Integrated Motion Control of a Wheelchair in the Longitudinal, Lateral and Pitch Directions

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***Abstract*— The motion of a wheelchair is different from any other vehicles. It needs controlling in three dimensions: the longitudinal direction, the lateral direction, and the pitch direction. This paper takes this point into consideration and provides three-dimensional control of a wheelchair.**

We focus on the control of a push-rim power assist wheelchair. The main objective of this paper is to provide a wheelchair rider with three-dimensional assistance which guarantees power and safety assistance. To this end, three kinds of assist controls are suggested: disturbance attenuation control is designed for the longitudinal and lateral directions and tip-over preventing control is designed for the pitch direction. These controls for three directions can be integrated appropriately taking advantage of system configuration. We demonstrate all these controls can work independently for each purpose with experimental results.

***Index Terms*— power assist wheelchair, three-dimensional control, impedance control, disturbance observer, phase plane, disturbance observer, tip-over prevention, power assist control**

I. INTRODUCTION

A. Necessity of Three-dimensional Control for a Power Assist Wheelchair

Wheelchairs have been a great help for handicapped people. Nowadays power assist wheelchairs draw new attention [1] as they are different from electric-power wheelchairs which use a joystick as an interface [2], in the point that the riders should apply their torque to be assisted [3], [4], [5], [6]. Although there are increasing demands for these power assist wheelchairs, when it comes to the control of the power assist wheelchairs, it cannot be said to be safe or easy to manipulate them. They adopt quite simple control algorithms; a feedforward control constituted by a gain to amplify the applied propulsion torque and a low-pass filter to smooth the torque signal. Consequently they can not recognize external environments nor suppress unexpected external forces. This adaptability to external environments is one requirement of human-friendly motion control [7], which is also necessary to control a wheelchair in a human-friendly way.

The effect of the gravity is a key factor in power assist control of a wheelchair. The conventional power assist wheelchair cannot remove this gravity's effect so that the user should provide all the force to hold his weight on a slope; e.g. the conventional power assist control algorithms cannot support the user on slopes.

Tips and falls incidents of a wheelchair which originate from exceeding the limits of the stabilities [8] will easily occur on slopes since the stability of a wheelchair/rider system can easily be impaired by the gravity when it is on a slope. Assisting torque in power assist wheelchairs, however, tends to increase the chance of tipping over on a slope, since the assisting torque works as torque to tip the wheelchair, with the center of mass of the wheelchair shifted near the axis of the rear wheels. This excessive torque should be addressed for safety of the user.

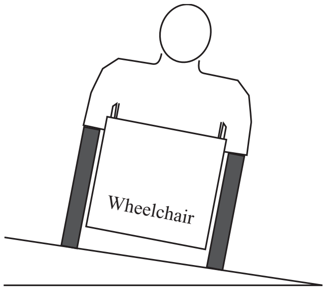

Fig. 1. Gravity acting laterally on a wheelchair

The gravity also interferes with the heading direction of a wheelchair. On a side slope described in Figure 1, the wheelchair will easily turn by the gravity. It needs quite large force to keep going straight. Also, the force or torque necessary to keep the heading direction is much larger than the torque to go forward. In order to assist this force, the gravity's effect in the lateral direction also should be removed.

B. Suggestion of Three-dimensional Power Assist Control for a Power Assist Wheelchair

From these facts, we can see that a power assist controller for a wheelchair should achieve three-dimensional control: disturbance rejection for the longitudinal and the lateral directions and prevention of tipping over for the pitch direction. A controller that can satisfy all these requests will be designed in this paper.

Another point we should notice is the strategic difference in the design of disturbance rejection control for each direction. The longitudinal disturbance should be rejected in the velocity level, and the lateral disturbance should be rejected in the position level. This difference can be implemented using the impedance concept; the relationship between the exerted disturbance torque and deviated values of position. Different types of feedback controllers will attain different impedances.

