# Realization of Fractional Order Impedance by Feedback Control


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*Abstract*— **Fractional order impedance control is proposed in this paper to control power assistive devices in more human friendly way. A novel discretization method of a fractional order integrator is proposed based on a multirate filter approach. Optimal coefficients for the multirate fractional order integrator are decided using golden-ratio particle swarm optimization. Based on this fractional order integrator, fractional order impedance control is realized using feedback control design. As another suggestion in this paper, force-sensor-less impedance control is introduced which uses only position information focusing on relationship between external force and motor force. The characteristics of fractional impedance are analyzed by simulations, and lastly future topics on fractional calculus application are explained.**



***Key Words*** : *fractional order impedance, fractional order calculus, multirate sampling, particle swarm optimization, golden-section search, human-friendly motion control, impedance control*


## I. Introduction

### A. Need for Novel Impedance Control in Assistive Devices

Impedance concept has been a quite important issue in robot control, and now its significance becomes clearer when robot applications are oriented toward assisting human. There are a number of researches on impedance control and impedance estimation. Many of the researches are on force control of robots, which generate position references for robots using force sensors. In human neuromuscular system, however, this impedance is also important concept. [1] has explored ability of human to perceive mechanical impedance, which means human perceives impedance as information of his external environment. Paying attention to this characteristic, controllers of robots which co-work with human are usually designed focusing on impedance design[2], [3].

There are other researches which indicate the impedance model used in general impedance control is insufficient to describe the impedance to which people are accustomed. Some researches adopt the neural networks to describe more complicated impedance [4], [5]. They point out the time invariant linear impedance model which is generally describe as $\frac{1}{Ms^2+Bs+K}$, is limited and can hardly represent complex task strategies.

### B. Proposal of Fractional Order Impedance

In this paper, as a promising impedance control method, fractional order impedance control is proposed. Fractional order calculus which realizes integration/differentiation in fractional order is an old topic. It, however, recently gains increasing attentions as a promising tools in mechatronics and biological systems[6], [7]. Especially [6] introduces the fractional order calculus in biomimetic actuator control and reveals the possibility of fractional order calculus to model the dynamics of biomimetic materials. Also [8] applied this fractional order calculus concept to the theory of electrical impedance of botanical elements. Although they focus on the possibility of fractional calculus as a new dynamic model, none of them have used it as a desired impedance of actuator.

This paper utilizes the fractional order calculus in impedance design of a robot. Concluding from the references, this fractional order impedance has more possibility to describe human-friendly impedance, since it can provide more tunable elements and realize various impedance: for instance, the middle phase of stiffness and damping. Especially, assistance in phase point of view can be highlighted in fractional impedance control.

In Section II, a novel discretization method of fractional integrator constiting of a multirate filter is suggestsand to propose this fractional order impedance control and its effectiveness is compared with other realization methods'. Then using the proposed discretization of fractional order integrator, fractional order impedance controller is designed in Section III. This control design only uses position feedback; force information is not measured in the proposed control. Lastly, a strategy which incorporates the fractional order calculus in force-sensor-less power assist control is give in Section IV as a future topic on fractional order calculus applications to human-friendly motion control.

## II. Novel Discretization Method of Fractional Order Integrator

The concept of fraction order calculus is not new. As recent developments of this fractional order calculus, it is applied to control design in order to improve performance, especially robustness. In these researches, the fractional order calculus is realized using certain approximation schemes[6]. Theoretically, the fractional order differentiation/integration needs an infinite dimensional linear filter, however, in practical point of view, a band-limit implementation is sufficient to use and this can be realized using a finite dimensional approximation.

### *A. Realization Methods of Fractional Order Calculus*

Among existing implementation methods of fractional order calculus, the CRONE(Contrôle Robuste d'Ordre Non Entier) method [7] and CFE(Continued Fraction Expansion) approach[9] are most well known methods.

The CRONE control achieves a band-limited fractional differentiator/integrator based on broken-line approximation approach. Figure 1 illustrates an example of the approximation of band-limit fractional order integration.

