(B) = n < m$. A group of linear derivative equations could be obtained by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_n \end{pmatrix} = \underbrace{\begin{pmatrix} a_{11} \dots a_{1n} \\ a_{21} \dots a_{2n} \\ a_{31} \dots a_{3n} \\ \vdots \ddots \vdots \\ a_{n1} \dots a_{nn} \end{pmatrix}}_{A_{n\times n}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \cdots\cdots (3)$$

$$+ \underbrace{\left(\begin{array}{ccc|ccc} b_{11} & \dots & b_{1n} & b_{1(n+1)} & \dots & b_{1m} \\ b_{21} & \dots & b_{2n} & b_{2(n+1)} & \dots & b_{2m} \\ b_{31} & \dots & b_{3n} & b_{3(n+1)} & \dots & b_{3m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{nn} & b_{n(n+1)} & \dots & b_{nm} \end{array}\right)}_{B_{n\times m}} \left(\begin{array}{c} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \\ \hline u_{n+1} \\ \vdots \\ u_m \end{array}\right)$$

Since the matrix $B_{n\times m}$ was not square, but full raw rank, as Eq(4) showed, colums of matrix B from $n+1$ to m were not linear independent and span a null space. The corresponding control inputs from u_{n+1} to u_m had no effect on the state variables $x_i, i = 1, ..., n$. Those control inputs $u_{n+1}, \dots, u_m$ were redundant.

Such redundancy could be used to combine different control inputs $u_i, i = 1, ..., m$ to obtain the same effect on state vector.

Since B is rank deficient, it can be factorized as

$$B_{n\times m} = B_v B_u \cdots\cdots (4)$$

where $B_v \in R^{n\times n}$ and $B_u \in R^{n\times m}$ both have rank n.

Introducing the virtual control effector

$$\begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} = B_u \begin{pmatrix} u_1 \\ \vdots \\ u_m \end{pmatrix} \cdots\cdots (5)$$

where $v \in R^n$, we can rewrite the systems dynamics Eq(1) as

$$\begin{pmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{pmatrix} = A_{n\times n} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + B_v \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix} \cdots\cdots (6)$$

As control design for an over actuated system, according to Eq(1)(6), variables $v_i, i = 1, ..., n$ should be looked on as total control effectors, which were specified by an upper control unit using state feedback control law and created by combinations of control inputs $u_j, j = 1, ..., m$ later.

Control effectors were used to keep the whole system stability, robust and obtain some steady and dynamic characteristics. For example, in order to control the yaw behavior of EV and keep maneuverbility, the control effectors such as added yaw moment, active front steering angle or active rear steering angle were required. For the direct yaw moment control of EV, as shown in Fig3, the state variables include slip angle of vehicle and yaw rate. The control inputs could consist of active steering angle and traction force command of each driving motor. The virtual control effector could be looked on as added yaw moment Mz, which might be calculated by state feedback.

$$v = \begin{pmatrix} \triangle\delta_f \\ Mz \end{pmatrix} \cdots\cdots (7)$$

$$Mz = K \begin{pmatrix} \beta - \beta^{ref} \\ \gamma - \gamma^{ref} \end{pmatrix} \cdots\cdots (8)$$

where

K	......	gain matrix of P-controller
β	......	vehicle slip angle
γ	......	yaw rate
$\triangle\delta_f$	......	active steering angle

According to Eq(5)(4), given the control effectors $v_i, i = 1, ..., n$, an optimal method was used to obtain the control input $u_i, i = 1, ..., m$ as follows

$$u = arg \min_{u\in\Omega} \|W_u(u - u_d)\|_2 \cdots\cdots (9)$$

$$\Omega = arg \min_{u_{min} \le u \le u_{max}} \|Wv(B_u u - v)\|_2 \cdots\cdots (10)$$

Here, u_d is the desired control inputs and W_u, W_v are weighting matrices.

Eq(9)(10) could be interpreted as follows [8]: When given Ω, the set of feasible control inputs that minimize $B_u u - v$ (weighted by W_v), pick the control inputs $u_i, i = 1, ..., m$ that minimizes $u - u_d$ (weighted by W_u).

u_d, W_u, W_v are design parameters. W_u allows for operating actuator prioritization, i.e., which u_i should be used primarily. Similarly, W_v allows for prioritization among the control effectors $v_i, i = 1, ..., n$.

Redundacy analysis and its resolution concept mentioned above could be summarized as what Fig2. showed.

