Prevention of Overturn of Power Assisted Wheelchair using Novel Stability Condition

Wei Li, Naoki Hata, Yoichi Hori

Department of Electrical Engineering
The University of Tokyo
Meguro 4-6-1, Tokyo, 153-8505, JAPAN
lucy@horilab.iis.u-tokyo.ac.jp, hata@horilab.iis.u-tokyo.ac.jp, hori@iis.u-tokyo.ac.jp

Abstract— **Last 10 years we have been experiencing a constant evolution in the field of power assisted wheelchairs, which allows more flexibility of use for disabled people. However, motor output is quite big, and without proper control, power assisted wheelchair are much easier to fall backward than traditional ones. Therefore, in this work, to prevent the overturn, three kinds of stability conditions, including the novel one based on Lagrange Equation have been proposed. They are compared by using experimental data and the best one is adopted. Its validity, as well as practicality, is authenticated by experiments on both horizontal plane and slope.**

I. Introduction

A power assisted wheelchair is lightweight, maneuverable, and simple to transport. It represents an entirely new class of wheelchair, and is studied by several researchers. However, motor output is quite big, and without proper control, power assisted wheelchair is much easier to fall backward than traditional ones. Meanwhile, the dynamics of wheelchair is nonlinear and till now there is no practical technique to prevent the overturn of power assisted wheelchair.

In this work, after introducing present assist technique [2] as well as the system model, three kinds of dynamical stability criteria on horizontal plane will be given. The third one is deduced from Lagrange Equation. It is more sensitive, and much easier for calculation. Experimental results are provided to verify these three kinds of criteria. Later, stability criteria on slope will be analyzed. Finally the data of experiment on slope will be given.

II. Power Assisted Wheelchair System

Large numbers of power assisted wheelchairs are based on two independently driven rear wheels and two front ones. Batteries and two BLDC motors are used to augment the power applied by the user to one or both pushrims.

In this work, the power-assisted wheelchair JWII made by YAMAHA in 1996 (Fig. 1), is used. In the following, present assist technique and system model will be introduced.

Fig. 1. The outlook of power-assisted wheelchair JWII.

A. Present Assist Technique

The assist force of motor is not just times of human torque. The reason is that if assist force descents to zero right after operator stops, the operator cannotfeel been assisted. So, as in [2], low pass filter is used for calculating assist force (1).

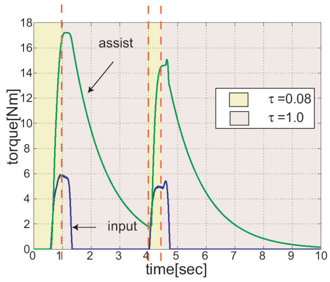

Fig. 2. Relationship of Torque Input and Assist.

$$T_{assist} = \alpha \frac{1}{1+\tau s} T_{human} \qquad (1)$$

Here α is power-assistance-ratio, T_{assist} is the assist force, T_{human} is the input torque from the pushrim, and τ is the time constant. For better ride quality, the ascent should be fast and the descent slow. So in this work time constant τ is decided as the following (2),

$$\tau = \begin{cases} \tau_{fast} = 0.08[s], & \frac{d}{dt}T_{human} > 0; \\ \tau_{slow} = 1.0[s], & \frac{d}{dt}T_{human} < 0. \end{cases} \qquad (2)$$

B. System Model

When a person is sitting, COM of the him and the wheelchair frame is supposed to be at the surface of his abdomen. And by neglecting two front wheels, the model of wheelchair can be represented as Fig. 3.

Please note that although two front wheels are not shown in figure, operator cannot fall "forward", and on horizontal plane there is $\varphi_0 \geq \varphi$.

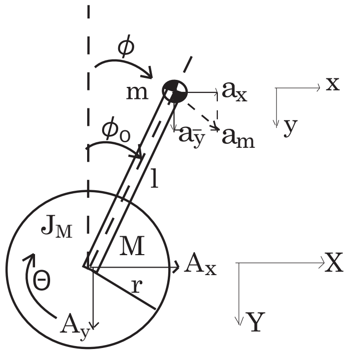

Fig. 3. Wheel Model.

