Development of a Novel Instantaneous Speed Observer and its Application to the Power-Assisted Wheelchair Control

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Abstract—Nowadays advanced power assisting tools are drawing people's attention as emerging control application. A power-assisted wheelchair is a good example of that kind of power assisting tools. Development of controllers for a power-assisted wheelchair has just started[1]. Though a variety of power assisting tools are being developed, there is little discussion on control methods for those tools. In this paper, we will design a controller for a power-assisted wheelchair considering necessary conditions for power assisting tools.

For advanced controls of a power-assisted wheelchair, we need to control the speed of power assisting motors. However, a wheelchair runs at very slow speed and even stops frequently, which means the instantaneous speed observer is necessary for the control of a power-assisted wheelchair.

From this point of view, we develop a novel instantaneous speed observer which has fast convergence speed, and apply it to a gravity compensation control of a power-assisted wheelchair.

We develop a novel disturbance attenuation controller that can be generally used for power assisting tools. Proposed controller has the characteristics that the disturbance response can be modified arbitrarily.

Using these observer and controller, we design a power assisting controller which can compensate gravity when a wheelchair goes on a hill.

Key Words : instantaneous speed observer, gravity compensation, two-degree-of-freedom control, power-assisted wheelchair, flexible disturbance attenuation, compliance control

I. INTRODUCTION

In conventional power-assisted wheelchairs, motors just multiply original human force to propel by up to several times. But, when a wheelchair goes on a hill, assisting motors can worsen the maneuverability, if the controller of motors does not consider the slope of ground. Besides that, when a wheelchair goes down a hill, power assistance does make dangerous situation because it increases the speed.

To prevent these problems, assistance system should distinguish the road condition. But how can a controller sense if the wheelchair is on a hill?

Gravity can be measured in a controller as a disturbance, and the response against disturbances can be modified using the two-degree-of-freedom control which feedbacks the error between real plant output and model output.

However, we should pay attention to the characteristics unique to power-assisting tool control. The perfect disturbance rejection strategy which has been usually adopted Yoichi Hori

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in industrial motor controls is not suitable for these powerassisting tools. Disturbance attenuation should be more flexible. From this point of view, we will propose a controller that can moderate the disturbance response arbitrarily in section IV

Precise speed information will improve the distinction of the road condition and control performance. To get this precise speed information even when the wheelchair runs at low speed, we first introduce a novel instantaneous speed observer in section II, and try to control gravity on a wheelchair.

II. DEVELOPMENT OF NOVEL SPEED OBSERVER

To control the velocity of a wheelchair, precise velocity information should be obtained. Generally, motor speed information is calculated from the increased pulse number of an encoder in a sampling period. However, when the rotary encoder for the measurement of motor speed has low resolution, the observation does not work well. If so, the observation speed should be slow to prevent to be unstable. Can not we get fast observation even if the encoder has low resolution ?

To this end, a novel instantaneous speed observer is proposed here.

A. Problems in Conventional Observers



Fig. 1. The Structure of Instantaneous Speed and Disturbance Observer

Figure 1 shows the structure of instantaneous speed observer[2],[3]. Its structure is same as general observer's, but it can not get precise value of the output information

of the plant at each sampling period, because of the low resolution of the encoder. Precise value is obtained in a few samples, which means the sampling time of observerstate update by output error several times as long as control sampling time.

Conventional observer feedbacks the output error information only one time after it gets a new encoder pulse. This feedback strategy has some problems. As is shown in figure 2, observation error does not decrease while the observer can not get any new pulse from the encoder, which makes the observation imprecise and even unstable.



Fig. 2. Error Transitions in Two Observers

To make this observer stable, variable gain decision strategy is employed in [3]. However it still uses the observation error only once after the encoder reads the pulse.

B. How to Realize Fast Response

To realize stable and fast observation, output error information should be feedback more frequently even while the encoder does not give out new pulse information. The transition of observation error can be calculated based on the observer dynamics. By feedback of calculated output error, fast and stable instantaneous speed observer can be realized.

