An Introduction of Fractional Order Control and Its Applications in Motion Control

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Abstract: Fractional Order Control (FOC) means controlled systems and/or controllers described by fractional order differential equations, which is beginning to attract considerable attention in recent years. In this paper, a brief introduction of FOC research is given, including its history, present situation and mathematical aspects. A real application is also used to show FOC's advantages in control design. In oscillatory system control, the tradeoff between stability margin loss and vibration suppression strength is a common problem. By introducing FOC approach, control system's phase and gain responses can be easily offset to any desired value. Design process and experimental results demonstrate a clear-cut and effective robust control design is possible based on FOC design method. Even the realization of fractional order controllers looks somewhat problematic. Experiment results show they can actually be realized quite acceptably.

Keywords: introduction, fractional order control, application, motion control

1 A BRIEF REVIEW OF HISTORY

Fractional Order Control (FOC) means controlled systems and/or controllers described by fractional order differential equations. Expanding calculus to fractional orders is by no means new and actually had a firm and long standing theoretical foundation. Leibniz mentioned it in a letter to L'Hospital over three hundred years ago (1695). The earliest more or less systematic studies seem to have been made in the beginning and middle of the 19th century by Liouville (1832), Holmgren (1864) and Riemann (1953), although Eular, Lagrange, and others made contribution even earlier [1] [2].

As one of fractional order calculus's applications, FOC was introduced by Tustin for the position control of massive objects half a century ago, where actuator saturation requires sufficient phase margin around and below the critical point [3]. Some other pioneering works were also carried out around 60's by Manabe [4]. However FOC was not widely incorporated into control engineering mainly due to the unfamiliar idea of taking fractional order, so few physical applications and limited computational power available at that time [5].

2 PRESENT SITUATION

In last few decades, researchers found that fractional order differential equations could model various materials more adequately than integer order ones and provide an excellent tool for describing dynamic processes [1] [2] [6]. The fractional order models need fractional order controllers for more effective control of dynamic systems [7]. This necessity motivated renewed interest in various applications of FOC [8] [9] [10]. And with the rapid development of computer performances, modeling and realization of FOC systems also became possible and much easier than before.

The researches on FOC are mainly centered in European universities at present. The CRONE (Non-integer order robust control in France) team in France is leaded by Alain Oustaloup and Patrick Lanusse from Bordeaux University, France. Their practices include applying FOC into car suspension control, flexible transmission, hydraulic actuator etc. Denis Matignon, a researcher from ENST, Signal Dept. & CNRS, URA, France, contributed to the theoretical aspects of FOC concept, such as stability, controllability, and observability of the fractional order systems. Slovak researchers, Ivo Petras and Igor Podlubny from the Technical University of Kosice, are well-known for their efforts in modeling, realization and implementation of fractional order systems. J. A. Tenreiro Machado and Yangquan Chen, from Polytechnic Institute of Porto, Portugal, and Utah State University, Logan, are playing important roles in developing the implementation methods for fractional order controllers and applying FOC in robotics control, disturbance observer, etc.

Fractional differentiation's applications in automatic control is now an important issue for the international scientific community. The First Symposium on Fractional Derivatives and Their Applications (FDTA) of the 19th Biennial Conference on Mechanical Vibration and Noise was held from September 2 to September 6, 2003 in Chicago, IL, USA [11]. This conference was part of the ASME 2003 Design Technical Conferences. 29 papers concerning FDTA in Automatic Control, Automatic Control and System, Robotics and Dynamic Systems, Analysis Tools and Numerical Methods, Modeling, Visco-elasticity and Thermal Systems were presented in the symposium. A sub-committee called "Fractional Dynamic System" under ASME "Multi-body Systems and Nonlinear Dynamics" committee was formed during the symposium. And the first IFAC Workshop on Fractional Differentiation and its Applications will be held in this year's summer, July 19-21, in Bordeaux, France [12]. The following areas will be covered by the workshop: Representation tools, analysis tools, synthesis tools, simulation tools, modeling, identification, observation, control, vibration insulation, filtering, pattern recognition, edge detection. Besides the presentation of theoretical works and applications, this workshop can also give rise to benchmark, testing bench and software presentations.

3 MATHEMATIC ASPECTS

The mathematical definition of fractional derivatives and integrals has been the subject of several different approaches [1] [2]. The most frequently encountered definition is called Riemann-Liouville definition, in which the fractional order integrals are defined as

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\xi)^{\alpha-1} f(\xi) d(\xi) \qquad (1) $$

while the definition of fractional order derivatives is

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{d^{\gamma}}{dt^{\gamma}} \left[{}_{t_0}D_t^{-(\gamma-\alpha)} \right] \tag{2}$$

where

$$ \Gamma(x) \equiv \int_0^{\infty} y^{x-1} e^{-y} dy \tag{3}$$

is the Gamma function, t_0 and t are limits and α ($\alpha > 0$ and $\alpha \in R$) is the order of the operation. γ is an integer that satisfies $\gamma - 1 < \alpha < \gamma$.