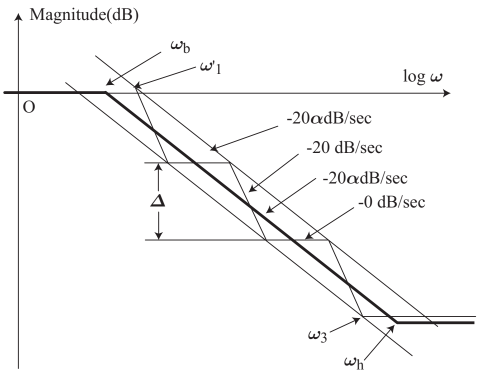



Fig. 1. An Example of Broken-line Approximation (N = 3)

$\omega_h$'s and $\omega_b$'s are the poles and zeros which determine the interval of fractional integration. The approximation is formulated as Equation (1) to (3).

$$\left(\frac{\frac{s}{\omega_h}+1}{\frac{s}{\omega_b}+1}\right)^r \simeq \prod_{i=0}^{N-1}\frac{\frac{s}{\omega'_i}+1}{\frac{s}{\omega_i}+1}, \qquad (1)$$

where

$$\frac{\omega_{i+1}}{\omega'_i}=\zeta, \frac{\omega'_i}{\omega_i}=\eta, \omega_0=\eta^{\frac{1}{2}}\omega_b, \omega'_{N-1}=\eta^{-\frac{1}{2}}\omega_h \qquad (2)$$

and

$$\zeta\eta=\left(\frac{\omega_h}{\omega_b}\right)^{\frac{1}{N}}, (\zeta\eta)^r=\zeta. \qquad (3)$$

This approach is based on the fractal concept and the greater N is the better the approximation becomes.

The other implementation method is to discretize the fractional order differentiator $s^r$ using continued fraction expansion. The Tustin transformation is one of the most widely used generating function and [6] uses this Tustin rule and the Al-Alaoui operator to discretize $s^r$, expanding it using the continued fraction expansion(CFE). Equation (4) describes the approximation $s^r$ in the form of the tustin rule.

$$D^r(z)=\left(\frac{2}{T}\right)^r \mathrm{CFE}\left\{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^r\right\} \qquad (4)$$

The CFE of $D^r(z)$ has the following form:

$$D^r(z)=1+\cfrac{z^{-1}}{\frac{1}{2r}+\cfrac{z^{-1}}{-2+\cfrac{z^{-1}}{\frac{3}{2}\frac{r}{r^2-1}+\frac{z^{-1}}{2+\cdots}}}} \qquad (5)$$

This expansion can produce a certain order IIR form filter that can discretize $s^r$ to a certain accuracy.

### *B. Discretization of Generating Function by Multirate Filter*

Motivated from the CFE of the Tustin transformation in Equation (5), we propose a novel discretization method of $\frac{1}{s^r}$ in this section using multirate sampling. For an example, with single rate sampling, Tustin transformation of a half order integration $\frac{1}{s^{0.5}}$ is approximated by a rational function of $z^{-1}$. [9] is one good example of this $z^{-1}$ function approximation. With multirate sampling, however, the same Tustin transformation can be approximated by $z^{-0.5}$ or other fractional power function of $z^{-1}$. This is the idea of our proposal, which is illustrated in Equation (6).

$$\begin{aligned}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^{-\frac{1}{2}} &\simeq \frac{n_0+n_1z^{-\frac{1}{2}}+n_2z^{-1}+\cdots+n_{2n}z^{-n}}{1+d_1z^{-\frac{1}{2}}+d_2z^{-1}+\cdots+d_{2n}z^{-n}} \\ &= f(z) \qquad (6)\end{aligned}$$

Odd terms in either numerator or denominator will be removed ($n_{2n+1}$ or $d_{2n+1}$) to make the filter a multirate sampling filter. The only problem is how to decide the coefficients $n_0$ to $d_{2n}$. Although expansions or other theoretical approaches can be employed for this, totally different one approach is adopted here: GSPO[10], an improved version of PSO (Particle Swarm Optimization).