C. Observer Design Based on Output Error Simulation

The instantaneous speed observer is described as

$$\hat{x}[n] = \mathbf{A}_d \hat{x}[n-1] + \mathbf{B}_d u[n-1] + \mathbf{L} (y[n] - \hat{y}[n])$$

$$\hat{y}[n] = \mathbf{C}_d \left(\mathbf{A}_d \hat{x}[n-1] + \mathbf{B}_d u[n-1] \right) + \mathbf{D}_d u[n]$$
(1)

$$\hat{x}[n] = \begin{pmatrix} \hat{\omega}[n] \\ \hat{\theta}[n] \\ \hat{d}[n] \end{pmatrix}, \mathbf{A}_d = \exp \begin{pmatrix} -\frac{B}{J}T_2 & 0 & \frac{T_2}{J} \\ T & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$B_d = \begin{pmatrix} \frac{T}{J} \\ 0 \\ 0 \end{pmatrix}, \mathbf{C}_d = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}, \qquad (2)$$

where ω is angular velocity, θ is rotated angle of a wheel, d is input disturbance, and T_2 is sampling time. $\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d$ correspond to $\mathbf{A}, \mathbf{B}, \mathbf{C}$, and \mathbf{D}_d is 0 here.

The term $\mathbf{L}(y[n] - \hat{y}[n])$ is the feedback of observation error. However, the measurement of output y[n] is not available at each sampling time if the encoder has low resolution. Using that quantized and inaccurate $(y[n] - \hat{y}[n])$ for feedback will make the observation inaccurate and even unstable.

For this reason, conventional instantaneous observer does not feedback output error while there is no new pulse reading from the encoder. It switches its feedback phase like below.

$$\hat{x}[n] = \mathbf{A}_d \hat{x}[n-1] + \mathbf{B}_d u[n-1] + \mathbf{L} (y[n] - \hat{y}[n])$$

... when there is new pulse reading from the encoder

$$\hat{x}[n+k] = \mathbf{A}_d \hat{x}[n+k-1] + \mathbf{B}_d u[n+k-1]$$

... otherwise.
(3)

Transition of observation error will be affected by this feedback switching. Let us assume that there is no new pulse reading for K sampling times. The encoder reads a new pulse when $t = nT_2$, and there has been no new pulse reading until $t = (n + K)T_2$. Then the observation error at $t = (n + K)T_2$ can be expressed as follows.

$$e[n+K] = \mathbf{A}^{K-1}(\mathbf{I} - \mathbf{L}\mathbf{C}_d)\mathbf{A}_d e[n] = \mathbf{G}_1(K)e[n] \quad (4)$$

The eigenvalues of $(\mathbf{I} - \mathbf{LC}_d)\mathbf{A}_d^K$ express convergence speed of observation. If the observer can feedback observation error at every sample, this error transition will be,

$$e[n+K] = \left((\mathbf{I} - \mathbf{L}\mathbf{C}_d)\mathbf{A}_d \right)^K e[n] = \mathbf{G}_2(K)e[n].$$
(5)

Figure 3 shows the transitions of eigenvalues of these two matrices ($\mathbf{G}_1, \mathbf{G}_2$) according to K. We can find out from this figure that convergence speed of conventional observer ($\mathbf{G}_1(K)$) is slower. The gain tends to be high for this reason.



Fig. 3. Eigenvalue Transitions of Two Matrices (left: $G_1(K)$, right: $G_2(K)$)

Our proposal for this problem is using simulated output error $\varepsilon_y[n]$ instead of real output error $(y[n] - \hat{y}[n])$ while accurate output measurement y[n] is not available. Compared with the conventional observer, proposed observer feedbacks observation error at each sampling time. This will result in fast observation.

There will be no drastic changes in the target system while the encoder can not get new pulse, which means it is reasonable to model observation error as a step during that period. It is necessary to distinguish two variables to explain this in more detail.