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II. DESIGN OF DISTURBANCE ATTENUATION CONTROL IN THE LONGITUDINAL DIRECTION

A. Appropriated Impedance Design for Disturbance Attenuation in the Forward and Reverse Directions

A controller that attenuates the gravity's effect in the longitudinal, or forward and reverse directions is developed in this section.

Feedback control can adjust the impedance between the disturbance and the position or velocity [9] of a wheelchair. In order to stop a wheelchair removing the gravity's effect on a slope, a wheelchair needs stiffness against the gravity that provides the same amount of force with the gravity in the opposite direction. However, small stiffness magnitude or just only damping factor will be enough to assist a wheelchair on a slope since it is not necessary to stop a wheelchair on a slope. Moreover, excessive amount of assisting torque can cause tipping over of a wheelchair on a slope. Taking this point into considerations, just to decrease the velocity of the wheelchair pulled down by the gravity is enough for assistance of a wheelchair on a slope, and this can be said to be more human-friendly.

Original impedance of a wheelchair in the longitudinal direction can be depicted as $\frac{1}{Js+B}$ (Note that the impedance used in this paper is defined as a transfer function from the disturbance force to the output velocity). If the controller includes a stiffness factor, this impedance will be changed to $\frac{s}{Js^2+Bs+K}$ (K is added stiffness by feedback control). However, this stiffness is not adequate as we discussed before, and it will be enough to increase the inertia J and damping B in the impedance just to decrease the velocity caused by the gravity.

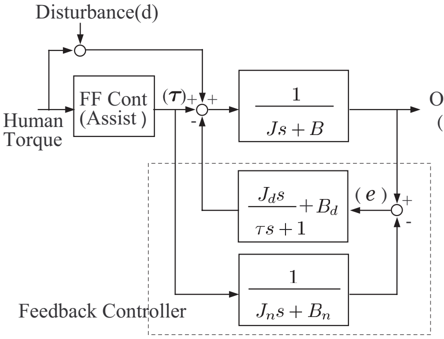


Fig. 2. Structure of proposed disturbance attenuation controller for a wheelchair

Figure 2 is the proposed feedback controller. $\frac{1}{Js+B}$ is the dynamics of the wheelchair, and "FF Cont.(Assist)" means a feedforward controller to amplify the user's propelling torque measured by torsion sensors. Further discussion on this feedforward assist control will be done in the next section. Controller in the dotted rectangular is the feedback controller for disturbance attenuation. Assuming $J_n \simeq J, B_n \simeq B$, this feedback control changes the impedance as following:

$$\frac{1}{Js+B} \rightarrow \frac{1}{(J+J_d)s+(B+B_d)}. \tag{1}$$

This increase in damping and inertia of a wheelchair makes the wheelchair react more heavily against the gravity. The amount of torque produced by the feedback controller can be modified arbitrarily based on J_d and B_d, and the frequency bandwidth of the torque also determined by $\frac{B+B_d}{J+J_d}$.

Note that though various design of feedback controller other than $\left(\frac{J_d s}{\tau s+1}+B_d\right)$, enables us to realize various kinds of control: position control, velocity control and force control [10], velocity control is adopted here. Although this velocity-control-based disturbance attenuation controller uses the model dynamics of a wheelchair, it is different from disturbance rejection by the disturbance observer which is force-control-based, since the input to the nominal model is not the control input which includes disturbance compensation but the reference torque before the disturbance compensation. The disturbance observer control will attempt to eliminate the disturbance - the gravity's effect here - in torque level, which is not necessary and moreover may cause degeneration in human manipulation.

Another important point is that human torque can be dealt with as disturbance form the controller viewpoint. However, since it is measured by torsion sensors, the controller will produce some torque input which amplifies human torque by the FF Cont.(Feedforward controller) in Figure 2 so that human torque is not attenuated but amplified.

Velocity information can be incorrect with low precision shaft encoders as a wheelchair moves at quite low speed. This problem can be addressed by adopting some speed observers [11], [12], [13]. The operation states observer proposed in [14] is adopted in this paper.