M	:	*Mass of the driving wheel*
m	:	*Mass of operator including wheelchair frame*
r	:	*Radius of the driving wheel*
J_M	:	*Moment of Inertia of the driving wheel*
J_m	:	*Moment of Inertia of operator and frame*
θ	:	*Rotational angle of the driving wheels*
φ	:	*Angle of the vertical to COM*
φ_0	:	*Initial angle of the vertical to COM*
a_x, a_y	:	*Accelerations of COM*
A_x, A_y	:	*Accelerations of the wheel*

III. Dynamical Criteria of Stability

A. Moment Criteria

Assume clockwise moment positive, counterclockwise negative, moment criteria of the system can be written as

$$m(a_y + g)l \sin\varphi - m(A_x - a_x)l \cos\varphi > 0 \tag{3}$$

Usually, when the total moment of system equals 0, system is stable. However, in this work, even when total moment is larger than 0, system is still stable because of two front wheels.

B. ZMP (Zero Moment Point) Criteria

The idea of ZMP was introduced by Mr. Vukobratovic at 1969 and 1972 [3]. ZMP (Zero Moment Point) is determined by the movement and gravity of the object. As shown in Fig. 4, the broomstick is influenced by gravitation and inertia force. These combined forces are called the total inertial force. Also, the point where the floor reaction force operates is called the floor reaction point. The intersection of the floor and the axis of the total inertial force have a total inertial force moment of 0, so it is called the Zero Moment Point. It is clear that when ZMP and floor reaction point are the same, system is stable [4].

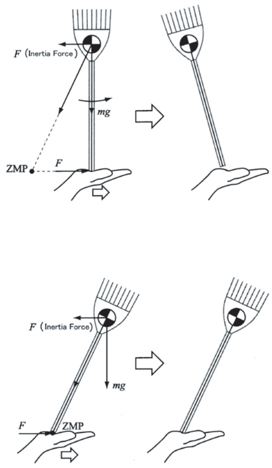

Fig. 4. Standing a Broomstick Upright.

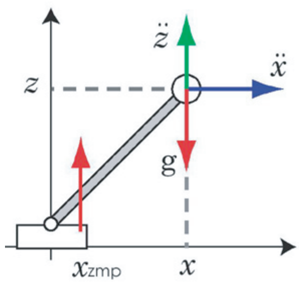

Fig. 5. Simple ZMP calculation model.

Fig. 5 in [5] shows the simplest case, and ZMP of wheelchair system can be calculated as (4).

$$\begin{aligned} X_{ZMP} &= \frac{\sum_{i=1}^{n} m_i[x_i(\ddot{z}_i + g) - z_i\ddot{x}_i]}{\sum_{i=1}^{n} m_i(\ddot{z}_i + g)} \\ &= \frac{m[l \sin\varphi(a_y + g) - l\cos\varphi a_x]}{m(a_y + g)} \\ &= l\sin\varphi - \frac{l\cos\varphi}{a_y + g}(A_x - a_x) > 0 \end{aligned} \tag{4}$$

Or

$$(a_y + g)l\sin\varphi - (A_x - a_x)l\cos\varphi > 0 \tag{5}$$

Remind of (3), it is clear that moment criteria and ZMP criteria are exactly the same.

C. Lagrange Criteria

The kinetic energy T of that system model can be calculated as below.

$$\begin{aligned} T = &\ \textit{Translational kinetic energy of the driving wheel} \\ &+ \textit{Rotational kinetic energy of the driving wheel} \\ &+ \textit{Translational kinetic energy of the body} \\ &+ \textit{Rotational kinetic energy of the body} \end{aligned}$$

The first two items are quite easy to calculate, while the other two are difficult. Assume the velocity of COM is v_G and its horizontal component is v_{mx} while vertical component is v_{my}. Then the translational kinetic energy of body is $\frac{1}{2}mv_G^2$. From the Pythagorean theorem, it becomes

$$v_G^2 = v_{mx}^2 + v_{my}^2. \tag{6}$$

Meanwhile, v_{mx}, v_{my} can be calculated by the following equations.

$$\begin{cases} v_{mx} &= \frac{d}{dt}(x + l\sin\varphi) \\ v_{my} &= \frac{d}{dt}l\cos\varphi \end{cases} \tag{7}$$

Also, the rotational kinetic energy of the body is $\frac{1}{2}J_m\dot{\varphi}^2$. So the total kinetic energy T is

$$T = \frac{1}{2}Mx^2 + \frac{1}{2}mv_G^2 + \frac{1}{2}J_M\dot{\theta}^2 + \frac{1}{2}J_m\dot{\varphi}^2 \tag{8}$$

