

Experimental Demonstration of Body Slip Angle Control based on a Novel Linear Observer for Electric Vehicle

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Abstract— In this paper, a body slip angle observer based on yaw rate γ and side acceleration a_y , and a novel control method of body slip angle β are proposed. Body slip angle observer is robust against parameter variation and change of road. Some experimental results by UOT MarchII (Fig. 1) are shown to verify the effectiveness of the proposed observer.

Next, we proposed new control methods for 2-Dimension control. We control β by yaw moment with PID controller. This method is known as DYC (Direct Yaw moment Control) in Internal Combustion engine Vehicles (ICVs). However, the torque difference can be generated directly with in-wheel motors. We performed experiments by UOT MarchII. The experimental results proved that our proposed method was good.

I. INTRODUCTION

Electric Vehicles (EVs) are environment-friendly and expected to be a promising solution for solving today's energy problems. Thanks to the dramatic improvement of motors' and batteries' performances, EVs will become more popular in the near future. It is predicted that before pure EVs, Hybrid EVs (HEVs) will be widely used in the next 10 years.

However, it is not well recognized that EVs have other advantages over Internal Combustion engine Vehicles (ICVs) [1]. Those advantages can be summarized in three aspects.

First, motor's torque generation is fast and accurate. Electric motor's torque response is only several milliseconds, which is 10-100 times as fast as combustion engine's. This advantage can enable us to realize high performance control of EVs.

Second, motor torque can be known precisely. Therefore we can easily estimate driving and braking forces between tire and road surface in real-time. This advantage can be used to realize novel control based on road condition.

Third, in-wheel motors can be installed in EVs' each rear and front tires. We can control each torques of the four motors so that it is easier to control EVs' slip angle β and yaw rate γ than ICVs'. In order to make full use of EVs' advantages, it is essentially important to research on β and γ control and β observer.

In order to estimate β , we utilize full order linear ob-



Fig. 1. UOT MarchII

server because nonlinear observer is too complex to control β and γ . We design observer's gain matrix and propose a new method based on γ and side acceleration a_y . We did experiments by using UOT March II. Experiments show the proposed observer can estimate β accurately and the observer is robust against parameter variation.

Next, we design controller to control only β . If β is too big, EVs cannot be controlled and spin off. β should be controlled. We utilize Model Following Control (MFC) and PID controller. Experiments by UOT March II demonstrated the effectiveness of our control method.

II. MODELING OF EVS

We use two-wheel model [2] for two-dimensional movement of EVs as shown in Fig. 2. Generally, in order to describe vehicle's two dimension movement exactly, four-wheel model is needed. However because four wheeled model is non-linear model, it cannot be used for linear observer design. Where P is the center of gravity, l_f is the distance from P to the front wheel, l_r is the distance from P to the rear wheel, α_f is the front wheel slip angle, α_r is the rear wheel slip angle and δ_f is actual steering angle at tire.

Usually, we express state equations with β , γ , and vehicle speed v . Motion equations are expressed in Eqs. (1).

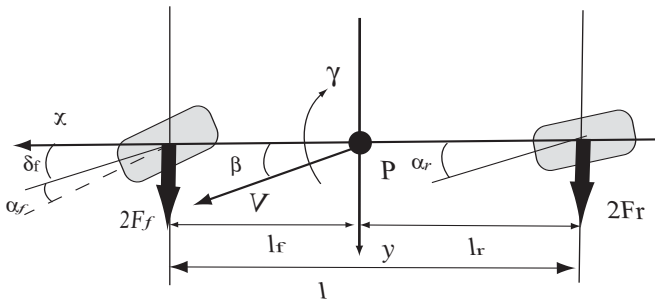


Fig. 2. Two-wheel model of vehicle motion

From Eqs. (1) we can get the state equation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (1)$$

$$\mathbf{A} = \begin{bmatrix} \frac{-2(C_f+C_r)}{mv} & \frac{-2(l_f C_f - l_r C_r)}{mv} - 1 \\ \frac{-2(l_f C_f - l_r C_r)}{I} & \frac{-2(l_f^2 C_f + l_r^2 C_r)}{Iv} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \frac{2C_f}{I} & 0 \\ \frac{2l_f^m C_f}{I} & \frac{1}{I} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \delta_f \\ N \end{bmatrix}$$

C_f and C_r are cornering power, which is defined as Eqs. (2) and (3).

$$C_f = \frac{\partial F_f}{\partial \alpha_f} \Big|_{\alpha_f=0} \quad (2)$$

$$C_r = \frac{\partial F_r}{\partial \alpha_r} \Big|_{\alpha_r=0} \quad (3)$$

Input N is yaw-moment by driving force distribution, which is expressed by Eqs. (4).

$$N = \frac{d}{2}(-F_{x-fl} + F_{x-fr} - F_{x-rl} + F_{x-rr}) \quad (4)$$

Because N cannot be measured, we must estimate N .