B. Experimental Results

On this disturbance attenuation control in the longitudinal direction, an experimental result is analyzed in the paper [14]. In addition to that experiment, another experiment is conducted on downward hill to demonstrate the effect of the controller on downward hill. Figure 3 describes the result.

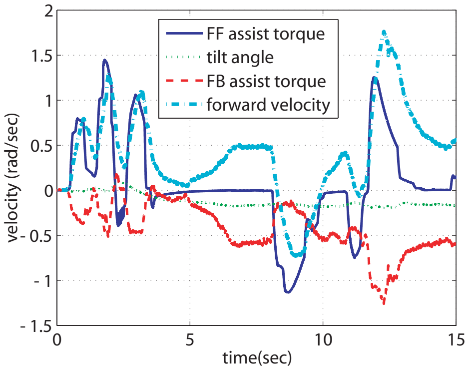


Fig. 3. Assist torque on a downhill

Here, the tilt angle described as dotted line in the Figure 3 shows that from 4 second, the wheelchair goes into a downhill. Then, the feedback assist torque described as the dashed line increases gradually and finally it reaches a certain value.

During the period from 7 second to 8 second, the wheelchair moves on at a constant velocity. This is the velocity caused by the gravity. Although the magnitude of the velocity is reduced by the assist torque provided by the proposed feedback control, the wheelchair does not stop.

Around 12 second, the user applies his torque to move forward, which means positive feedforward assist torque that accelerates the velocity and endangers the user is applied. However, the negative feedback assist torque also increases, and it leads to slow acceleration so that it can ensure safety of the user.

During level road operation from 0 second to 3 second, and backward propulsion on the downhill from 8 second to 10 second, the feedback assist torque is generated in the opposite direction of the feedforward torque. This is due to the difference between the nominal dynamics $\frac{1}{J_n s+B_n}$ in Figure 2 and the real dynamics of the wheelchair. The proposed controller produces feedback torque according to the difference between simulated velocity and the actual velocity; that is, not only external force but also modeling error will cause the feedback assist torque. If the nominal parameters J_n and B_n are set bigger than the real values and the simulated velocity is smaller than the actual velocity, negative feedback torque is produced to track the actual velocity to the simulated velocity. This is the reason why the torque opposite to the feedforward torque is produced by the feedback controller in the above experiment. Nevertheless, due to the small magnitudes of impedance implemented by the feedback controller, this opposite feedback torque is not so large.

III. COMBINATION WITH TIP-OVER PREVENTION CONTROL

A. *Tip-over Prevention Control*

In the last section, the proposed feedback controller compensated the pull of gravity on a slope. However, there is another problem when we control a wheelchair on a slope. The wheelchair will be tilted and its center of gravity will shift to the unstable area on a slope, that is, inadequate power assisting torque makes the wheelchair unstable and tip over, working in the same direction with the gravity.

To cope with this problem, the assisting torque should decrease during driving on a slope. The assisting torque in power assist wheelchair consists of feedforward portion and feedback portion which we employed in the last section. Between these two assist torques, the feedforward torque accounts for tipping over of the wheelchair, since its magnitude is much larger and its rate of rise is faster than that of feedback torque.

The feedforward assist control in Figure 2 amplifies the torque applied by a rider. It consists of a first-order time delay given as

$$\alpha\frac{1}{1+\tau s}, \tag{2}$$

where α is a power-assist ratio and τ is the time constant of first order delay. τ is set smaller at the beginning of propelling and larger at the ending:

$$\tau = \begin{cases} \tau_{fast} & \frac{d}{dt}T_{human} > 0 \\ \tau_{slow} & \frac{d}{dt}T_{human} < 0 \end{cases}, \quad (\tau_{fast} < \tau_{slow}) \tag{3}$$

Since two τs are time constants in the feedforward controller and will not affect the stability of system, they can be decided only focusing on the assistance performance. Some experiments have been done to decide the τs and the following values are adopted in this paper as they provide most satisfying assist torque pattern.