Here, there exists a constraint as

$$r\theta = x. \tag{9}$$

To calculate the potential energy U, we assume that the horizontal plane passing through the rear axle as the potential energy reference plane.

$$U = mgl\cos\varphi \tag{10}$$

Now the Lagrangian of the system turns out to be

$$\begin{aligned} L &= T - U \\ &= \tfrac{1}{2}Mx^2 + \tfrac{1}{2}mv_G^2 + \tfrac{1}{2}J_M\dot{\theta}^2 + \tfrac{1}{2}J_m\dot{\varphi}^2 - mgl\cos\varphi \\ &= \tfrac{1}{2}(M+m)r^2\dot{\theta}^2 + \tfrac{1}{2}ml^2\dot{\varphi}^2 + mlr\dot{\theta}\dot{\varphi}\cos\varphi \\ &\quad + \tfrac{1}{2}J_M\dot{\theta}^2 + \tfrac{1}{2}J_m\dot{\varphi}^2 - mgl\cos\varphi \\ &= \tfrac{1}{2}[(M+m)r^2 + J_M]\dot{\theta}^2 + \tfrac{1}{2}(J_m + ml^2)\dot{\varphi}^2 \\ &\quad + mlr\dot{\theta}\dot{\varphi}\cos\varphi - mgl\cos\varphi \end{aligned}$$

And the equations of motion can be obtained from Lagrange's equations,

$$\tau + d\theta = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} + B_M\dot{\theta} \tag{11}$$

$$-\tau + d\varphi = \frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \frac{\partial L}{\partial\varphi} + B_m\dot{\varphi} \tag{12}$$

Here $d\theta$ and $d\varphi$ are disturbances, and $B_M\dot{\theta}$, $B_m\dot{\varphi}$ are resistances. Or more concisely,

$$\begin{aligned} \tau + d\theta = {} & [(M+m)r^2 + J_M]\ddot{\theta} + mlr\ddot{\varphi}\cos\varphi \\ & - mlr\dot{\varphi}^2\sin\varphi + B_M\dot{\theta} \end{aligned} \tag{13}$$

$$\begin{aligned} -\tau + d\varphi = {} & (Jm + ml^2)\ddot{\varphi} + mlr\ddot{\theta}\cos\varphi \\ & - mgl\sin\varphi + B_m\dot{\varphi} \end{aligned} \tag{14}$$

From above we can derive the third stability criteria now. For simplicity, we just take into account the wheelchair system model with no wheelie, which means that $\varphi = \varphi_0, \dot{\varphi} = 0, \ddot{\varphi} = 0, d\varphi = 0$. (The experimental results later will show

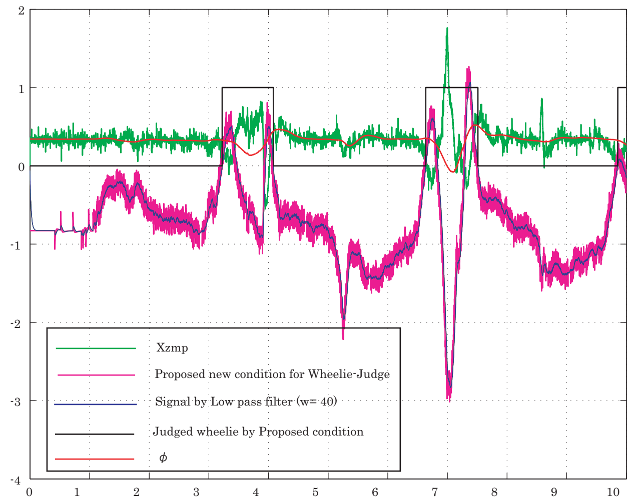

Fig. 6. Experiment with 2 times of wheelie.

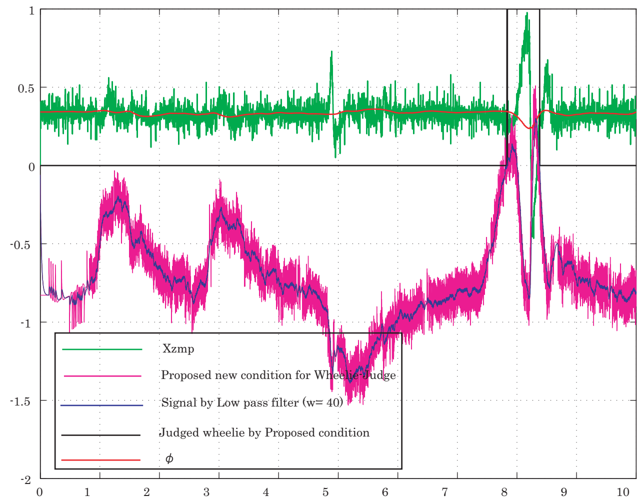

Fig. 7. Experiment with 1 time of wheelie.