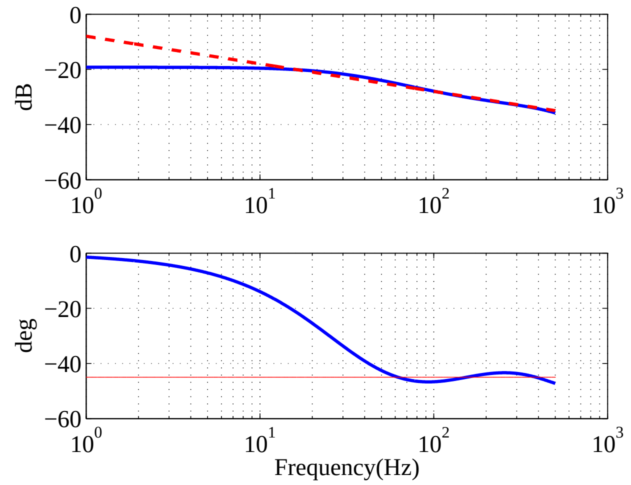
This GPSO can optimize the approximation performance of the filter (6) producing the optimal coefficients of $n_0$ to $d_{2n}$ which will decrease the approximation error. This approximation error is defined as the difference between the frequency responses of ideal $\frac{1}{s^{0.5}}$ and the multirate filter $f(z)$ in Equation (6) in the frequency domain. Equation (7) and (8) are the error definition used in the GPSO.

$$\sum_{k=1}^{K}\left\{20\log 10\left|\left(\frac{1}{j\omega_k}\right)^{\frac{1}{2}}\right| - 20\log 10|f(j\omega_k T)|\right\} \qquad (7)$$

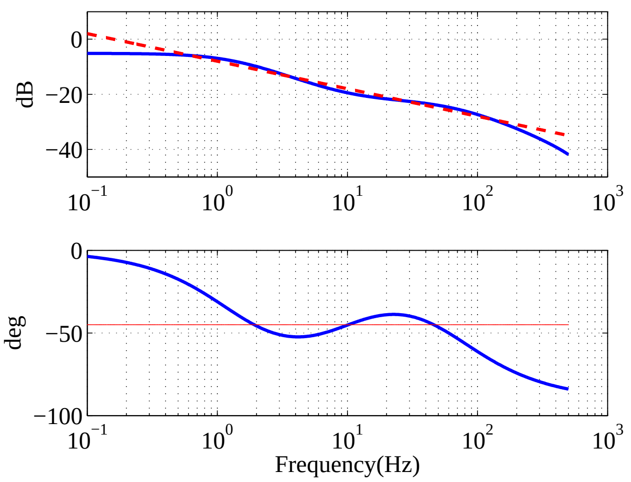
$$\sum_{k=1}^{K}\left\{\arg\left(\frac{1}{j\omega_k}\right)^{\frac{1}{2}} - \arg\left(f(j\omega_k T)\right)\right\} \qquad (8)$$

Based on this definition, the GPSO evaluates the gain and phase error at $K$ numbers of frequencies and finds the optimal coefficients which will minimize these errors. At first, the optimal coefficients for a first order multirate filter($n=1$ in Equation (6)) are calculated. Input data is sampled at 1msec and output at 0.5msec, which makes the filter multirate. Three types of frequency bandwidth (50Hz to 500Hz, 1Hz to 50Hz, and 1Hz to 500Hz) are evaluated and three filters are produced. Figure 2 are the result described in the Bode diagrams.

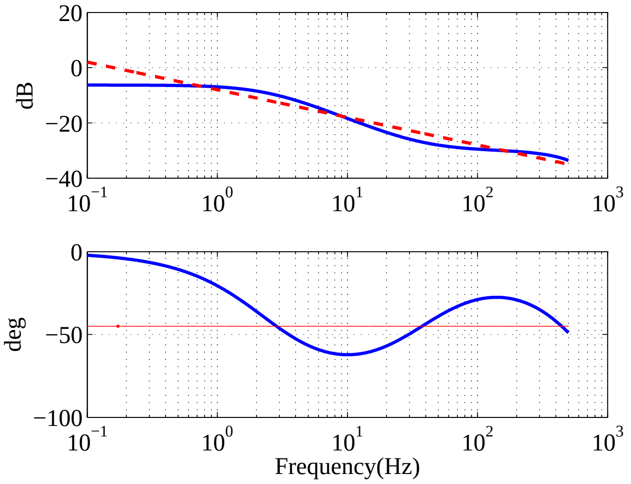
In Figure 2, the red dotted lines are the ideal frequency responses of $\frac{1}{s^{0.5}}$ and the blue solid lines are the produced