$$\tau_{fast} = 0.08[s], \quad \tau_{slow} = 1.0[s]. \tag{4}$$

For the tipping over problem, we modeled wheelchair pitch motion as an inverted pendulum dynamics and analyzed its stability [15]. Based on that analysis, we proposed a time-varying assist ratio control. The assist ratio described as α is changed as follows;

$$\alpha = \alpha_{\mathrm{max}} \exp\left(\beta\frac{\dot{\varphi}_{CG}}{\varphi_{CG}}\right), \tag{5}$$

where β is the time constant which decides the decreasing speed of the assist ratio α, and α_{max} is the maximum assist ratio. φ_{CG} is the tilted angle of the center of gravity.

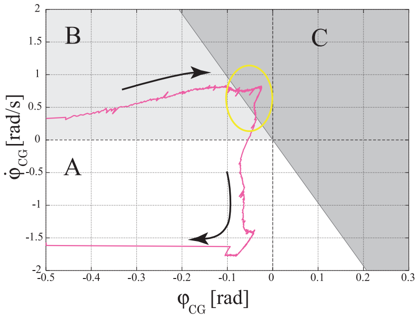


Fig. 4. Tip-over protection control using phase plane of φ_{CG}

Figure 4 shows the phase plane of φ_{CG}. According to the inverted pendulum dynamics, it can be divided into three regions based on the level of danger: A) proper safety zone ($\dot{\varphi}_{CG} < 0$ and below the stability line), B) semi-safety zone ($\dot{\varphi}_{CG} < 0, \ \ \varphi_{CG} > 0$ and below the stability line), and C) dangerous zone (above the stability line). The stability line is an asymptotic line calculated by solving the dynamics of an inverted pendulum model. If the phase is below this line, φ_{CG} will be rotated oppositely to the tipping direction by the gravity. During stable operation, the state of a wheelchair generally stays in the safety zone A. But it will shift to the C region through B region when the wheelchair tips over by excessive torque. The trajectory in Figure 4 shows the result of stabilizing control which decreases the torque to remove the rotating momentum in the pitch direction. The time-varying $\alpha(\varphi_{CG})$ decreases the assist torque according to the state of the wheelchair; φ_{CG}.

In this research, the observed variable $\hat{\varphi}$ estimated by the operation state observer proposed in [14] and the angular velocity φ of the chassis measured by a gyroscope substitute φ_{CG} and $\dot{\varphi}_{CG}$. Assuming that the rider does not shift his center of gravity, φ_{CG} can be calculated based on $\hat{\varphi}$, the tilted angle of the wheelchair body around the real axle. Equation (6) shows this strategy. $\hat{\varphi}_0$ is the constant angle between the center of gravity and the location of the accelerometer on the wheelchair chassis.

$$\alpha = \alpha_{\mathrm{max}} \exp\left(\beta \frac{\dot{\varphi}}{\hat{\varphi}-\hat{\varphi}_0}\right), \tag{6}$$

Another noticeable point is the independence of this tip-over prevention control from the feedback disturbance attenuation control suggested in the last section. The tip-over prevention control is a feedforward control, while the control attenuating the pull of the gravity is a feedback control. This control configuration is the two-degree-of-freedom control [16], [17] which allows respective design of two controllers (See Figure 2).

Moreover, from the viewpoint of the time domain, they are also independent. The tip-over prevention control changes the ratio α especially when the rider begins propelling $\left(\frac{d}{dt}T_{\mathrm{human}} > 0\right)$. According to the equation (3), the time delay of the feedforward assist control, τ_{fast} is set small, which makes the feedforward tip-over prevention control work fast. The center of gravity will be stabilized as soon as the feedforward assisting torque decreases which means the duration of the tip-over prevention control is short. On the other hand, the gravity attenuation control provides slow and steady torque.