A. yaw moment estimation

To estimate N , It is necessary to estimate driving forces F_d . We proposed this yaw-moment estimation method, which is defined as Eqs. (5) and expressed by Fig. (3).

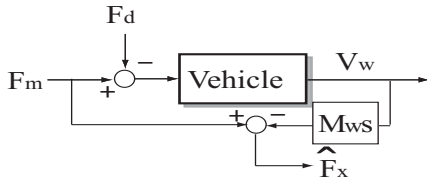


Fig. 3. driving force estimator

$$\hat{F}_x = F_m - M_w \frac{dV_w}{dt}$$

$$\hat{N} = \frac{d}{2}(-\hat{F}_{x-fl} + \hat{F}_{x-fr} - \hat{F}_{x-rl} + \hat{F}_{x-rr}) \quad (5)$$

III. DESIGN OF PROPOSED LINEAR OBSERVER

A. Restructuring of output equation by using side acceleration a_y

Various methods for estimating β were proposed previously. For example, direct integral method [3] estimates β based on Equ. (6). In this method, estimated β contains steady state error, therefore it can't estimate β exactly. Nonlinear observers [4] [5] [6] [7] aim to design an accurate model based on actual vehicle's dynamics and to estimate β . These methods are suitable for simulation. But due to the complexity of the models, these methods are difficult for β estimation in real-time.

$$v(\dot{\beta} + \gamma) = a_y \Leftrightarrow \beta = \int \left(\frac{a_y}{v} - \gamma \right) dt \quad (6)$$

The advantage of conventional linear observers is its simple structure. However they are not robust enough against model error. Moreover it cannot estimate β exactly in non-linear region.

In order to overcome these disadvantages, we propose a novel linear observer in this paper. Unlike conventional observers using only γ as measurable signal, we utilize a_y together with γ to construct the linear observer [8], which can estimate β in non-linear region.

To design the observer, it is necessary to restructure output equation by measurable parameters.

Parameters we can measure are

- longitudinal acceleration (a_x)
- side acceleration (a_y)
- yaw rate (γ)

Because a_x cannot be expressed by linear equation, we use γ and a_y to restructure output equations. Using Eqs. (1) and (6), a_y can be restructured as:

$$a_y = v(a_{11}\beta + a_{12}\gamma + b_1\delta + \gamma) \quad (7)$$

The output equation is:

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \quad (8)$$

$$\mathbf{C} = \begin{bmatrix} 0 & 1 \\ va_{11} & v(a_{12} + 1) \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ vb_{11} & 0 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}$$

B. Full order linear observer

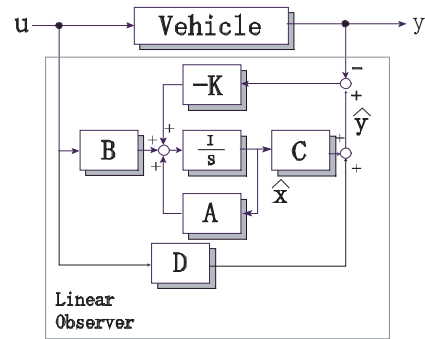


Fig. 4. Full order linear observer

We use full order observer (Fig. 4), which is defined by the following equations.

$$\dot{\hat{x}} = A\hat{x} + Bu - K(\hat{y} - y) \quad (9)$$

$$\hat{y} = C\hat{x} + Du \quad (10)$$

where K is observer matrix gain. \hat{x} is estimated x . The estimation error equation $e = \hat{\beta} - \beta$ should satisfy the following error equation:

$$\dot{e} = (A - KC)e \quad (11)$$

Full order observer's characteristic is decided by value of matrix gain K .

