### Design of Body Slip Angle Observer Based on Nonlinear Tire Model for Electric Vehicle Stabilization Control with In-Wheel Motors

Geng Cong, Hori Yoichi (The University of Tokyo)

Abstract: Body slip angle is important for electric vehicle stabilization control with in-wheel-motors. However, as sensors to measure body slip angle are very expensive, it needs to estimate  $\beta$  from variables measurable. The most difficult for  $\beta$  estimation is that the strong non-linear characteristics of vehicle, especially non-linear characteristics of tire force makes vehicle model parameters change greatly as vehicle running in nonlinear conditions. This paper proposes an observer based on nonlinear tire model which can describe the tire lateral force in its nonlinear region. Simulation and experimental analysis demonstrate the observer's effectiveness, especially when the vehicle running in nonlinear cornering regions.

Key words: electric vehicle, vehicle stabilization control, body slip angle, nonlinear observer, nonlinear tire model

#### 1. Introduction

In recent years, considering environmental protection and energy conservation, electric vehicles (EVs) are intensively developed. Due to the improvement of motor design and control technology, modern configurations are developed for EV. One of the latest configurations is motor-in-wheels, which means motors can be fitted into all the driving wheels of EV independently. For vehicle chassis control, such configuration brings much more advantages. All the wheel motors torque can be measured and controlled independently. Also, electric motor can generate driving torque quickly and accurately. Hence, it can provide more flexible and novel ideas for vehicle motion control [1]. One of such motion control technologies is Direct Yaw Moment Control (DYC), which is much more conveniently implemented in motor-in-wheel EVs than that in traditional internal-combustion-engine vehicles.

The main purpose of DYC is to control the vehicle body slip angle ( $\beta$  angle) to its safety value. However, as sensors to measure  $\beta$  angle are very expensive, it needs to estimate  $\beta$  from only variables measurable.  $\beta$  observer has been researched on in recent years. Some methods of robust design are also put forward to improve the estimation accuracy [2].

As the unstable movement of vehicle often happens in nonlinear cornering region, it needs to ensure  $\beta$  observer's accuracy and robustness in nonlinear cornering region. However, the most difficult for  $\beta$  estimation is that the strong non-linear characteristics of vehicle, especially non-linear force characteristic of tire makes vehicle model parameters change greatly as vehicle cornering in nonlinear region. This makes the observer parameters error get so large that it is difficult to maintain the robustness of observer. This paper proposes a nonlinear observer which has the similar structure as the previous researched linear observer but adopting a nonlinear tire model. For observer feedback gain matrix design, the observer model is changed into the form of an equivalent linear two freedom model by adopting the value of extended tire cornering power. Simulations and field tests analysis demonstrate the observer effectiveness and its robustness against tire stiffness parameter errors, especially in nonlinear cornering region.

## 2. $\beta$ Observer structure and description:



Fig. 1  $\beta$  observer with nonlinear tire model

The nonlinear observer structure is shown as fig.1. The observer model is described by nonlinear states equations with 2 states variable as body slip angle  $\beta$  and yaw rate  $\gamma$ . Vehicle lateral acceleration  $a_y$  and yaw rate  $\gamma$  are 2 measurable variables in vehicle and are chosen as output variables of the observer. The estimate of  $\beta$  angle is computed as predicted value from states equation corrected by output feedback.

The observer's state equation is:

$$\dot{\hat{x}} = f(u, \hat{x}) - K(\hat{y} - y)$$
 (1)

In the observer,  $f(u, \hat{x})$  describes the states equation with nonlinear tire model. The observer's states variables, input variables and output variables are:

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ N \end{bmatrix}, \quad y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}$$

In which:

 $\delta_{\rm f}$ : steering angle of front wheel;

N: yaw moment caused by differential longitudinal forces among tires.

K: feedback matrix of observer.

The observer's output equation is:

$$\begin{cases} \hat{\gamma} = \hat{\gamma} \\ \hat{a}_{y} = v(\hat{\beta} + \hat{\gamma}) \end{cases}$$
(2)

In which, v is the velocity of vehicle.

The following tire model is adopted in the observer to describe the nonlinear characteristics of tire lateral force [2]:

$$F_{yi} = k_{xi} \frac{2}{\pi} \mu F_{zi} \tan^{-1}(\frac{\pi}{2\mu F_{zi}} C_i \alpha_i)$$
(3)

In which:

 $F_{vi}$ : tire lateral force;

 $\alpha_i$ : side slip angle of tires;

C<sub>i</sub>: tire cornering stiffness;

F<sub>zi</sub>: wheel vertical load;

 $\mu$ : road friction coefficient.