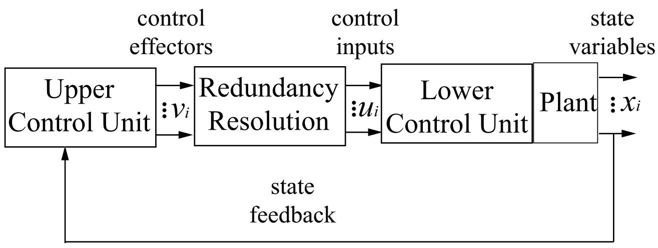

Fig. 2. Outline of redundancy resolution concept

2.2 **Dynamic redundancy resolution using sequential quadratic programming** Several methods, such as direct control allocation, daisy chaining and redistributed pseudoinverse have been proposed for redundancy resolutions in literatures [6] [7]. According to Eq(5), the control inputs $u_i, i = 1, ..., m$ could be calculated as

$$\begin{pmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{pmatrix} = f(B_u) \begin{pmatrix} v_1(t) \\ \vdots \\ v_n(t) \end{pmatrix} \cdots\cdots\cdots\cdots (11)$$

However these methods are static resolutions, which mapped from control effectors v to control inputs u by $f(B_u)$. The mapping is a kind of inverse of matrix B and not a time variable one.

Using a static mapping, the frequency of command change for operating motors is not considered. The dynamics, especially response characteristics of actuator are not considered. So, it need to use a dynamic mapping as follows

$$u(t) = f[v(t), u(t-T), v(t-T), ..., u(t-kT), v(t-kT), ...] \cdots\cdots (12)$$

According to Eq(9)(10), dynamic redundancy resolution using sequential quadratic programming can be formulated as constrainted quadratic programming problem.

$$\min_{u(t)} ||W_1(u(t) - u_d(t))||_2 + ||W_2(u(t) - u(t-T))||_2 \cdots\cdots\cdots (13)$$

subject to

$$v(t) = B_u u(t) \cdots\cdots\cdots\cdots\cdots\cdots (14)$$

$$u_{min} \leq u \leq u_{max} \cdots\cdots\cdots\cdots\cdots (15)$$

Where, $u_d(t)$ is the desired control inputs and W_1, W_2 are weighting matrices.

The basic idea of sequential quadratic programming (SQP) resolution is to solve quadratic programming subproblem in each iteration [9]. And in this paper, Eq(12) was simplified as a first order filter as follows

$$u(t) = \tau_u u(t-T) + \tau_v v(t) \cdots\cdots\cdots\cdots (16)$$

Where τ_u was time delay coefficient and τ_v was proportional coefficient.

3. Application of dynamic redundancy resolution method for optimal control design for 4WD EV

3.1 **Model of motorized vehicle with four wheels** The vehicle model was shown as Fig3. Consider in the linear case, the lateral dynamic of EV

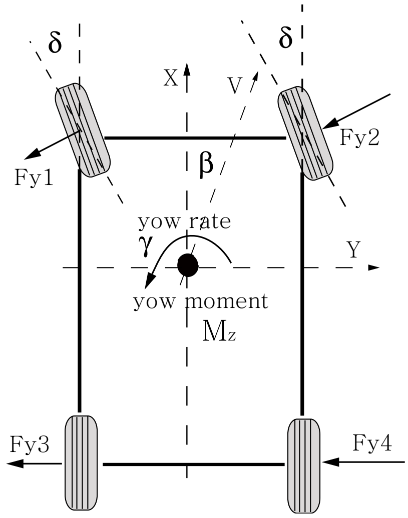

Fig. 3. Motorized vehicle with four wheels

was expressed as follows:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \cdots\cdots\cdots\cdots\cdots\cdots (17)$$

where

$$\mathbf{x} = [\beta, \gamma]^T, \mathbf{u} = [\delta_f, M_z]^T \cdots\cdots\cdots\cdots (18)$$

$$A = \begin{bmatrix} -2\frac{C_f + C_r}{MV} & -1 - 2\frac{l_f C_f - l_r C_r}{MV^2} \\ -2\frac{l_f C_f - l_r C_r}{I} & -2\frac{l_f^2 C_f + l_r^2 C_r}{IV} \end{bmatrix} \cdots (19)$$

$$B = \begin{bmatrix} 2\frac{C_f}{MV} & 0 \\ 2\frac{l_f C_f}{I} & \frac{1}{I} \end{bmatrix} \cdots\cdots\cdots\cdots (20)$$

where β is slip angle which between the chassis's velocity and wheel's velocity. γ is the yaw rate. δ_f is the front steer angle. M_z is the yaw moment. m is the vehicle mass and I is the moment of inertia about the Z axis, C_f and C_r is the front and rear wheel cornering stiffnesses which assumes a linear relationship between the wheel slip angle and the lateral force generated by the tires.