C. Design of gain matrix for robustness

If the selected matrix gain is inadequate, the linear observer will have a poor robust performance against model error and sometimes cannot estimate β exactly. To decide matrix gain, we must consider two important factors. First, the observer must be designed robust against model error. Since two-wheel model is used in our design, some model error exists more or less. Especially, cornering power C_f and C_r depend on road condition and loads on each tires. Therefore, their values are changing and cannot be measured. Second, all eigenvalues of $A - KC$ must be located in stable region. $A - KC$ is the state transition matrix of Equ. (10). The positions of $A - KC$ eigenvalues will affect control system's time response performances, such as overshoot, rising time and settling time. To make the observer robust, we referred [9]. By calculate Eqs. (7) and (8), we can get $\hat{\beta}$:

$$\dot{\hat{\beta}} = a_{11}\hat{\beta} + a_{12}\hat{\gamma} + b_{11}\delta_f - k_{11}(\hat{\gamma} - \gamma) - k_{12}(\hat{a}_y - a_y) \quad (12)$$

State equation of β is expressed as:

$$\dot{\beta} = a'_{11}\beta + a'_{12}\gamma + b'_{11}\delta_f \quad (13)$$

a'_{11} , a'_{12} and b'_{11} are the real values. Any model error is not contained in this equation.

By Eqs. (12) and (13), the state equation for $\hat{\beta} - \beta$ is given by following equation.

$$\begin{aligned} \dot{\hat{\beta}} - \dot{\beta} &= a_{11}(1 - k_{12}v)(\hat{\beta} - \beta) \\ &+ [a_{12} - k_{12}v(a_{12} + 1) - k_{11}](\hat{\gamma} - \gamma) \\ &- (1 - k_{12}v)(a'_{11} - a_{11})\beta \\ &- (1 - k_{12}v)(a'_{12} - a_{12})\gamma \\ &- (1 - k_{12}v)(b'_{11} - b_{11})\delta_f \end{aligned} \quad (14)$$

The best condition for robustness in Equ. (14) is:

$$1 - k_{12}v = 0 \Leftrightarrow k_{12} = \frac{1}{v} \quad (15)$$

Based on consideration of pole assignment and robustness against cornering power, K is decided as:

$$K = \begin{bmatrix} \frac{\lambda_1\lambda_2}{C_f} \frac{(lf-lr)I}{2(lf^2+lr^2)+4lflr} - 1 & \frac{1}{v} \\ -\lambda_1 - \lambda_2 & \frac{m(lf^2+lr^2)}{(lf-lr)I} \end{bmatrix} \quad (16)$$

λ_1 and λ_2 are the assigned poles of the observer.

PC to control	Pentium MMX 223[MHz]
	AMD K6-233[MHz]
OS	Slackware Linux 3.5
	RTLinux rel. 9K
encoder pulse number	3600[ppr]
acceleration sensor	ANALOG DEVICES ADXL202
Yaw rate sensor	HITACHI OPTICAL FIBER GYROSCOPE HOFG-CLI(A)
Noncontact Optical sensor	CORREVIT S-400

TABLE I
SENSORS OF UOT MARCHII

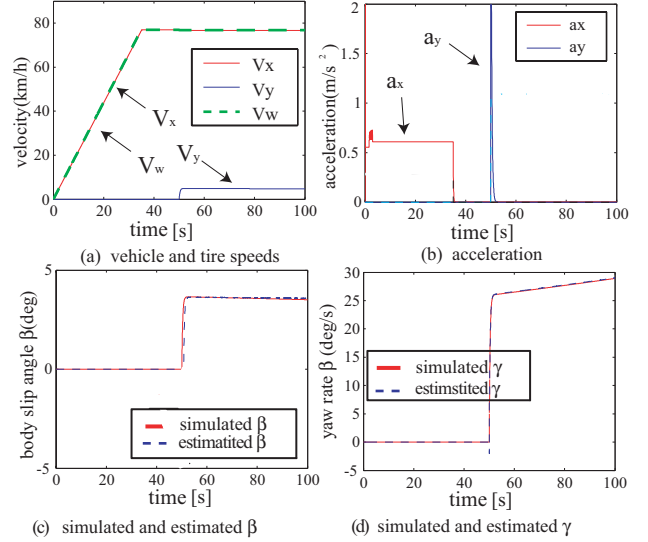


Fig. 5. Simulation result

IV. SIMULATION RESULTS

A. Simulation setup

Before experiments, we simulated the behavior of EV using the four-wheel model. β will be estimated by using the proposed observer. In simulation, all conditions can be set freely. Road type is supposed to be dry asphalt. Simulation time length is 100 [s]. In 0 ~ 35 [s], front and rear tires torques are 556 [N] at each. Over 35 [s], torques become 0 [N]. Over 50 [s], δ_f is changed to be 5 [deg]. EV's parameters are supposed to be UOT MarchII's parameters.