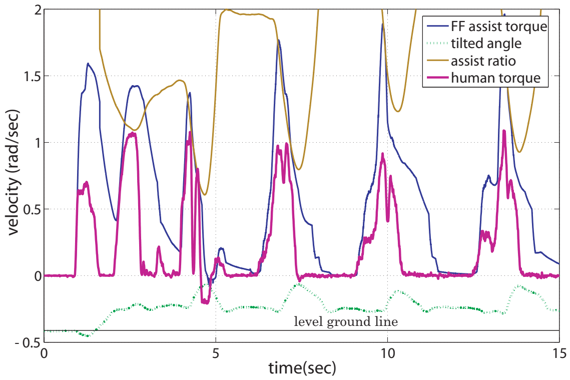
These points are studied later using the experimental results.

B. Experimental Results

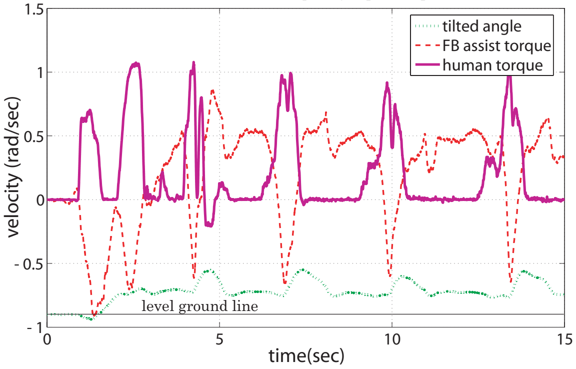
Experiments to investigate the effectiveness of the integration of the proposed controllers are conducted. A power assist wheelchair climbed a slope with assistance of the proposed controllers. Figure 5 shows the results. In order to help understanding, the magnitudes of data are rescaled.

The dotted line means the tilted angle of the wheelchair chassis $\hat{\varphi}$ from the horizontal. This $\hat{\varphi}$ is changed by the condition of terrains on which a wheelchair traverses and also by tipping of the wheelchair. As the wheelchair goes up to a slope, the estimated $\hat{\varphi}$ increases and arrives at a certain constant value representing the angle of the slope. On the slope, the rider exerts some torque to propel the wheelchair and it causes tipping, which is described by a peak of $\hat{\varphi}$ in Figure 5.

The feedforward assist torque around these peaks of $\hat{\varphi}$ reveals the performance of tipping over prevention control. Soon after human torque is exerted, $\hat{\varphi}$ is about to rise. Then, the assist ratio decreases by the tip-over prevention control, leading to the decrease in the feedforward assist torque. During this period, the feedback controller offers negative torque due to the modeling error in $\frac{1}{J_n s+B_n}$. Figure 5 (b) shows the result of feedback disturbance attenuation control. While the rider does not propel the wheelchair, the feedback torque provides a constant value to compensate the gravity. These changes in feedforward assist torque and feedback assist torque verifies that the strategy we adopt is right; assisting torque satisfies two requirements for power assist control on sloping surfaces: prevention of tipping over and compensation of the gravity.



(a) Decrease in feedforward assist torque by tip-over prevention control



(b) Independence between feedforward and feedback assist torque

Fig. 5. Experiment of tip-over prevention control with disturbance attenuation control

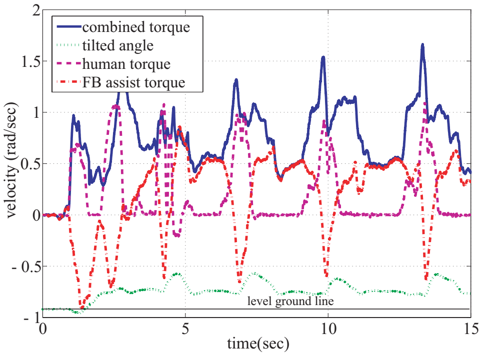


Fig. 6. Integrated assist torque during wheelies

However, with these control parameters, the decrease in the feedforward is insufficient, and the prevention of tipping over is insufficient, resulting in increase in $\hat{\varphi}$. The rider also applies negative torque around 5 second in Figure 5 (a) since he feels danger. Figure 6 depicts the same experimental result with Figure 5 but with the combined torque of the feedback and feedforward control. These figures represents that increase in $\hat{\varphi}$ is caused by the late decrease in the feedforward assist torque illustrated in Figure 5 (a).