3.2 **Optimal control design** The control objective was to track a dedsired yaw rate trajectory while minimizing vehicle sideslip. The control algorithm was shown in Fig4. There were three main parts included in this algorithm. First is the force feedfoward control, second is the state feedback control and the last is the distribution control using sequential quadratic programming method. The feedforward controller was designed as a P-controller

$$M_{ff} = k_f \delta \cdots\cdots\cdots\cdots\cdots\cdots (21)$$

Feedback controller was designed as a linear quadratic regulator (LQR) [9]. The state variables, which were γ and β, were fed back. The control effector v was calculated as

$$\min_{u} \int_0^{\infty} ((x - x^*)^T Q_1 (x - x^*)$$
$$+(v - v^*)^T R_1 (v - v^*))dt \cdots\cdots\cdots\cdots (22)$$
$$v = k_q x \cdots\cdots\cdots\cdots\cdots\cdots (23)$$

where Q_1 and Q_2 were weithting matrics. x^* and v^* were desired values. k_q was the solution of Ricatti equation, which was calculated as

$$A^T k_q + k_q A + Q_1 - k_q B R_1^{-1} B^T k_q = 0 \cdots\cdots (24)$$

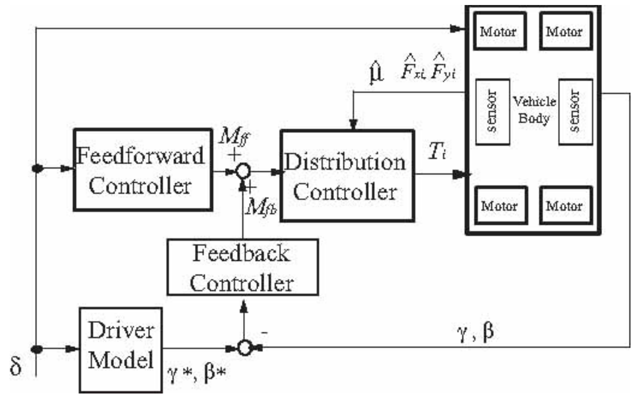

Fig. 4. Optimal control algorithm for vehicle

The distribution controller, which used sequential quadratic programming method, calculated an optimal combination of input commands u_i for motors. When those commands applied to the vehicle, it will produce the desired control effector v.

As for controller design in this paper, given a commanded braking moment M_z from the feedforward and feedback controller, the primary goal of the control distribution module was to obtain $M = M_z$ by commanding the appropriate wheel slip ratio to each of the four wheels. In particular, it make sense to apply as small wheel slip ratio as possible.

4. Simulation Result

A double lane change manoeuver test of "UOTII" was simulated. The input steering angle $\delta$was a sine wave signal. The desired yaw rate was calculate by using desired driver model and result was shown as line in Fig5. The vehicle was simulated at a constant velocity of $45m/s$. The friction coefficient μ was 0.8. The controlled yaw rate was shown as line in Fig5.

5. Conclusion

Resolving redundancy methods were examined in this paper. An optimal design outline used for EV control was presented. An explicit approximate optimal calculation based on the sequential quadratic progarmming

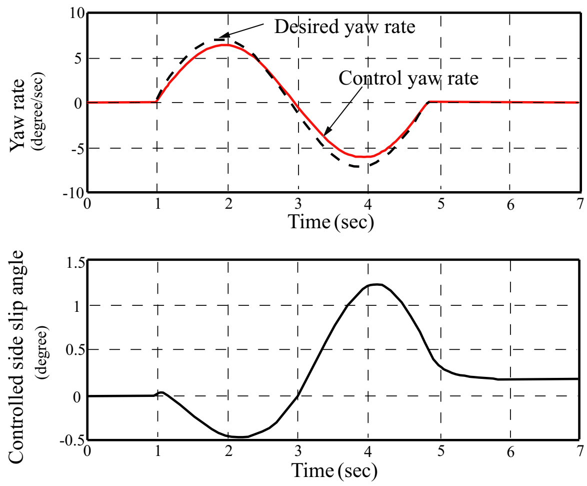

Fig. 5. Lane change simulation result of yaw rate control

was simulated. By the simulation results, it was shown that the stability of the yaw dynamics was enhanced by the use of proposed controller.

To motion control of the over actuated EV, this kind of redundancy would bring quiet advantages, especially during critical situations to retain controlability of the EV over a wider operating domain.

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