The other approach is Grünwald-Letnikov definition:

$$ {}_{t_0}D_t^{\alpha} f(t) = \lim_{\substack{h \to 0 \\ nh = t - t_0}} h^{-\alpha} \sum_{r=0}^{n} (-1)^{\alpha} \begin{pmatrix} \alpha \\ r \end{pmatrix} f(t - rh) \tag{4}$$

where the binomial coefficients are

$$ \begin{pmatrix} \alpha \\ 0 \end{pmatrix} = 1, \begin{pmatrix} \alpha \\ r \end{pmatrix} = \frac{\alpha(\alpha-1)\ldots(\alpha-r+1)}{r!} \tag{5}$$

The Laplace transform of Riemann-Liouville fractional order derivative with order $\alpha > 0$ is

$$ L\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s) - \sum_{j=0}^{n-1} s^j \left[{}_0D_t^{\alpha-j-1} f(0) \right] \tag{6}$$

where $(n-1) \le \alpha < n$ [1] [2]. If

$$ {}_0D_t^{\alpha-j-1} f(0) = 0, \quad j = 0, 1, 2, \ldots, n-1 \tag{7}$$

then

$$ L\left\{ {}_0D_t^{\alpha} f(0) \right\} = s^{\alpha} F(s) \tag{8}$$

Namely, the Laplace transform of fractional order calculus is fractional order Laplace operator s. Obviously, its Fourier transform can be exactly obtained by substituting s with $j\omega$ in its Laplace transform just like its integer order counterpart.

4 TORSIONAL SYSTEM SPEED CONTROL

The testing bench of torsional system is depicted in Fig. 1, which is a typical oscillatory system. Two flywheels are connected with a long torsional shaft. Driving force is transmitted from driving servomotor to the shaft by gears with gear ratio 1:2. Some system parameters are adjustable, such as gear inertia, load inertia, shaft elastic coefficient and gear backlash angle. The encoders and tacho-generators are used as position and rotation speed sensors.

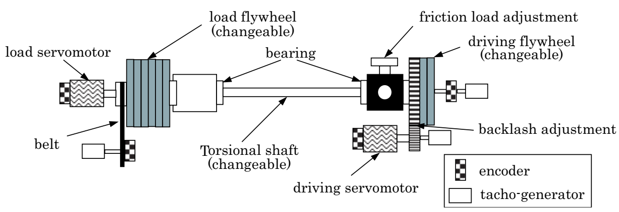


Figure 1. Experimental setup of torsional system

4.1 Multi-mass Model

The simplest model of torsional system with gear backlash is the three-inertia model depicted in Fig. 2 and Fig. 3, where J_m, J_g and J_l are driving motor, gear (driving flywheels) and load inertias, K_s shaft elastic coefficient, ω_m and ω_l motor and load rotation speeds, T_m input torque and T_l disturbance torque. In the modeling, gear backlash is simplified as a deadzone factor with backlash angle band [-δ, +δ] and elastic factor whose coefficient is K_g. Frictions between components are commonly neglected due to their small values.

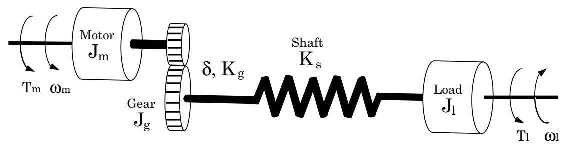


Figure 2. Torsional system's three-inertia model

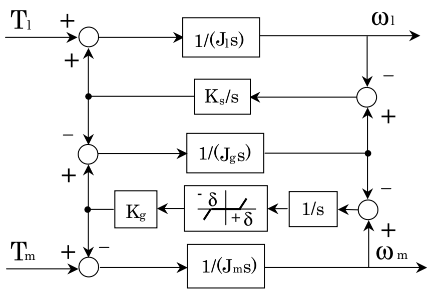


Figure 3. Block diagram of the three-inertia model

4.2 Classical PI Control

As depicted in Fig. 4 and Fig. 5, a well designed set-point-I PI controller can give a satisfactory performance for speed control in nominal case, where gear backlash is totally neglected [13]. The PI controller is designed by Coefficient Diagram Method (CDM) with $K_i = 119.78$ and $K_p = 1.6187$ [15] [16]. Since the driving servomotor's input torque command T_m has a limitation of maximum ±3.84 Nm, K_i is reduced to 18.032 by trial-and-error to avoid large over-shoot caused by input torque saturation.