B. Simulation result

Fig. 5 is the simulation results. It shows that the proposed observer can estimate β exactly. Rising time for estimation is short enough and overshoot doesn't occur. When δ_f is changed, inverse response occurred. It happened because $\frac{1}{v}$ is included in the matrix gain K and near-zero-divide problem appeared. This problem can be solved by limiting the value of β .

V. EXPERIMENTAL DEMONSTRATION FOR OBSERVER BY UOT MARCH II

A. Experiment setup for the proposed observer

UOT MarchII is our experimental EV built to prove EVs' advantages. We made this EV by ourselves, which is remodeling of Nissan March. The EV equips acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure β . Table. I explains specification.

We did various experiments and some experimental results were shown you. Vehicle velocity, driver's input steering wheel angle δ and road type were changed to test the effectiveness and robustness of the proposed observer and controller. While the experiment was done, road type was changed from dry road to wet road. But observer's and controller's parameters are kept unchanged. Steering angle was changed freely by the test driver.

We recorded β , γ , δ , v , V_w , F_m and a_y in hard disk drive by the sampling time of 1 [ms] and calculated by Matlab.

B. Experimental results for the proposed observer

Figs. 6 and 7 show two experiments' results under the difference conditions.

Fig. 6 shows experimental results in linear region. Fig. 6 (a) is measured and estimated β . Fig. 6 (b) and (c) are measured and output a_y and γ . In Fig. 6 (a), EV is in linear region because steering angle δ is small. Fig. 6 shows us that the novel proposed observer can estimate β well in linear region.

Fig. 7 is experimental results in nonlinear region. Because δ is larger than δ in Fig. 6, β becomes larger and EV enters non-linear region. Fig. 7 demonstrates that the novel observer can estimate β even in non-linear region.

Fig. 8 is same experiment condition as Fig. 7. But two linear observers were used in this experiment. One observer is the proposed observer, the other observer has different matrix gain (see Equ. (17)). The novel observer can estimate β exactly, but the other observer cannot. This experimental results shows us that our design of gain matrix can make the linear observer more robust.

$$K = \begin{bmatrix} \frac{\lambda_1(a_{12}+1)}{a_{11}} - 1 & \frac{a_{11}-\lambda_1}{va_{11}} \\ a_{22} - \frac{a_{21}(a_{12}+1)}{a_{11}} - \lambda_2 & \frac{a_{21}}{va_{11}} \end{bmatrix} \quad (17)$$

VI. DESIGN OF β CONTROLLER

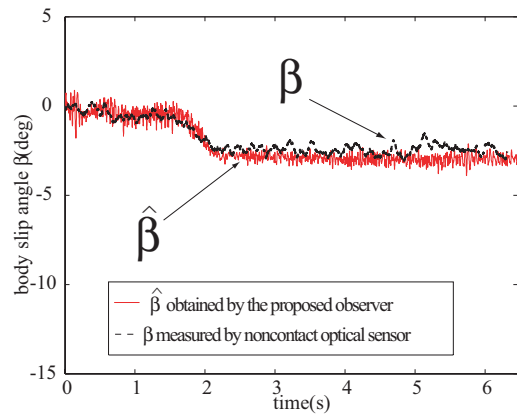
Body slip angle β must be controlled for driver's safety. We proposed this method based on DYC (Direct Yaw Moment control), which makes full use of EVs' advantages.

Proposed method is based on PID controller and MFC (Model Following Control).

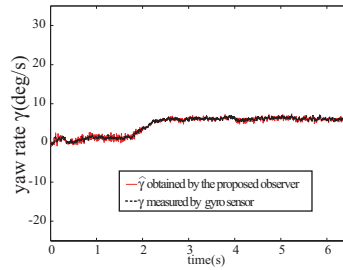
We did experiments by UOT March II to demonstrate the effectiveness of our control method.