The combined torque soon after the feedforward torque decrease tends to keep a larger value than the value of the feedback torque provided while there is no human input. This additional torque is due to the large time constant τ_{slow} in Equation (3). This variable is set large for driving on level ground where the rider wants the assist torque to last long. However, the result suggests that the parameter be small when the wheelchair is on a slope.

These experimental results validate our design of two-degree-of-freedom power assist control for wheelchair achieves stable and satisfying power assist on a slope.

IV. LATERAL DISTURBANCE REJECTION CONTROL

As is explained in the introduction, when a wheelchair crosses a side slope as described in Figure 1, it is difficult to manage the direction; since the gravity works as a disturbance that changes the direction. This section focuses on this problem.

A. Lateral Dynamics of a Wheelchair

First, the lateral disturbance that causes changes of the direction should be defined mathematically so that we can control it. Figure 7 shows the definition we adopt in this paper; the lateral disturbance is defined as the difference between the external disturbances on the left and right wheels. Disturbance, here, means the other external torque than the motor produces.

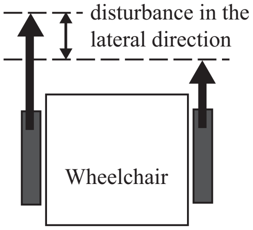


Fig. 7. Definition of lateral disturbance

This definition of lateral disturbance may seem too simple to explain the lateral motion in detail. The lateral dynamics of a four-wheel vehicle such as a car is, in general, described in a more complicated way [18]. However, the lateral dynamics of a wheelchair is simpler than that of a car. The front wheels of a wheelchair are casters and not restricted. Thus, the front wheels produce negligible amount of cornering forces [19]. This justifies the definition of lateral disturbance in Figure 7.

B. Design of Lateral Disturbance Rejection Control

The control design for motion in the lateral direction needs different strategy from the control for the longitudinal direction. A rider controlling a vehicle is accustomed to velocity control in the forward direction and position control in the lateral direction. This is made clear when we investigate to which kind of steady-state people are most sensitive.

This is the point we take into considerations when we design lateral control for a wheelchair. Based on this idea, a position controller is suggested. Figure 8 is the proposed controllers for disturbance rejection in the lateral direction. The integrator is included in order to make the lateral control a position controller.

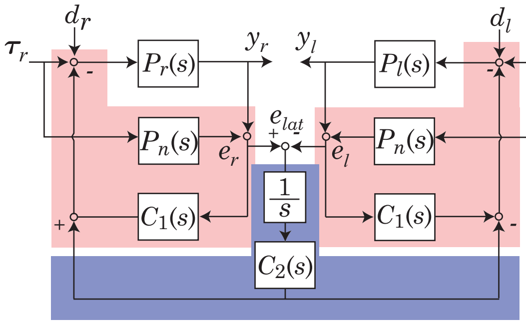


Fig. 8. Structure of a lateral disturbance rejection controller - position control type

Small difference between the angles of two wheels results in a turn of a wheelchair. The lateral controller described in Figure 8 is designed to remove this difference, making high stiffness against the lateral disturbance force.

Note that the longitudinal disturbance attenuation controllers are included too. Controllers in the upper two colored area are the longitudinal disturbance attenuation controllers which has the same structure described in Figure 2; each y, e and τ correspond y, e and τ in Figure 2. The subscripts l and r represent left and right. The controller in the below colored area is the lateral disturbance rejection controller. These two kinds of controllers constitute two dimensional disturbance suppression control of a wheelchair. A power assist wheelchair has two motors, one for each wheel so that it can achieve this two dimensional control.