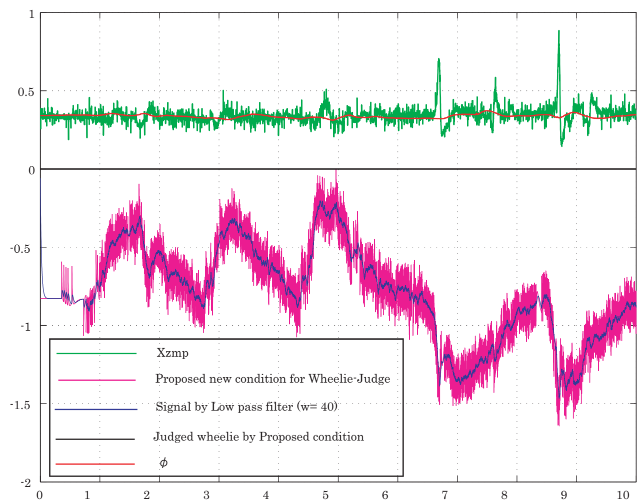

Fig. 8. Experiment with no wheelie.

such simplification is feasible and valid.) Equation (14) turns out to be

$$mlr\ddot{\theta}\cos\varphi_0 - mgl\sin\varphi_0 + \tau = 0 \tag{15}$$

Based on (15), we calculated by using the experimental data, and the results are shown in Fig. 6,7 and 8.

So it is clear that when

$$\tau < mgl\sin\varphi_0 - mlr\ddot{\theta}\cos\varphi_0 \tag{16}$$

is satisfied, there is no wheelie and wheelchair system runs safely. Here in this work it is called Lagrange Criteria.

D. Remark

It has been stated before that moment criteria and ZMP criteria are essentially the same. On the contrast, Lagrange criteria are much simple, with only two variables, and are much easier to calculate. Furthermore, in the former 2 criteria, information of accelerations are used while in the Lagrange criteria, information of torque is used. Acceleration is the result of torque, and as a result Lagrange criteria are much "sensitive" and changes quicker. Most importantly, element "Torque" is included and wheelie can be prevented by regulating it directly.

IV. Experiment

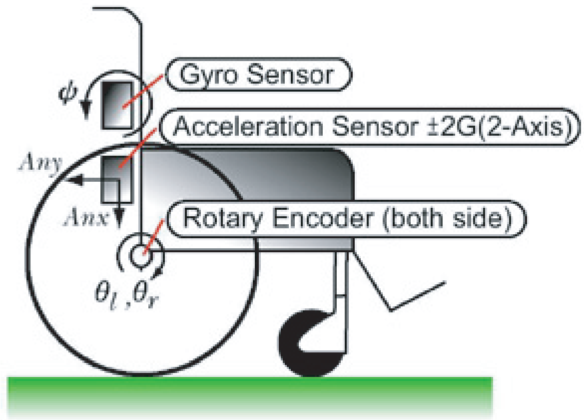

Fig. 9. The construction of sensor system.

As shown in Fig. 9, in this system there are 5 sensors for: rotating speed (ψ) of the wheelchair frame, vertical and horizontal accelerations (A_{nx}, A_{ny}), angle of both wheels (θ_l, θ_r).

Experiments on horizontal plane have been carried out and the limit of torque is calculated. During the experiment, if human and motor torque is bigger than this limit, motor output decreases so that the total torque is equal to the limit. Data are shown in Fig. 10-12.

It is obvious that φ vibrates around φ_0, but it never exceed 0. Similarly, Xzmp does not cross over 0, it can be said that there is no wheelie during the experiment.