 $\boldsymbol{k}_{xi}$  : influence coefficient of tire longitudinal force.

i: index of tires

Compared with linear tire model, this nonlinear model can describe the saturation characteristics of tire lateral force as tire slip angle gets large, and also it can reflect the influence of tire vertical load and road friction to the tire lateral force.

For simplification, a two-freedom vehicle model is adopted as observer' vehicle model (fig.2).



Fig.2 two-freedom vehicle model for observer design

The dynamics of vehicle is described as:

$$ma_{y} = F_{Xf} \sin \delta_{f} + F_{Yf} \cos \delta_{f} + F_{Yf}$$

$$I_{z} \dot{\gamma} = -I_{f} F_{Xf} \sin \delta_{f} I_{f} + I_{f} F_{Yf} \cos \delta_{f} - I_{r} F_{Yr} + N$$
(4)

In which:

m: mass of vehicle

Iz: yaw inertia moment

l<sub>f</sub>: distance between mass center and front axle

l<sub>r</sub>: distance between mass center and rear axle

 $F_{xf}$ : longitudinal forces of front tires

 $F_{\rm Yf}$  and  $F_{\rm Yr}$ : lateral forces of front and rear tires which can be calculated according to above nonlinear tire model.

Considering the kinematics relationship as equation 2, the observer's nonlinear states equations

are derived as:

$$\begin{cases} \dot{\hat{\beta}} = \frac{1}{mv} (F_{Yf} + F_{Yr}) - \hat{\gamma} \\ \dot{\hat{\gamma}} = \frac{1}{I_z} (l_f F_{Yf} - l_r F_{Yr} + N) \end{cases}$$
(5)

# 3. Equivalent linear model and feedback gain design

To get the high response and robustness of the observer, the feedback gain matrix is designed as a linear observer according to the method in the reference paper [3].

First, the observer model is changed into the form of an equivalent linear two freedom model by adopting the value of extended tire cornering power  $c'_{p}$ , which is defined as:

$$c'_{p} = \frac{F_{y}}{\alpha}$$
(6)

In which,  $F_y$  is the lateral force and  $\alpha$  is the tire slip angle at its operating point .

Then the equivalent linear model can be described as:

$$\dot{x} = Ax + Bu \tag{7}$$

In which,

$$A = \begin{bmatrix} \frac{-(C'_{fl} + C'_{fr} + C'_{rr} + C'_{rl})}{mv} & \frac{-l_{f}(C'_{fl} + C'_{fr}) + l_{r}(C'_{rl} + C'_{rr})}{mv^{2}} - 1 \\ \frac{-l_{f}(C'_{fl} + C'_{fr}) + l_{r}(C'_{rl} + C'_{rr})}{I_{z}} & \frac{-l_{f}(C'_{fl} + C'_{fr}) - l_{r}^{2}(C'_{rl} + C'_{rr})}{I_{z}v} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{C'_{fl} + C'_{fr}}{mv} & 0\\ \frac{l_f \left(C'_{fl} + C'_{fr}\right)}{I_z} & \frac{1}{I_z} \end{bmatrix}, \quad x = \begin{bmatrix} \beta\\ \gamma \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f\\ N \end{bmatrix}$$

 $C'_{fl} \sim C'_{rr}$  are the extended cornering power

values of tires

The above equations have the same structures as the linear observer of reference paper [3]. So the same design method of gain matrix K can also be adopted.

According to [3], K is selected as following, for

high response and robustness purposes.

$$K = \begin{bmatrix} \frac{[l_{f}(C'_{fr} + C'_{fl}) - l_{r}(C'_{rr} + C'_{rl})]\lambda_{1}\lambda_{2}I_{z}}{(C'_{fr} + C_{fl})(C'_{rr} + C'_{rl})(l_{f} + l_{r})^{2}} - 1 & \frac{1}{\nu} \\ -\lambda_{1} - \lambda_{2} & \frac{m((C'_{fr} + C'_{fl}))^{2}_{f} + (C'_{rr} + C'_{rl})l^{2}_{r}}{[(C'_{fr} + C'_{fl})]_{f} - (C'_{rr} + C'_{rl})l_{r}]I_{z}} \end{bmatrix}$$

In which,  $\lambda_1$  and  $\lambda_2$  are the assigned pole values of

the observer.