- $\begin{array}{rcl} \varepsilon_y[n] & : & \text{observation error to be feedback} \\ \varepsilon_{yn} & : & y[n] \hat{y}[n] \\ \varepsilon_y[n] \text{ will be same as } \varepsilon_{yn} = (y[n] \hat{y}[n]) \text{ only} \end{array}$
- 1) when the encoder reads a new pulse.
- 2) when observation error $(y[n] \hat{y}[n])$ is bigger than the resolution.

This is the condition for the update of $\varepsilon_y[n]$.

Except these two cases, $\varepsilon_y[n+k]$ will be calculated value based on the last output error ε_{yn} .

$$\varepsilon_{y}[n+k] = \mathbf{C}_{d}\mathbf{A}_{d}\left(\left(\mathbf{I}-\mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)^{k-1}\mathbf{E}_{1}\varepsilon_{yn}$$
 (6)

Here we introduced the matrix \mathbf{E}_1

$$\mathbf{C}_d \mathbf{E}_1 = \mathbf{I}_m = 1, \quad \mathbf{E}_1 = \begin{pmatrix} 0\\1\\0 \end{pmatrix} \tag{7}$$

Using this predicted observation error, proposed observer will be designed like figure 4. This observer employs a latch instead of a switch which is adopted in the conventional observer.



Fig. 4. Structure of Proposed Observer

A brief description of proposed observer is as follows,

$$\hat{x}[n] = \mathbf{A}_{d}\hat{x}[n-1] + \mathbf{B}_{d}u[n-1] + \mathbf{L} (y[n] - \hat{y}[n])
\cdots \text{ when the update condition of } \varepsilon_{y}[n] \text{ is satisfied}
\hat{x}[n+k] = \mathbf{A}_{d}\hat{x}[n+k-1] + \mathbf{B}_{d}u[n+k-1] + \mathbf{L}\varepsilon_{y}[n+k]
\cdots \text{ otherwise.}$$
(8)

Error transition by this observer is :

$$e[n+K+1] = \mathbf{G}_{3}(K+1)e[n]$$

= $\left(\mathbf{A}_{d}\mathbf{G}_{3}(K) - \mathbf{L}\mathbf{C}_{d}\mathbf{A}_{d}\left((\mathbf{I} - \mathbf{L}\mathbf{C}_{d})\mathbf{A}_{d}\right)^{K}\mathbf{E}_{1}\mathbf{C}_{d}\right)e[n]$
(9)

 $\mathbf{G}_3(K+1)$ will change according to the equation (10)

$$\mathbf{G}_{3}(K+1) = \mathbf{A}_{d}\mathbf{G}_{3}(K) - \mathbf{L}\mathbf{C}_{d}\mathbf{A}_{d}\left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)^{K}\mathbf{E}_{1}\mathbf{C}_{d}$$
(10)

To calculate the matrix $\mathbf{G}_3(K+1)$, we introduce a matrix \mathbf{E}_2 :

$$\mathbf{E}_{2} = \mathbf{I} - \mathbf{E}_{1} \mathbf{C}_{d} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(11)

Multiplying on the left by this E_2 , the first and third columns of G_3 can be calculated as follows,

$$\mathbf{G}_{3}(K+1)\mathbf{E}_{2} = \mathbf{A}_{d}\mathbf{T}_{pr}(K)\mathbf{E}_{2} = \mathbf{A}_{d}^{K}\mathbf{T}_{pr}(1)\mathbf{E}_{2}$$
$$= \mathbf{A}_{d}^{K}\left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)\mathbf{E}_{2}$$
(12)

In the same way, multiplying on the left by E_1 , the second column of G_3 can be calculated as follows,

$$\mathbf{G}_{3}(K+1)\mathbf{E}_{1}$$

$$= \mathbf{A}_{d}\mathbf{G}_{3}(K)\mathbf{E}_{1} - \mathbf{L}\mathbf{C}_{d}\mathbf{A}_{d}\left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)^{K}\mathbf{E}_{1}$$

$$= \left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)^{K}\mathbf{G}_{3}(1)\mathbf{E}_{1}$$

$$= \left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d}\right)\mathbf{A}_{d}\right)^{K+1}\mathbf{E}_{1}$$
(13)