A. Model Following Control (MFC)

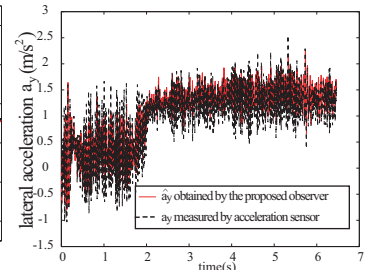
We utilize Model Following Control (MFC) in order to generate the reference of the vehicle dynamics, or the "de-



(a)measured value and estimation of β

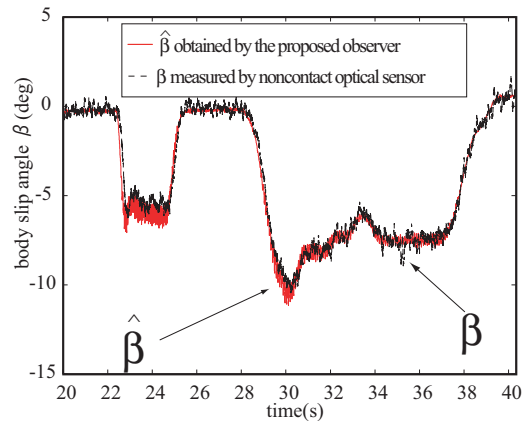


(b)measured value and observer's output of γ

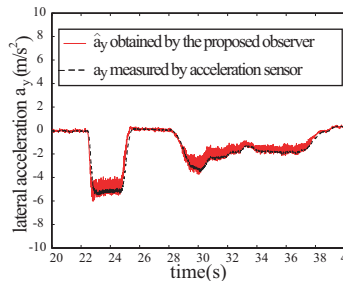


(c)measured value and observer's output of a_y

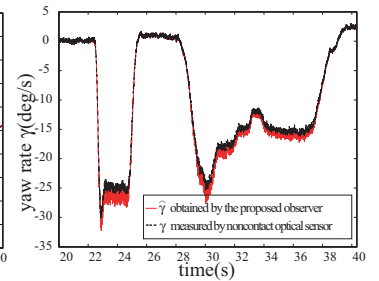
Fig. 6. estimation results in linear region



(a)measured value and estimation of β



(b)measured value and observer's output of γ



(c)measured value and observer's output of a_y

Fig. 7. estimation results in nonlinear region

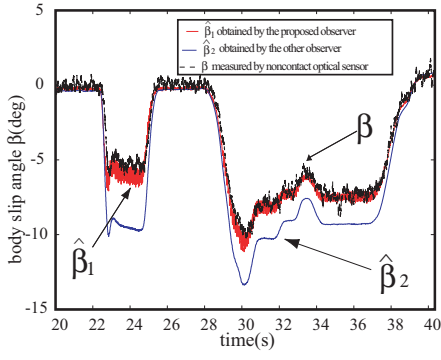


Fig. 8. Measured value and estimation of β with two different gain matrixes when observer model has error

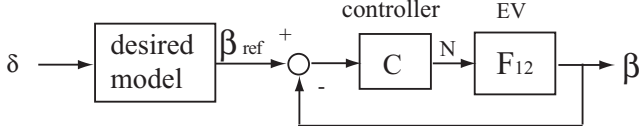


Fig. 9. the proposed control diagram

sired" response to driver's input [2]. In this paper, MFC calculates the desired value β_{ref} .

We use linear state equations expressed by Equ. (1) as the desired model. Using Equ. (1), β_{ref} is:

$$\beta_{ref} = \frac{(s - a_{22})b_{11} + a_{12}b_{21}}{(s - a_{11})(s - a_{22}) - a_{12}a_{21}} \delta_f \quad (18)$$

where matrix A and B are defined in Equ. (1)

If we can keep β following the desired value β_{ref} freely, EVs' movements and safety will be improved dramatically.

B. Design of body slip controller

The proposed control diagram is Fig. 9. We control body slip angle β by yaw rate N . We use linear two wheel (Equ. 1) model for PID controller to make EV's advantage, that is "motor's torque generation is fast and accurate".