In order to design $C_2(s)$ in Figure 8, the lateral disturbance is defined d_{lat} like Equation (7). d_r and d_l are the disturbance acting on the right wheel and the left wheel respectively.

$$d_{lat} = d_r - d_l \tag{7}$$

The purpose of this lateral direction control is to make the effect by this d_{lat} on $\int (e_r - e_l)\, dt$ as small as possible. Let us define this $e_r - e_l$ as e_{lat}. The controller $C_2(s)$ can render the transfer function from d_{lat} to e_{lat}.

$$T_{lat}(s) = \frac{e_r - e_l}{d_{lat}} = \frac{P}{1 + P(C_1 + 2\frac{C_2}{s})} \tag{8}$$

To achieve position control, $C_2(s)$ should provide some stiffness against the lateral disturbances.

$$C_2(s) = \frac{1}{2}(K_D s + K_P) \tag{9}$$

This $C_2(s)$ makes the transfer function

$$T_{lat}(s) = \frac{1}{(J + J_d)s^2 + (B + B_d + K_D)s + K_P}. \tag{10}$$

Here, K_P works as the stiffness against the lateral disturbances which is a key parameter determining how much the controller suppress lateral disturbance. In spite of this existence of stiffness K_P, a constant lateral disturbance can cause a constant position error. If we want to reject that error, an integrator $\frac{K_I}{s}$ should be added in $C_2(s)$, but it may bring about some troublesome problems such as a wind-up to which integration is subject.

A disturbance observer can be adopted for this lateral control, and [20] have suggested that disturbance observer as a lateral disturbance rejection controller for a wheelchair. However, the disturbance observer has an inverse dynamics of a wheelchair and is quite sensitive to unknown dynamics. To achieve robust performance, the provided position-control-type disturbance rejection control may be preferred.

For suppression of the gravity in the longitudinal direction, $C_1(s)$ employs the controller described in Figure 2 and just increases the inertia and damping against the gravity. But the $C_2(s)$ allows us a strict disturbance rejection control in the lateral direction. This is the control strategy for the two dimensional disturbance suppression control of the wheelchair.

C. *Experimental Verification of Proposed Method*

In order to verify the effectiveness of the proposed control, two kinds of experiments are conducted: one is to see the disturbance rejection performance in the lateral direction, and the other is to see the independence between two controllers in the lateral and longitudinal directions.

In these experiments, the feedforward assist control in Figure 2 is not utilized so that applied human torque works only as disturbance. In order to measure this disturbance, we use torsion sensors to measure the torque applied by the rider. Since the feedforward assist control is not implemented, the applied human torque will not be assisted and just works as disturbances. These disturbance torques are used to see how our proposed controller works. Figures 9 to 11 are the results.

Figure 9 shows applied disturbances in the lateral direction, and these disturbances are calculated by subtracting measured human torques. The dotted torque is applied to a wheelchair without the lateral control, and the solid line is the torque applied to a wheelchair with the lateral control. At the beginning (to 5 second in the without control case, and to 7 second in the with control case), the right and left disturbances are exerted in the same direction so that the lateral disturbance is not so large. After that period, the torque is applied in the opposite directions, producing quite large lateral disturbance.

Figure 10 is the output of this applied torques and shows the differences between the right and left wheel angles. Without the lateral control, the difference becomes large, and this will make the wheelchair turn. While, with the control the difference does not become so large that the lateral disturbance does not interfere with the moving direction.

Independence between the longitudinal control and the lateral control is also verified with an experiment. In this experiment, longitudinal disturbance attenuation control is not designed so strong. J_d and B_d in Figure 2 is set small to decrease the performance of the longitudinal disturbance attenuation control. In contrast, lateral disturbance control is designed strong enough to suppress the lateral disturbance. This design will make distinction between performances of two controllers, and by this, independence between the lateral disturbance rejection control and the longitudinal disturbance attenuation control can be verified.