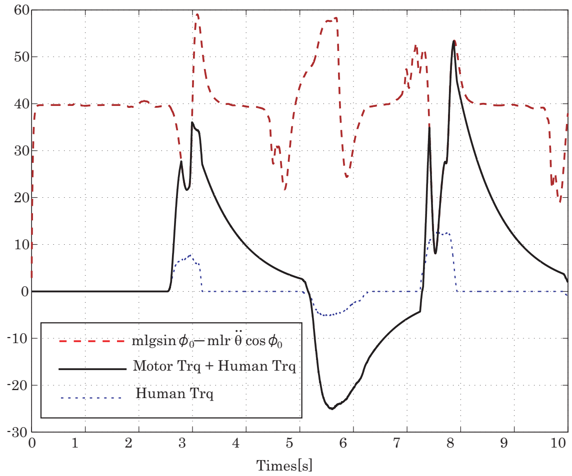

Fig. 10. Experiment Data.

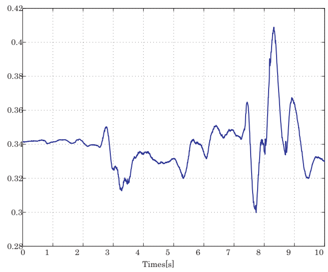

Fig. 11. φ (Integration of Gyro).

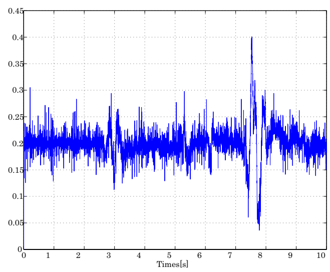

Fig. 12. Xzmp.

V. Lagrange's Equation of System on Slope

It is on the upslope, that the wheelchair is easy-to-overturn. So we will deduce the Lagrange's Equation of System on slope, and try to find a universal criteria, which can be used both on horizontal plane and slope.

The total kinetic energy T is

$$T = \frac{1}{2}Mx^2 + \frac{1}{2}mv_G^2 + \frac{1}{2}J_M\dot{\theta}^2 + \frac{1}{2}J_m\dot{\varphi}^2. \tag{17}$$

Now it will be proven that no matter how the coordination system of COM is defined the result will not be affected. As shown in Fig. 14, here we define the coordination system of COM as vertical and horizontal, then the velocity of COM can be calculated as below.

$$\begin{cases} v_{mx} = \frac{d}{dt}(x\cos\zeta + l\sin(\varphi-\zeta)) \\ v_{my} = \frac{d}{dt}(x\sin\zeta + l\cos(\varphi-\zeta)) \end{cases} \tag{18}$$

Or

$$\begin{cases} v_{mx} = \dot{x}\cos\zeta + l\cos(\varphi-\zeta)\dot{\varphi} \\ v_{my} = \dot{x}\sin\zeta - l\sin(\varphi-\zeta)\dot{\varphi} \end{cases} \tag{19}$$

So velocity of COM v_G is

$$\begin{aligned} v_G^2 &= v_{mx}^2 + v_{my}^2 \\ &= \dot{x}^2 + l^2\dot{\varphi}^2 + 2\dot{x}l\dot{\varphi}(\cos\zeta\cos(\varphi-\zeta) \\ &\quad - \sin\zeta\sin(\varphi-\zeta)) \\ &= \dot{x}^2 + l^2\dot{\varphi}^2 + 2\dot{x}l\dot{\varphi}\cos\varphi \end{aligned} \tag{20}$$

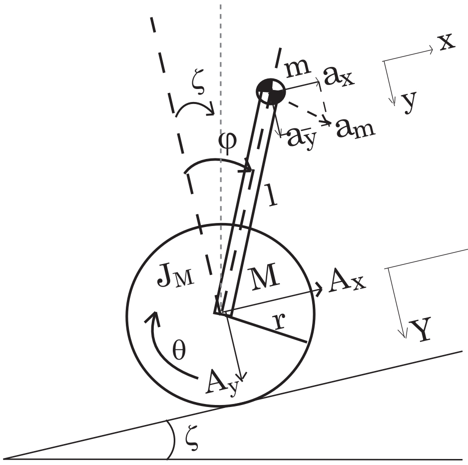

Fig. 13. Wheel Model On Slope.

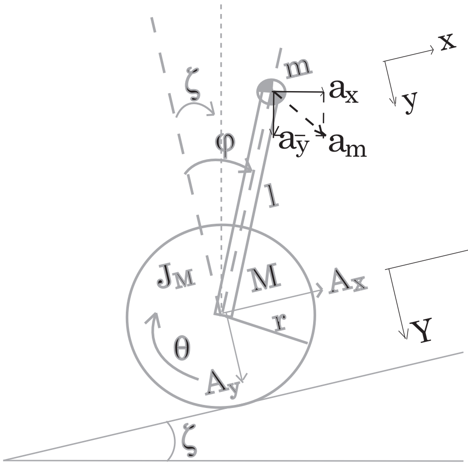

Fig. 14. New Coordinate System of a_x,a_y.