(a) Optimized in the range of 50Hz to 500Hz
($n_0 = 0.709, n_1 = -0.271, d_1 = -0.992, d_2 = 0.082$)




(b) Optimized in the range of 1Hz to 50Hz
($n_0 = 0.338, n_1 = -0.318, d_1 = -1.81, d_2 = 0.807$)




(c) Optimized in the range of 1Hz to 500Hz
($n_0 = 0.739, n_1 = -0.583, d_1 = -1.07, d_2 = 0.0724$)

Fig. 2. Optimized Multirate Fractional Order Integrator

frequency responses of optimized multirate filters. Gain and phase are well fitted in the evaluated frequency ranges, which shows the optimization performance of the GPSO is excellent.

### *C. Comparison with Other Realization Methods*

Here, performance of other fractional order calculus implementation methods are compared with our proposed method. First, Bode diagram of $s^{0.5}$ approximated by the CFE in [9] is plotted in Figure 3.

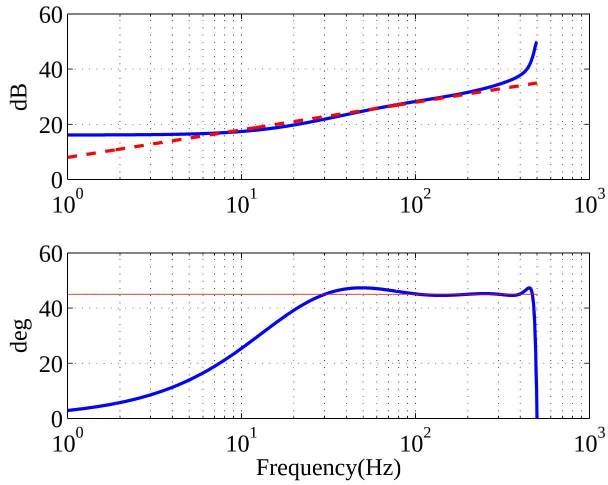



Fig. 3. $s^{0.5}$ Discretized by the CFE Approximation

Third order approximation, $44.72\frac{1-0.5z^{-1}-0.5z^{-2}+0.125z^{-3}}{1+0.5z^{-1}-0.5z^{-2}-0.125z^{-3}}$ is used for this approximation. Compared with Figure 2 (a), the result in Figure 3 is fit with the ideal response with wider range of frequency; higher order, almost double order of our proposed method enables better approximation. However, due to the multirate sampling, the proposed method has better approximation performance around the nyquist frequency. Additionally, our proposed method can adjust the frequency ranges where the response needs to be approximated.

Another implementation the CRONE also can adjust this approximated frequency range. Figure 4 shows Bode diagram of the CRONE method which approximates $s^{-0.5}$ with a second order filter. $w_b$ is set as 10Hz, and $w_h$ as 500Hz. The result shows the CRONE implementation is good at gain-fitting (a proper gain is multiplied to bias the gain), while its phase is the most distant from the ideal phase among three methods.

These comparison clarifies that our proposed method is effective when designing a digital filter which can discretize fractional order calculus in a certain range of frequencies including the nyquist frequency.

## III. Design of Fractional Order Impedance by Position Feedback Control

Using the proposed fractional order calculus discretization, fractional order impedance controller is designed in this section.