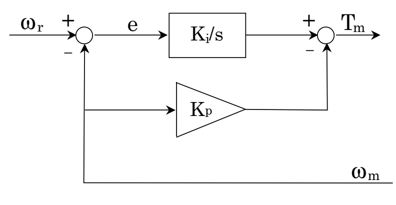


Figure 4. Set-point-I PI controller

For the nominal three-inertia model $P_{3m}(s)$, the close-loop transfer function of integer order PI control system from ω_r to ω_m is

$$ G_{close}(s) = \frac{C_I(s)P_{3m}(s)}{1 + C_I(s)P_{3m}(s) + C_P(s)P_{3m}(s)} \tag{9}$$

where $C_I(s)$ is I controller and $C_P(s)$ is P controller in minor loop; therefore $G_{close}(s)$ is stable if and only if $G_l =$

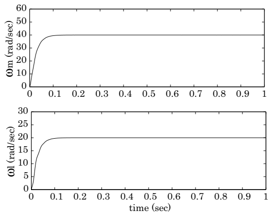


Figure 5. Simulation results with nominal three-inertia model

$C_I(s)P_{3m}(s) + C_P(s)P_{3m}(s)$ has positive gain margin and phase margin. At the same time, for torsional system's speed control, suppressing vibration caused by the gear backlash must be concerned.

As depicted in Fig. 6, the PI speed control system has enough stability margin; while in order to improve vibration suppression performance, additional factors with negative slope and phase-lag are needed. However introducing these factors will inevitably lead to phase margin loss. A tradeoff exists between stability margin loss and vibration suppression strength in control design.

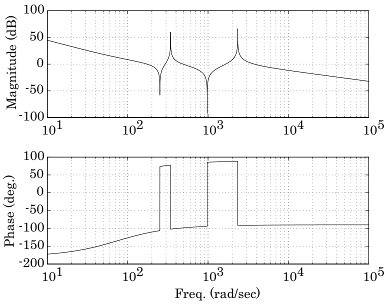


Figure 6. Bode plot of PI control system

4.3 Disturbance Observer

Disturbance observer can be applied to improve the PI control system's robustness. As depicted in Fig. 7, a simple inverse plant model Js is used, where J equals the sum of J_m, J_g and J_l. Q-filter is a low-pass filter to restrict the effective bandwidth of disturbance observer:

$$Q(s) = \frac{1}{(\tau s + 1)^n} \tag{10}$$

where τ is the cutoff frequency and n is the relative degree of Q-filter.

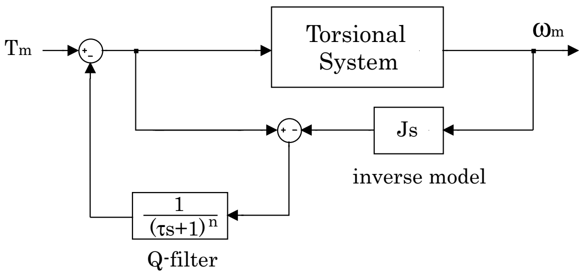


Figure 7. Conventional disturbance observer

5 FRACTIONAL ORDER CONTROL APPROACH

5.1 Fractional Order Low-pass Filter

In order to achieve a proper controller, which is neither conservative nor aggressive, a fractional order low-pass filter $\frac{1}{(Ts+1)^\alpha}$ is introduced (see Fig. 8). By choosing proper fractional order α, the tradeoff between stability margin loss and vibration suppression strength can be adjusted in a clear-cut way, as depicted in Fig. 9.