Consequently, no matter how to define the coordination system of COM, the total kinetic energy T does not change. For simplicity, the coordination system is set as Fig. 13.

To calculate the potential energy U, similarly we assume that the horizontal plane passing through the rear axle as the potential energy reference plane.

$$U = mgl\cos(\varphi - \zeta) + Mgx\sin\zeta \tag{21}$$

So the Lagrange's equation can be written as

$$\begin{aligned} \tau + d\theta &= [(M+m)r^2 + J_M]\ddot{\theta} + mlr\ddot{\varphi}\cos\varphi \\ &\quad -mlr\dot{\varphi}^2 sin\varphi + Mgr\sin\zeta + B_M\dot{\theta} \\ -\tau + d\varphi &= (Jm + ml^2)\ddot{\varphi} + mlr\ddot{\theta}\cos\varphi \\ &\quad -mgl\sin(\varphi - \zeta) + B_m\dot{\varphi} \end{aligned}$$

Similarly to the previous section, Lagrange Criteria on slope is

$$\tau < mgl\sin(\varphi_0 - \zeta) - mlr\ddot{\theta}\cos\varphi_0 \tag{22}$$

Let $\zeta = 0$ in (22), it turns out to be the same as (15). Therefore, (22) can be used both on horizontal plane and slope.

VI. Calculation of Gravity Vector

Theoretically, by using gravity vector, the angle of slope (ζ) can be calculated and used for preventing wheelchair from falling backward.

$$A_{ny} = g\sin\zeta \simeq g\zeta,\ g = \sqrt{A_{nx}^2 + A_{ny}^2} \tag{23}$$

$$\zeta = \frac{g\zeta}{g} = \frac{A_{ny}}{\sqrt{A_{nx}^2 + A_{ny}^2}} \tag{24}$$

However, there are many disturbances, and in this work main disturbances are considered as:

- Forward and Backward Accelerations (Effecting on A_{ny})
- Centrifugal Acceleration (Effecting on A_{ny})
- Acceleration of Wheelchair Frame Rotation (Effecting on $\dot{\theta}_l, \dot{\theta}_r, A_{nx}, A_{ny}$)

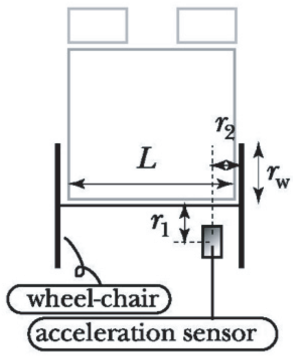

Fig. 15. System.

A. Forward and Backward Accelerations

The forward and backward accelerations affect directly on A_{ny}. The following equation can be obtained.

$$A_{ny} = \ddot{\theta}_r r + g\zeta \tag{25}$$

B. Centrifugal Acceleration

The turning radius of right wheel R is defined as:

$$R = \frac{\dot{\theta}_r L}{\dot{\theta}_r - \dot{\theta}_l} \tag{26}$$

While the angle of velocity w is given as the following equation.

$$w = \frac{\dot{\theta}_r - \dot{\theta}_l}{L} r \tag{27}$$

From above, the centrifugal force that affects A_{ny} is $r_1 w^2$. So the difference between acceleration of right wheel and acceleration by sensor is $r_2\dot{w}$. Consequently

$$A_{ny} = g\zeta + \ddot{\theta}_r r + r_1 w^2 - r_2\dot{w}^2 \tag{28}$$

C. Acceleration of Wheelchair Frame Rotation

Again, the centrifugal force will be generated when the wheelchair frame rotates. The force that affects A_{ny} is $r_1\psi^2$, and that affects A_{nx} is $r_1\dot{\psi}$.

From all of the above, the real values of A_{nx}, A_{ny} can be expressed as

$$\begin{aligned} A_{nx} &= g + r_1\dot{\psi} + D_x \\ A_{ny} &= g\zeta + \ddot{\theta}_r r + r_1\psi^2 + r_1 w^2 - r_2\dot{w}^2 + D_y \end{aligned} \quad (29)$$

Furthermore, the angular velocities of wheels ($\dot{\theta}_{nr}$, $\dot{\theta}_{nl}$) are

$$\dot{\theta}_{nr} = \dot{\theta}_r - \psi,\ \dot{\theta}_{nl} = \dot{\theta}_l - \psi \quad (30)$$

As stated above, the gravity vector g and angle of slope ζ can be inferred by using the flow diagram below.