F_{12} is transfer function from input yaw moment N to body slip angle β .

$$F_{12} = \frac{a_{12}b_{22}}{(s - a_{11})(s - a_{22}) - a_{12}a_{21}} \quad (19)$$

Transfer function from β_{ref} to β in Fig. 9 is:

$$\frac{\beta}{\beta_{ref}} = \frac{CF_{12}}{1 + CF_{12}} \quad (20)$$

To decide poles of the transfer function in Equ. (20) freely, we use PID gain (Equ. (21)).

$$C = K_1s + K_2 + \frac{K_3}{s} \quad (21)$$

By Eqs. (19), (21) and (20), we can get following equation.

$$\frac{\beta}{\beta_{ref}} = \frac{(K_1s^2 + K_2s + K_3)a_{12}b_{22}}{s^3 - s^2(a_{11}a_{22} - a_{12}a_{21}) + (K_1s^2 + K_2s + K_3)a_{12}b_{22}} \quad (22)$$

This denominator of Equ. (22) is:

$$\begin{aligned} \text{denominator} = & s^3 + s^2(-a_{11} - a_{22} + K_1a_{12}b_{22}) \\ & + s(a_{11}a_{22} - a_{12}a_{21} + K_2a_{12}b_{22}) \\ & + K_3a_{12}b_{22} \end{aligned} \quad (23)$$

$$(24)$$

By pole placement method, we design this denominator as:

$$\text{denominator} = (s - \alpha)(s - \beta)(s - \gamma) \quad (25)$$

α , β and γ are poles of Equ. (20)

By Eqs. (23) and (25), PID gains are expressed in Equ. (28).

$$\alpha\beta\gamma = -K_3a_{12}b_{22}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = a_{11}a_{22} - a_{12}a_{21} + K_2a_{12}b_{22} \quad (26)$$

$$\alpha + \beta + \gamma = -(-a_{11} - a_{22} + K_1a_{12}b_{22})$$

$$\Updownarrow$$

$$K_1 = \frac{a_{11} + a_{22} - (\alpha + \beta + \gamma)}{a_{12}b_{22}}$$

$$K_2 = \frac{\alpha\beta + \beta\gamma + \gamma\alpha - a_{11}a_{22} + a_{12}a_{21}}{a_{12}b_{22}} \quad (27)$$

$$K_3 = -\frac{\alpha\beta\gamma}{a_{12}b_{22}}$$

VII. EXPERIMENTAL DEMONSTRATION FOR CONTROL AND ESTIMATION BY UOT MARCHII

A. Experiment setup for the proposed control

We did some experiments by UOT MarchII to verify the effectiveness of proposed controller. Two patterns of experimental conditions were shown. One experiment was did in linear region. Another experiment was did in non linear region.

B. Estimation and control results

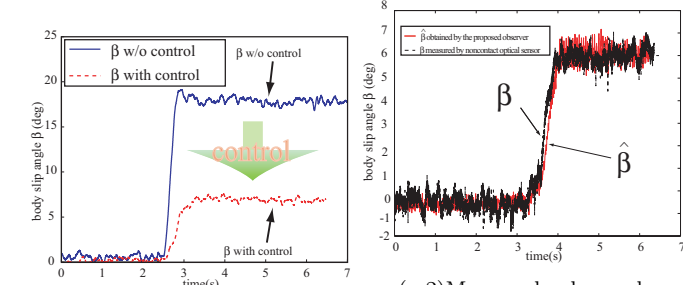
Two experiments' results under the difference conditions are shown as Figs. 10 and 11.

Fig. 10 is experimental results in linear region. Speed is 40km/h and driver's input steering wheel angle δ is 80[deg]. Fig. 10 (a-1) is body slip angle β with control and without control to verify the proposed control. Fig. 10 (a-2) is estimation and measured body slip angle β with control. Fig. 10 (b) and (c) are measured and output a_y and γ . Fig. 10 (d) is reference and estimation by Equ. 3. Fig. 10 (e) is actual steering angle δ_f at tire.

Fig. 10 (a-1) shows us that body slip angle β is suppressed by the proposed control. This figure demonstrates the effectiveness of the novel control. In Fig. 10 (a-2), estimated β corresponds with measured β . This figure demonstrates that the novel proposed observer can estimate β in linear region even if β is controlled by yaw moment N .

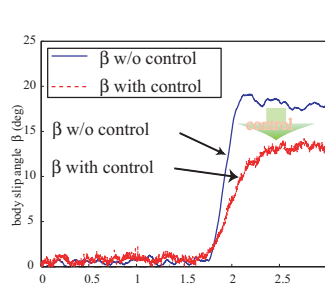
Fig. 11 is experimental results in non-linear region. Speed is 40km/h and driver's input steering wheel angle δ is 150[deg].