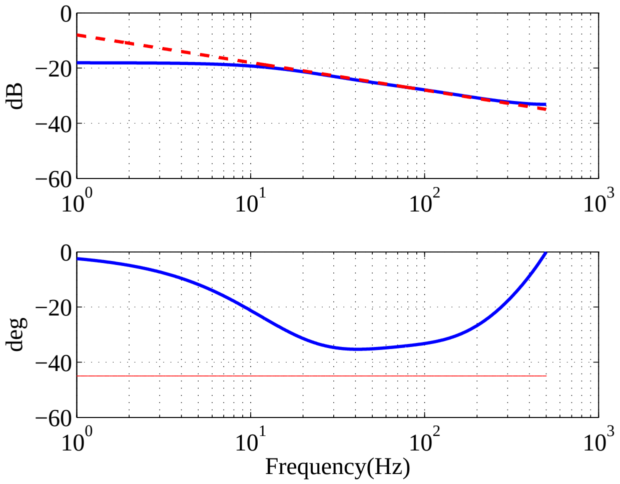



Fig. 4. $s^{-0.5}$ Discretized by the CRONE Approximation

### A. Design of Impedance Generated by a Motor Force

Impedance that a man will feel when he applies his force to the motor is determined by the reaction force that the motor generates. This reaction force is decided by a feedback controller based on the displacement. The relationship between the reaction force and the displacement from the reference position or velocity by external force is determined by the sensitivity function or disturbance response of a controller.

This fact enables us force control without a force sensor; force control can be achieved by design of the sensitivity function which is identified with the impedance. As will be discussed in the next section, the position control gain works as stiffness element in the impedance concept and the velocity control gain works as viscosity element; by adjusting these gains, the impedance in force control can be designed.

### B. Impedance Control by Feedback Control

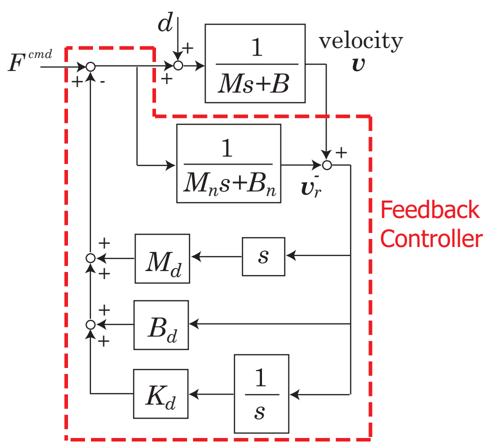



Fig. 5. Impedance Control by Velocity Feedback Control

Figure 5 is the proposed feedback controller structure where $M$ is the true mass of a target plant, $B$ is its true viscosity and $J_n$ and $B_n$ are the nominal model of the mass and the viscosity in the feedback controller. This controller can adjust the disturbance response in terms of impedance. Assuming $J_n = J$, $B_n = B$, the controller generates force $F_m$, the amount of which can be described like Equation (9)

$$F_m = \left( (M_d + M)\,\Delta\dot{v} + (B_d + B)\,\Delta v + K_d \int \Delta v dt \right), \tag{9}$$

where $\Delta v$ is the deviation in velocity by the disturbance.

This relationship is the mass-viscosity-elasticity model which is widely-used in the impedance control; the gains $M_d, B_d, K_d$ work as the mass, viscosity and elasticity. By adjusting these gains, the impedance of the motor can be designed.

The feedback control strategy has been used in disturbance attenuation control in human-friendly motion control[11] where the degree of disturbance attenuating torque should be designed in terms of impedance. [11] shows the usefulness of this impedance design approach in disturbance attenuation control design of power assistive devices.

### C. Analysis of Characteristics of Fractional Order Impedance

If the integrator or differentiator in Figure 5 is substituted with fractional order calculus, that can realize fractional order impedance. Figure 6 is one example of this frictional order impedance control.

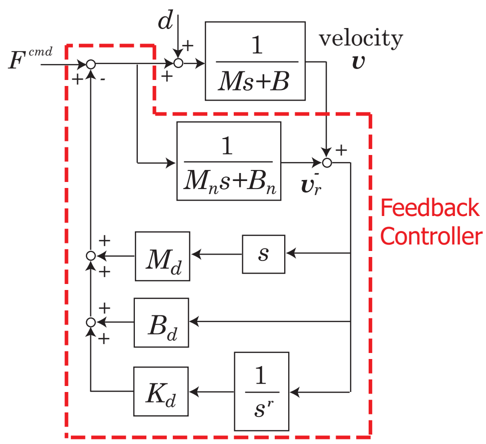



Fig. 6. Structure of Fractional Impedance Control

If $r$ is set as 0.5 at a certain range of frequency, it can realize the middle state of viscosity and elasticity, which can generate a novel reaction force that can be felt by human.