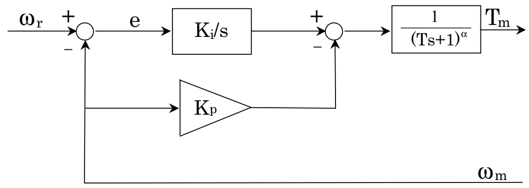


Figure 8. PI controller with fractional order low-pass filter

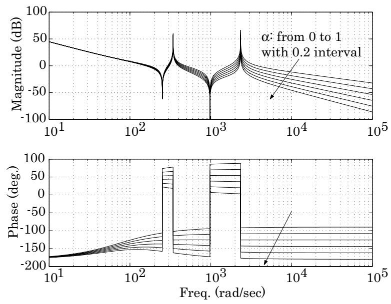


Figure 9. Bode plots of $G_l(s)$ with fractional order low-pass filter

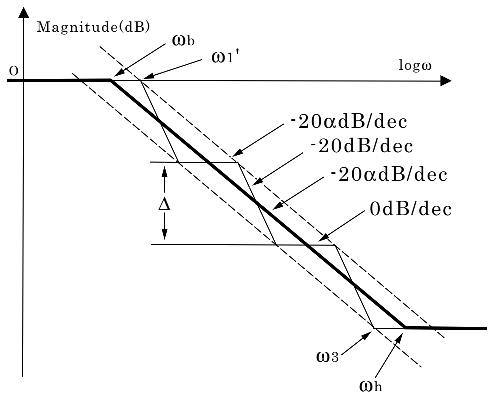


Figure 10. Broken-line approximation ($N = 1$)

5.2 Realization Method

Fractional order systems have an infinite dimension while integer order systems are finite dimensional. Proper approximation by finite difference equation is needed. There are various way to realized designed fractional order controllers, such as Short Memory Principle, Sampling Time Scaling, Tustin Taylor Expansion and Lagrange function interpolation [17].

In this paper, a broken-line approximation method is introduced to realize frequency-band fractional order I^α controller between $[\omega_b, \omega_h]$. Let

$$D(s) = \left(\frac{\frac{s}{\omega_b} + 1}{\frac{s}{\omega_h} + 1}\right)^\alpha \approx \prod_{i=-N}^{N} \frac{\frac{s}{\omega_i'} + 1}{\frac{s}{\omega_i} + 1} \tag{11}$$

Based on Fig. 10, ω_i and $\omega_i^{'}$ can be calculated in following form:

$$\omega_i^{'} = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+N+\frac{1}{2}-\frac{\alpha}{2}}{2N+1}} \omega_b,\ \omega_i = \left(\frac{\omega_h}{\omega_b}\right)^{\frac{i+N+\frac{1}{2}+\frac{\alpha}{2}}{2N+1}} \omega_b \quad (12)$$

Figure. 11 shows the Bode plots of ideal frequency-band $D(s)$ $(\alpha = 0.4,\ \omega_b = 200Hz,\ \omega_h = 10000Hz)$ and its 1st-order, 2nd-odes and 3rd-order approximations by the broken-line approximation method. Even taking $N = 2$ can give a satisfactory accuracy in frequency domain.

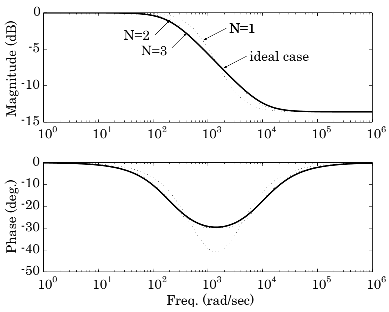
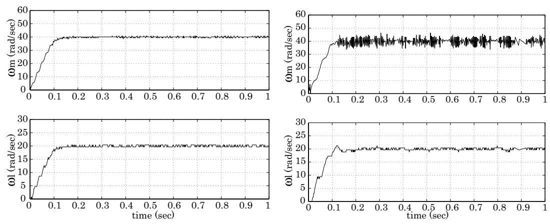


Figure 11. Bode plots of ideal case and approximations

6 EXPERIMENTAL RESULTS

Experiments are carried out with sampling time T=0.001sec. Two encoders (8000$pulse/rev$) are used as rotation speed sensors with coarse quantization $\pm 0.785 rad/sec$.

Firstly, integer order PI speed control experiment is carried out. Obviously, PI control only can not provide enough strength for suppressing backlash vibration. persistent vibration occurs when gear backlash non-linearity exists (see Fig. 12).

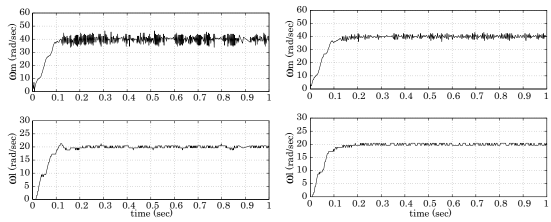


(a) No backlash (b) with backlash

Figure 12. Time responses of PI control

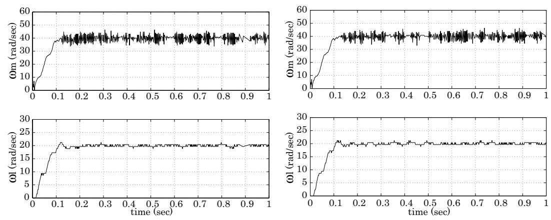
Figure 13 shows that compared with PI-only control, introducing disturbance observer can give better vibration suppression performance. However, this performance improvement is not enough to effectively suppress the vibration cause by gear backlash.