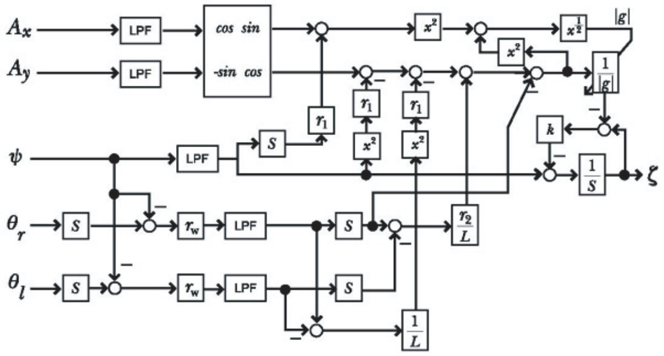

Fig. 16. Data Flow Diagram.

VII. Experiment on slope

In this experiment, wheelchair runs on a horizontal plane at first, and then goes to a 10^o slope. The angle of slope (ζ) is calculated online using the data from sensors.

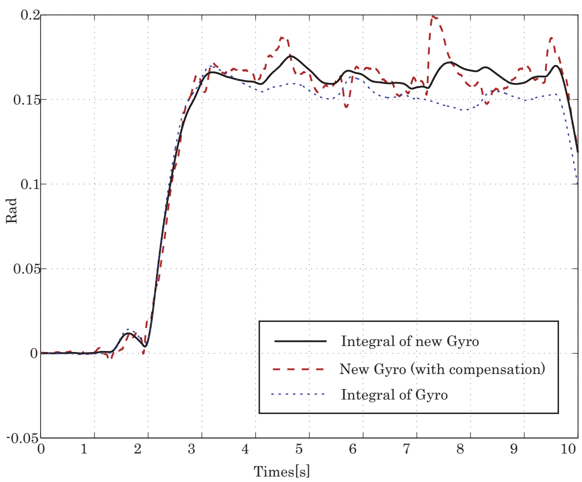

Fig. 17. Inferred angle of slope.

As shown in the experiment data, the angle of slope (ζ) can be correctly calculated. It is clear that proposed stability condition varies automatically as ζ changes, which means that such stability condition can be used on both horizontal plane and slope. Furthermore, even if front wheels of wheelchair rise from the ground, system will not fall back because the bigger frame rotation is, the smaller torque limit will be, and without big motor output, wheelchair can easily return to its original states.

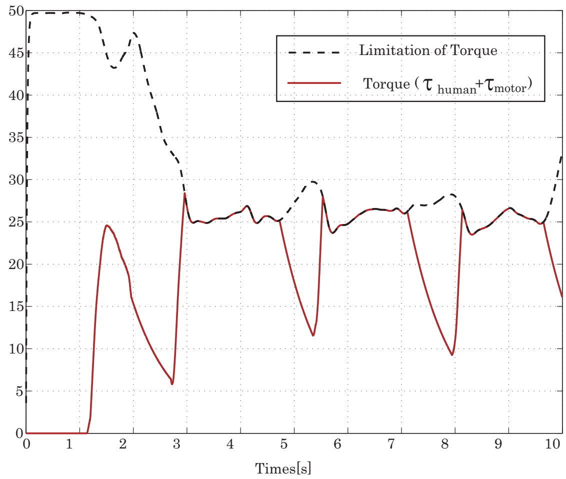

Fig. 18. Experiment data.

VIII. Conclusion and Future Work

As shown in Fig. 10, at 3s and 8s, there is a sharp increase of torque. With this increase, user may feel that wheelchair accelerates on its own. To deal with this phenomenon, one simple idea is to add a low-pass filter on torque.

In the experiment, mass of operator (m), initial angle of the vertical to COM (φ_0), as well as length from rear axle to COM (l) are treated as constants. And $m = 55$kg, $\varphi_0 = 0.3415$rad, $l = 0.6$m. However, these parameters vary from person to person. So the next research theme is to find out the adequate parameters, and to accomplish the robust controller.

Furthermore, this stability condition is just the utmost limit of torque. For other control objectives, such as better ride quality and so on, other control technologies are needed.

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