Then, we can find out that G_3 has the first and third columns of G_1 in equation (4) and the second column of G_2 in equation (5).

$$\mathbf{G}_{3}(K+1) = \mathbf{G}_{3}(K+1)\mathbf{E}_{1} + \mathbf{G}_{3}(K+1)\mathbf{E}_{2}$$

= $\mathbf{A}_{d}^{K} \left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d} \right) \mathbf{A}_{d} \right) \mathbf{E}_{2} + \left(\left(\mathbf{I} - \mathbf{L}\mathbf{C}_{d} \right) \mathbf{A}_{d} \right)^{K+1} \mathbf{E}_{1}$
(14)

Figure 5 shows the transition of eigenvalues of G_3 according to K. It shows smaller eigenvalues compared with G_1 in figure 3. These smaller eigenvalues of G_3 will result in the fast observation speed.



Fig. 5. Eigenvalue Transitions of \mathbf{G}_3

D. Verification of Convergence Speed

Fast convergence speed of the proposed observer is verified by a computer simulation. Figure 6 shows observations of velocity and disturbance by a conventional observer and proposed observer. We can make sure that frequent output error feedback results in fast and good estimation.

A wheelchair which is the object of next section drives at slow velocities and decelerates to a stop frequently. It is an important requirement to get accurate velocity information of the wheelchair even when it runs at slow speed. From the above simulation, the proposed observer is proved to



Fig. 6. Differences in Convergence Speed

have fast convergence speed even at low angular speed, and satisfy the requirement.

III. GRAVITY COMPENSATION CONTROL IN THE POWER-ASSISTED WHEELCHAIR

We have designed an observer for measurement of wheelchair velocity. In this section, we move on to the control design for gravity compensation in a wheelchair.

A. Necessity of Gravity Compensation in the Power-assisted Wheelchair

Propulsion of a wheelchair on hills is heavier burden than on level ground. Figure 7 compares necessary torques for propulsions in these two cases. It shows that necessary propulsion torque on hills is much larger than the torque for level ground propulsion, which means on hills, the power assist control is more necessary. However, many of commercial power-assisted wheelchairs do not assist the propulsion on hills because of difficulties of control.

On a hill, a wheelchair will be tilted and its center of balance will shift to the unstable area. This is a problem of the power assist control of a wheelchair on a hill. Inadequate power assistance makes the wheelchair unstable and fall backward, because assisting power will work in the same direction with gravity.

Elimination of the effect of gravity and amplification of human torque are the requirements for the power assist control. This is the very feature of the two-degree-offreedom(TDOF) control. The TDOF control can design these two responses of the wheelchair respectively, one against gravity and the other against human torque.



Fig. 7. Necessary torques to drive a wheelchair (Upper: drive on level ground, Lower: drive on hill)

B. Disturbance Attenuation Problem Unique to Powerassisting Tools

Gravity compensation problem in power assist control is a typical disturbance rejection problem. However, it should be different from the disturbance rejection used in industrial motor control.



Fig. 8. The Structure of the Disturbance Observer

The disturbance observer in figure 8 is a typical method of the disturbance rejection in industrial motor controls. It aims at perfect disturbance rejection up to high frequency ranges. This perfect disturbance rejection is not suitable for the power assist control. Disturbance in power assisting tools can be related to human activities in many cases. Stiff rejection of disturbance can worsen the operational performance and even make dangerous situation.

The disturbance attenuation should be flexible when it is applied to the power assisting tools. Considering this characteristic, we propose a flexible disturbance attenuation control for the gravity compensation in next section.

IV. GRAVITY COMPENSATION CONTROLLER DESIGN

A. Flexible Disturbance Attenuation Control

How can we make the disturbance attenuation flexible? As a solution we propose a feedback controller in figure 9. We can design the disturbance response arbitrarily using this feedback.