Figure 14 depicts the experimental results with different α order filters. The fractional order low-pass filters $\frac{1}{(Ts+1)^\alpha}$ are realized with the 2nd-order broken-line approximation. Vibration occurred in PI-only and PI+DOB control is effectively suppressed by choosing proper α. Taking α as 0.2 gives best time response. From the time responses, it can be seen that large order leads to poor stability; while small order gives poor vibration suppression strength. This observation verifies a continuous tuning of the tradeoff can be easily achieved through FOC approach.

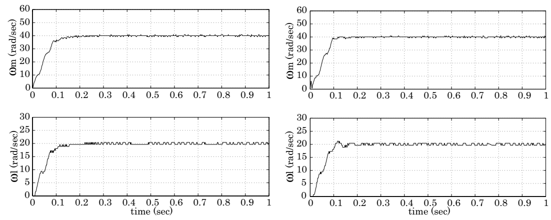


(a) PI only (b) PI with DOB

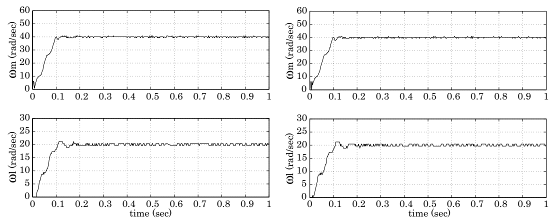
Figure 13. Improvement by introducing disturbance observer



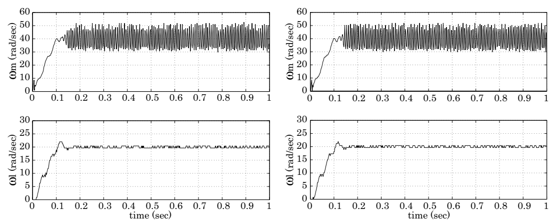
(a) $\alpha = 0$ (b) $\alpha = 0.01$



(c) $\alpha = 0.2$ (d) $\alpha = 0.4$



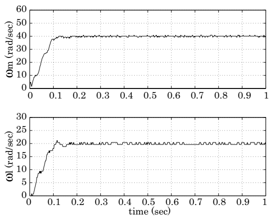
(e) $\alpha = 0.6$ (f) $\alpha = 0.8$



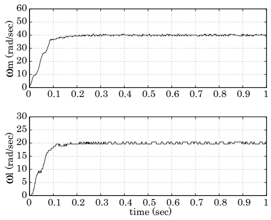
(g) $\alpha = 0.99$ (h) $\alpha = 1$

Figure 14. Time responses with fractional order $\frac{1}{(Ts+1)^\alpha}$ filter

Figure. 15 depicts experimental results with the 1st-order and 3rd-order approximation of 0.2 order low-pass filter by the broken-line method. Even taking 1st-order approximation can give a good backlash vibration suppression performance.



(a) 1st-order



(b) 3rd-order

Figure 15. Time responses with 1st- and 3rd-order approximations

7 CONCLUSIONS

Generally, there are three main advantages for introducing FOC to control design:

- More adequate modeling of dynamic systems
- More clear-cut robust control design
- Reasonable implementation by approximation

In this paper, design and implementation issues of FOC are mainly mentioned. In oscillatory system control, the tradeoff between stability margin loss and vibration suppression strength is a common problem. By introducing FOC approach, control system's phase and gain responses can be easily offset to any desired amount. Design process and experimental results demonstrate a clear-cut and effective robust control design is possible based on FOC design method. On the contrary to FOC control design, the implementation of fractional order controllers is not such straightforward. Some proper approximations are needed. However, as verified in experimental results, implementation issue is actually not problematic.

FOC should not be an independent concept of the well-developed IOC. Knowledge and design methods developed in IOC can still be made full use of in FOC research, as demonstrated in this paper. It is interesting to notice that even the theoretical analysis and design are based FOC approach, the implementation of fractional order controllers are certainly integer order controllers. Therefore, FOC should not be thought as a novel and conceptually difficult idea, but actually a natural and more effective control design tool. By FOC, control system's responses can be designed with much more flexibility. The integer order controller's structure and parameters can be decided by one parameter, the fractional order. This enlarged flexibility will provide more possibility to find excellent solutions with less design effects.

Namely, the tuning knob in FOC can be reduced significantly compared to high-order transfer functions obtained by classical IOC approaches. The authors do believe some well-designed IOC system might in fact be a unconscious approximation of FOC system. If this hypothesis can be established, it will further verify FOC's advantages in control field.

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