#### 4. Simulations

To test the effectiveness of the nonlinear observer, Simulations are conducted with different steering angle input modes. The comparison between the estimated  $\beta$  angle values and the real ones are shown in fig. 3 to fig. 5.

When the lateral acceleration is small, which means tires lateral forces are in their linear working region,  $\beta$  estimate values of both the linear and nonlinear observers have relatively high accuracy. When the lateral acceleration getting larger, which means tires lateral forces are in their nonlinear region,  $\beta$  estimate values of linear observers deviate from the real values. Comparatively, the nonlinear observer still has enough accuracy in such situations.





b) Nonlinear area ( $\delta_{\rm f}=2.7^\circ$ )

Fig.3 Simulation results of slip angle observer with  $\,\delta_{
m f}\,$  step input



Fig.4 Lane change maneuver simulation results of slip angle observer



Fig.5 simulation results of slip angle observer ( Vehicle accelerates with constant  $~\delta_{\rm f}$  = 3°)

#### 5. Field tests

To test the proposed  $\beta$  observer in real vehicle running conditions, field tests are also conducted in our experimental EV, UOT March II. UOT March II is equipped with acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure real ay,  $\gamma$  and  $\beta$  values. Table. 1 explains the sensors specifications.

Results of field test shown in fig. 6 demonstrate the effectiveness of the nonlinear observer in both the linear and nonlinear cornering situations of the experimental vehicle.





a) Linear area (driver steering angle= $90^{\circ}$ , v=40Km/h)



b) Nonlinear area (driver steering angle=90  $^\circ\,$  , v=60Km/h)

Fig.6 Field test results of slip angle observer

Acceleration sensor	ANALOG DEVICES
	ADXL202
Yaw rate sensor	HITACHI OPTICAL FIBER
	GYROSCOPE HOFG-CLI(A)
Slip angle senser	Noncontact Optical sensor
	CORREVIT S-400

### 6. Accuracy analysis influenced by model parameter errors.

In the adopted nonlinear tire model, cornering stiffness and road surface friction coefficient  $\mu$  are the two model parameters. Cornering stiffness can be changed with the factors such as tire pressure and tire's weariness. Road surface friction coefficient may vary with the different road conditions. For these uncertain factors, there exist model parameter errors for the real situation application of the observer.

Fig.7 and fig.8 show the simulation results when different cornering stiffness and road surface friction coefficient values are applied in  $\beta$  observer.

As far as the nonlinear tire model adopted in the proposed observer is concerned, for any initial cornering stiffness given, the tire lateral force tends to be its fixed saturation value when tire slip angle get large. Therefore, the estimate  $\beta$  value in nonlinear region is insensitive with the deviation of cornering stiffness. The robustness over tire cornering stiffness error is one of the main advantages of nonlinear observer over the linear ones.

The results also show the estimate  $\beta$  are more sensitive to with the road surface friction coefficient errors. This is for the reason that the tire lateral force saturation values are determined by road friction conditions. As tire forces approach the road friction limitation, the calculated tire forces with incorrect  $\mu$ value may be quite different from the real ones. Therefore,  $\mu$  value needs to be identified for further improving the observer's precision.



Fig.7 simulation results with different stiffness values

(Vehicle accelerates with constant  $\delta_{\rm f}=3^\circ$ )



 $\mu_{ob}=0.85, \mu_{v}=0.7, \delta_{f}=4^{\circ}$ 

Fig.8 simulation results when the observer model's value ( $\mu$ \_ob) is different from real one ( $\mu$ \_v)

#### 7. Conclusions and future work

In this paper, by adopting nonlinear tire model, a nonlinear  $\beta$  observer is proposed, which is demonstrated to be effective by simulation and field tests, especially in nonlinear cornering region. This is because that the nonlinear tire model can describe the saturation characteristics of tire lateral force in nonlinear region, and also it can reflect the influence of tire vertical load and road friction to the tire lateral force.

Another advantage of the proposed observer is its robustness over tire cornering stiffness parameter errors. For any initial cornering stiffness given, the tire lateral force tends to be its saturation value in tire's nonlinear region. Therefore, the estimate  $\beta$  value in nonlinear region is insensitive with the deviation of cornering stiffness.

The estimate  $\beta$  are sensitive to the road surface friction coefficient  $\mu$  errors for the reason that road friction conditions determine the tire lateral force saturation values. So  $\mu$  value identification is an important topic to make the observer more practical in future researches.

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