A simulation which shows characteristic of the fractional order impedance in time domain can reveal what reaction force a human can feel. As a more fundamental simulation, difference in step response between an integrator and fractional order integrator is conducted. Figure 7 is the simulation result. With a unit step input, three calculation: integration, low pass filtering, and fractional order integration approximated by a first order multirate filter ((c) in Figure 2), are conducted.

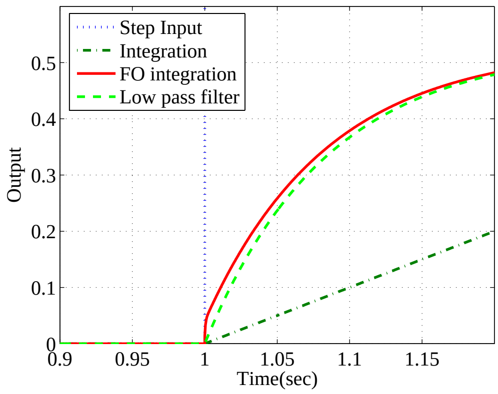



Fig. 7. Step Response of Approximated Frational Order Integrator

The response of fractional order integrator is quite fast at first; almost equal to proportional calculation. After quite short period, however, its response becomes slower, and it finally converges a certain value same with a low pass filter output. This result explains that the smaller phase delay in fractional order calculus can realize fast reaction in short time range and smooth reaction in long time range, which can not be accomplished by an integrator nor a low pass filter.

## IV. Fractional Order Calculus Application to Force-sensor-less Power Assist Control

In power assist control of assistive devices, force-sensor-less power assist control is one of most promising technology [12], [13]. The disturbance observer is employed to estimate the applied human torque. Although the estimation is subject to noise and modeling error, estimation speed is quite faster than that of a force sensor. This is one of the advantage in using disturbance observer[12].

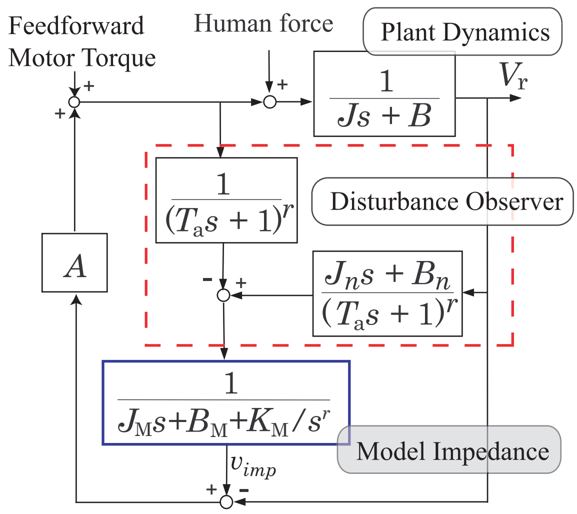



Fig. 8. Force-sensor-less Power Assist Control Incorporating Fractional Order Calculus

Figure 9 illustrates this power assist controller using blocks.

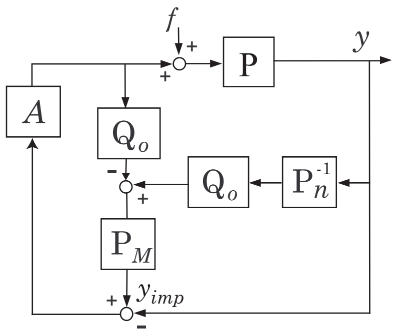



Fig. 9. Blocks of Force-sensor-less Power Assist Control

Fractional calculus can be applied in this power assist control in two ways: fisrt, model impedance $P_M$ in Figure 9 can be designed as a fractional order impedance. Figure 8 is one example of the fractional order impedance; middle state of elasticity and viscosity. Since this force-sensor-less power assist control increases the sensitivity of the system to assist external force, it degenerates system's robustness to modelling error. Figure 10 shows this tradeoff.