Fig. 11 (a-1) shows us that body slip angle β can be suppressed by the proposed control even if in non linear

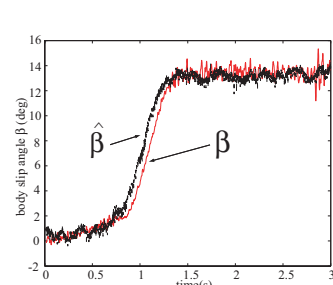


(a-1) Comparison β with normal and the proposed control

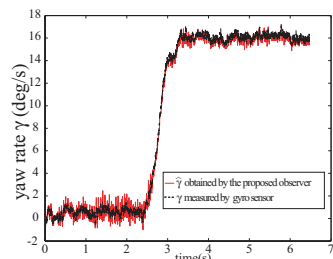
(a-2) Measured value and estimation of β



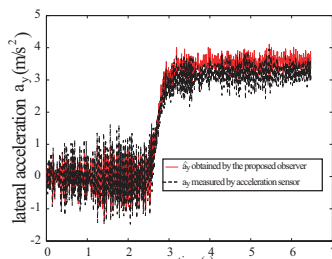
(a-1) Comparison between β with normal and the proposed control



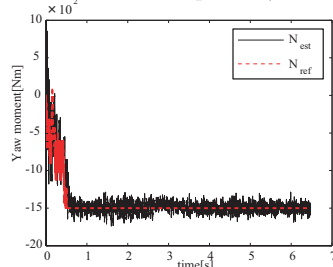
(a-2) measured value and estimation of β



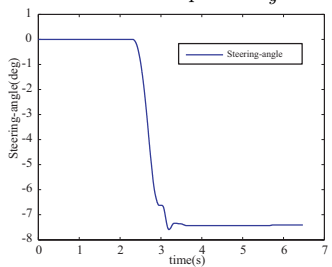
(b) measured value and observer's output of γ



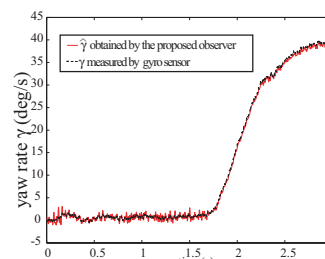
(c) measured value and observer's output of a_y



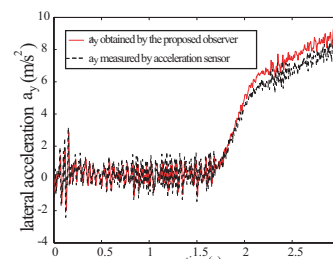
(d) reference and estimation of N



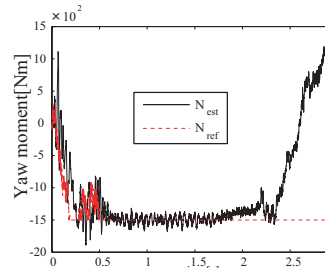
(e) input δ_f



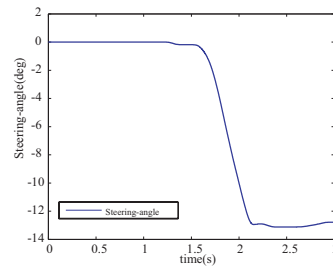
(b) measured value and observer's output of γ



(c) measured value and observer's output of a_y



(d) reference and estimation of N



(e) input δ_f

Fig. 10. β Control and estimation results in linear region

region. In Fig. 11 (a-2), estimated β corresponds with measured β . This figure demonstrates that the novel proposed observer can estimate β exactly in non linear region under the proposed control.

VIII. CONCLUSION

In this paper, we proposed novel methods to estimate β and to control β for EVs. The improved observer is based on acceleration and yaw rate γ sensors. By this method, we can estimate β robustly and accurately. The results of experiments by using UOT March II demonstrated that the proposed observer could estimate β exactly and robustly.

Next, We utilize MFC and PID controller for EVs' motion control by yaw moment N . Experimental results by UOT March II demonstrate the effectiveness of the proposed method and show us that the proposed linear observer can estimate β under the novel control in any region.

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Fig. 11. β Control and estimation results in nonlinear region

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