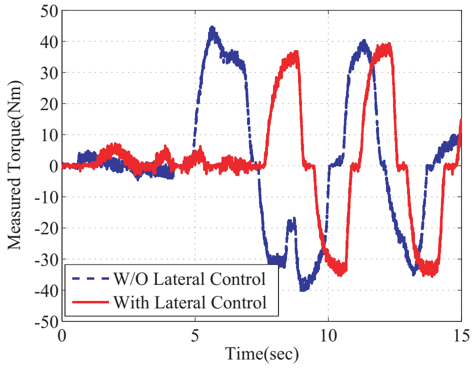


Fig. 9. Exerted lateral disturbances

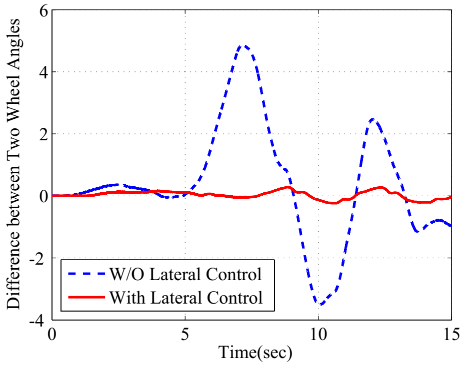


Fig. 10. Differences between the right and left wheel angles

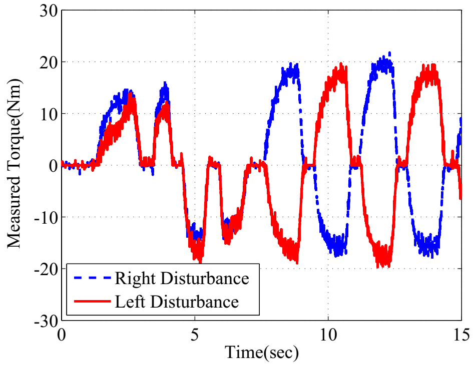


Fig. 11. Disturbances in the same and opposite directions

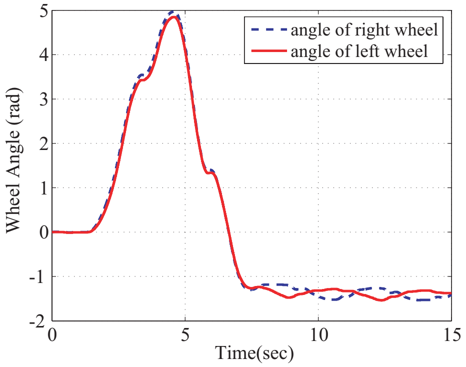


Fig. 12. Independence of the control on the direction: wheel angles

Figure 11 and 12 show that the proposed lateral control rejects only the lateral disturbance. Figure 11 is the measured disturbances, and it shows the disturbances on two wheels respectively. Until 7 second, both right and left disturbances are working in the same direction, which drives the wheelchair in the linear direction. Angles described in Figure 12 shows that the wheelchair goes forward until 7 second and the disturbance is not removed. From 7 second, the torque works in the opposite direction. Although the magnitude of the torque is much larger, the angle driven by that torque is quite small.

This represents that the right and left disturbances are eliminated, making clear that the proposed controller only rejects disturbances when they are applied in the lateral direction, which proves that proposed controller removes the disturbances in different ways according to their directions. This result ensures that the proposed controller that adopts two different strategies for two directions can be successfully implemented.

V. CONCLUSION

We suggested three controllers for a power assist wheelchair and a method to integrate those controllers. Controllers for the longitudinal and lateral directions are designed in terms of human-friendliness: velocity control for the longitudinal direction and position control for the lateral direction. Experimental results shows that they can function as desired respectively.

Taking advantage of the two-degree-of-freedom controller, feedforward power assist control can be designed independently from feedback controller. By changing the assist ratio with respect to the center of the gravity of a wheelchair, tipping over problem can be addressed.

The controller proposed in this paper proves to be a safe and satisfying integrated assist control for a power assist wheelchair.

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