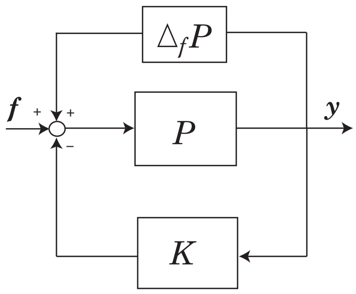



Fig. 10. Tradeoff between Robustness and Power Assistance

The transfer function from the external force to be assisted, $\frac{P}{1+PK}$ is same with the transfer function to the feedback modeling error $\Delta_f P$. The amplitude of transfer function should be set large in order to assist or amplify the external force $f$. This large amplitude, however, leads to small model uncertainty margin. According to the small gain theory, Equation (10) should be satisfied for the system to be stable under the modeling error $\Delta_f P$.

$$\left\| \frac{P\Delta_f P}{1+PK} \right\|_\infty < 1 \tag{10}$$

Equation (10) clarifies the limitation of force-sensor-less power assist control; the tradeoff between power assistance performance and robustness to model uncertainty. We cannot increase both of $\frac{P}{1+PK}$ and $\Delta_f P$. This point explains that more elaborate design of feedback control is necessary in the force-sensor-less power assist control. The design of model impedance including fractional order calculus can be a solution for the tradeoff design.

The other solution is the desing of low pass filter incorporation fractional order calculus. A Research [14] reveals that the relative degree of the Q-filter (which is also described as $Q_o$ in Figure 9) is the major tuning knob for the tradeoff between the phase margin loss and the strength of the low frequency vibration suppression.

$$T_{OL} = \frac{A(1 - Q_o P_M P_n^{-1})}{1 + A Q_o P_M} P \quad (11)$$

$$T_{yf} = \frac{(1 + Q_o P_M A) P}{1 + AP + A Q_o P_M (1 - P_n^{-1} P)} \quad (12)$$

Equation (11) is the open loop transferfunction and Equation (12) is the closed loop transfer function of Figure 9 respectively. Under the condition $A \gg 1$, in the low frequency range where $Q_o$ works as 1, the closed loop system will be $\frac{P_M P}{P + P_M (1 - P_n^{-1} P)}$. In this range, the feedback controller can achieve power assistance succesfully with the possibility to be unstable due to the term $(1 - P_n^{-1} P)$. While in the high frequency range, where $Q_o$ works as 0, the system will be $\frac{P}{1+AP}$. In this range, we cannot hope for any power assistance although there is no threat to the stability since there is no negative term. These facts explain that we can tune the tradeoff of power assistance and robust stability by designing an adequate low pass filter $Q_o$.

To investigate this design in more detail, each block is substituted as follows:

$$Q_o = \frac{1}{D_o}, P_M = \frac{1}{D_M}, P_n = \frac{1}{D_n}, P = \frac{1}{D} \quad (13)$$

With this substitution and under the same condition $A \gg 1$, the open loop transferfunction and the closed loop transfer function are given by

$$T'_{OL} = \frac{D_M D_o - D_n}{D} \quad (14)$$

$$T'_{yf} = \frac{1}{D_M D_o + (D - D_n)}. \quad (15)$$

Roughly speaking, in the frequency range where, for power assistance, the gain of $D_M$ is set smaller than that of $D_n$, the phase of Equation (14) is near 180 degree. This is the reason why the force-sensor-less power assist control easily becomes robust unstable. As the coefficients of $D_M$ are set small to obtain large assistance, the less gain margin it will have because the gain of Equation (14) approaches to 1 with the phase of 180 degree.

To deal with this robustness problem, the low pass filter $D_o$ should be designed elaborately based on Equation (15). If there is some information on the model variation of $(D - D_n)$, the filter $D_o$ can be designed based on that information to make the denominator $D_M D_o + (D - D_n)$ stable in spite of the existence of model variation. In this robust control design, the fractional order low pass filter described in Equation (16) can be a good solution since it has the relative degree as the tuning knob as well as the time constant $\tau_a$.

$$D_o = (\tau_a s + 1)^r \quad (16)$$

## V. Conclusion

A novel implementation method of fractional order integrator is suggested and how to realize that fractional order impedance is also suggested in this paper. Simulation results shows that the fractional order calculus can generate different time response and phase characteristics than integrator or low pass filter.