Fig. 9. Proposed Flexible Disturbance Attenuation Control

By this feedback controller, the response of the wheelchair will be :

$$y = P\left(\frac{1+CP_n}{1+CP}r + \frac{1}{1+CP}d\right).$$
 (15)

r is a reference input and d is a disturbance, which are assisted torque and gravity respectively. Note that if $P_n = P$, the response by r will be Pr, which means the feedback controller does not affect this response. However the response by d is adjusted to $\frac{P}{1+CP}d$ from Pd.

This structure is the same with the passive adaptive control in [4]. While [4] adopted P_n^{-1} for C to realize the perfect disturbance rejection, our proposed method decides C considering physical characteristics.

Flexible disturbance attenuation does not reject disturbance perfectly. It just modifies the physical characteristics of the plant against disturbance. This point is similar with the compliance control[5] used in robot controls.

Utilizing various filters as C, we can realize various flexible disturbance attenuations. We will design a gravity compensation controller as one example of this flexible disturbance attenuation.

B. Flexible Gravity Attenuation Control

Figure 10 shows the structure of flexible gravity attenuation controller applied to a power-assist wheelchair.

 $\frac{1}{J_{s+B}}$ is the dynamics of the wheelchair, and "FF Cont.(Assist)" means a feedforward controller for a power assistance which will be $\frac{K}{\tau_f+1}$, where K is an assistance ratio. Controller in the dotted rectangular is the gravity compensation controller.

Increasing the friction and inertia of the wheelchair makes the wheelchair seem heavy to gravity. This can be flexible gravity attenuation. The controller will produce just a certain amount of power to attenuate the effect of gravity on the wheelchair. The amount can be modified arbitrarily based on the inertia and friction of the wheelchair changed



Fig. 10. Structure of flexible gravity attenuation controller applied to a power-assisted wheelchair

by the filter C. To this end, we adopt $J_d s + B_d$ as C, and this will change the dynamics of the wheelchair from equation (16) to (17).

$$\frac{1}{Js+B}$$
(16)

$$\frac{1}{(J+J_d)s + (B+B_d)}$$
(17)

For the feedback controller, velocity of the wheelchair is used as plant output. This velocity information will be obtained by the proposed instantaneous speed observer.

C. Experimental verification of proposed method

The proposed instantaneous speed observer and gravity compensation controller are verified by experiments. A rotary encoder and a microprocessor were added to a YAMAHA JW II to enable a PC to control the wheelchair. Figure 11 shows an appearance of this experimental setup.



Fig. 11. Experimental Setup (YAMAHA JW II)

Two kinds experiment were done. One is done on level ground and the other is done on a hill. The result is shown in figure 12. In contrast to the propulsion on level ground, the controller produces a certain amount of motor torque on a hill while there is no human torque input. Almost same torque is produced even when the wheelchair descend the hill.

The amount of produced torque can be adjusted by the B_d parameter, and the parameter J_d will adjust the response time against gravity.



Fig. 12. Experimental Results(Upper: on level ground, Lower : on hill)

V. CONCLUSION

In this paper, we suggested a novel design of the instantaneous speed observer and a gravity compensation controller using that observer. By simulation and experiment, we made sure that proposed observer is adequate for control of power-assisted wheelchairs which move at low speed. And the gravity compensation controller using that velocity information could change the response against gravity without worsening the operational performance.

Off course, the method used for the design of the instantaneous speed observer can be used in various ways. Especially it plays an important role when plural sensors have different resolution, because it eliminates or weakens the limit of measurement by low resolution of sensor and make all sensors able to be used in same way. This characteristic is important for sensor fusion.

The other suggestion was the flexible disturbance attenuation that designs the response to external disturbance. The disturbance response will be adjusted by the parameters: inertia, damping and stiffness. This flexible disturbance attenuation is a key when we design a controller for power assisting tools, because the stiffness to the disturbance should be modified arbitrarily.

What we have suggested in this paper showed that advanced motion control theory can improve existing power assistance systems. This research is one example of the application of the advanced motion control to power assisting tools.

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