Control of phase is important function in the control of power assistive devices; however, there is no precedent research. Most power assist control are focusing on amplification of power not phase compensation. The fractional order impedance concept must be a key technology in control of phase in power assist devices.

This paper also analyzed robust stability in force-sensorless power assist control which has not been researched. It is revealed that fractional order calculus can realize more elaborate design of low pass filter and improve the robustness of the controller.

## References


[1] Y. Tanaka, T. Tsuji, H. Miyaguchi, “Analysis of human perception ability for robot Impedance”, in *Proc. 16th IFAC World Congress*, Prague, Czech Republic, July, 2005.

[2] S.A. Schneider, R.H. Cannon Jr., “Object impedance control for cooperative manipulation: theory andexperimental results”, *IEEE Trans. Robotics and Automation*, vol. 8, no. 3, pp. 383-394, Jun. 1992.

[3] R. Ikeura, H. Inooka, “Variable impedance control of a robot for cooperation with a human”, in *Proc. IEEE International Conference on Robotics and Automation*, Nagoya, Japan, May, 1995, vol.3, pp.21-27.

[4] H. Asada, “Teaching and learning of compliance using neural nets: representation and generation of nonlinear compliance”

[5] J. Seul, T.C. Hsia, “Neural network impedance force control of robot manipulator”, *IEEE Trans. on Industrial Electronics*, vol. 45, no. 3, pp. 451-461, Jun. 1998.

[6] Y. Chen, D. Xue, H. Dou, “Fractional calculus and biomimetic control”, in *Proc. IEEE International Conference on Robotics and Biomimetics*, Shenyang, China, Aug. 2004, pp. 901- 906

[7] A. Oustaloup, J. Sabatier, and P. Lanusse, “From fractal robustness to CRONE control”, *Fractionnal Calculus and applied Analysis*, vol. 2, no. 1, pp. 1-30, 1999

[8] I.S. Jesus, J.A.T. Machado, J.B. Cunha, M.F. Silva , “Fractional order electrical impedance of fruits and vegetables”, in *Proc. the 25th IASTED international conference on Modeling, indentification, and control*, Lanzarote, Spain, 2006, pp. 489 - 494

[9] Y. Chen, B.M. Vinagre, and I. Podlubny, “Continued fraction expansion approaches to discretizing fractional order derivatives-an expository review”, in *Nonlinear Dynamics*, vol.38, no. 1-2, pp.155-170, Dec. 2004.

[10] S. Oh, Y.Hori, “Control parameter optimization in the hardware-in-the-loop system using novel search algorithm”, in *Proc. Annual Conference of the IEEE Industrial Electronics Society*, Paris, France, Nov. 2006, pp.5240-5245.

[11] S. Oh, N. Hata, Y. Hori, “Integrated motion control of a wheelchair in the longitudinal, lateral and pitch directions”, in *Proc. IEEE International Workshop on Advanced Motion Control*, Istanbul, Turkey, Mar. 2006, pp.344- 349.

[12] S. Katsura, Y. Matsumoto, K. Ohnishi, “ Analysis and experimental validation of force bandwidth for force control”, in *Proc. IEEE International Conference on Industrial Technology*, Maribor, Slovenia, Dec. 2003, vol.2, pp. 796- 801

[13] S. Oh, Y. Hori, “Sensor free power assisting control based on velocity control and disturbance observer”, in *Proc. Proceedings of the IEEE International Symposium on Industrial Electronics*, Dubrovnik, Croatia, June, 2005. vol.4, pp. 1709- 1714

[14] Y. Chen, B. M. Vinagre,I. Podlubny, “Fractional Order Disturbance Observer for Robust Vibration Suppression”, *Nonlinear Dynamics*, vol. 38, no. 1-4, pp. 355-367, 2004