Development of Noise Robust State Observer for Power Assisting Wheelchair and its Applications

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This paper suggests a state observer for a power assisting wheelchair. In order to control a power assisting wheelchair in the way that makes the user safe and comfortable, we need some information of wheelchair motion.

Three sensors (encoder, gyroscope, and accelerometer) are employed to obtain driving velocity and inclination angle of a wheelchair. Each sensor has inherent defect. An LQG designed observer is designed to overcome these defects. Proposed observer turns out very robust to sensor noise and to provide good estimations.

As control applications of the observer this paper suggests three control designs for a wheelchair. Overturn prevention control, gravity compensation control on a slope, and force sensor-less assisting control.

Keywords: state observer, LQG method, power assisting wheelchair, sensor fusion, power assisting control

1. Introduction

Emerging Power Assistance Tools

Nowadays advanced power assistance tools are drawing people’s attention as emerging control application. These tools are usually located near a man or attached to one’s body, and amplify human power. This operational environment makes the control difficult and unique to these tools. Though a variety of power assistance tools are being developed, there is little discussion on control methods for those tools.

A power assisting wheelchair is a good example of that kind of assistance tools. Development of controllers for a power assisting wheelchair has just started. In conventional power assisting wheelchairs, motors just multiply original human force to drive by up to several times.

But, if the controller can sense some information on driving situation, advanced power assistance controls can be achieved. To this end, we need an observer that can estimate some important physical values.

Necessity of State Observer in the Control of Power-assisted Wheelchair

Motion of a wheelchair is different from that of a car. As for the wheelchair, the motion in the pitch direction is important from the stability viewpoint. In order to obtain the inclination angle of the wheelchair, the gyroscope is adopted for the measurement. But the integration calculation is necessary to get the angle, and this procedure makes the estimation weak to the noise in the gyroscope.

Furthermore, the accurate velocity of the wheelchair is necessary for the advanced power assisting control. With the feedback control of velocity, we can design the response by disturbance and human force respectively. This can realize various power assisting controls. Encoders are adopted for this measurement of the velocity. But, if the equipped encoder has low resolution, slow velocity of the wheelchair results in noisy and incorrect velocity estimation by the encoder.

In this paper, an observer which is robust to sensor noises is proposed in order to obtain this pitch information or inclination angle and the velocity of a wheelchair.

In section 2, two different mathematical models of wheelchair are suggested. Based on these models, observers are designed and verified by experiments in section 3, and 4. In section 5, control examples that can make use of this observer are introduced.

2. Mathematical Modeling of a Wheelchair

Two descriptions of wheelchair motions are adopted for observer designs.
Inverted Pendulum Model  
Wheelchair system consists of a wheelchair and a man who rides on it. This system analogizes to a cart with an inverted pendulum
\(^{(1)}\). Considering this point, motion equations of \(\theta, \phi\) can be described in equation (1), (3) using the Euler-Lagrange Differential Equation. Generally, the inclination angle \(\varphi\) will not change so largely that it can be assumed to be around 0 rad. Under this assumption, some approximations in equation (5) can be taken. These approximations linearize the equations (1), (3) into (2), (4).

\[
\tau + d_\theta = \{(M + m)r + J_M\} \ddot{\theta} - m lr \varphi \cos \varphi + m lr \varphi^2 \sin \varphi + B_M \dot{\theta} \quad \ldots \ldots \ldots \ldots \ldots (1)
\]

\[
\begin{aligned}
\varphi &\approx \{(M + m)r + J_M\} \ddot{\varphi} - m r \varphi \cos \varphi + mg l \sin \varphi + B_M \dot{\varphi} \quad \ldots \ldots \ldots \ldots \ldots (2)
\end{aligned}
\]

\[
\begin{aligned}
d_\varphi &= (J_m + ml^2) \ddot{\varphi} - m lr \varphi \cos \varphi - m gl \sin \varphi + B_m \dot{\varphi} \quad \ldots \ldots \ldots \ldots \ldots (3)
\end{aligned}
\]

\[
\begin{aligned}
\cos \varphi & \approx 1, \quad \sin \varphi \approx \varphi, \quad \varphi^2 \ll 1 \quad \ldots \ldots \ldots \ldots \ldots (5)
\end{aligned}
\]

Because there are front wheels in a wheelchair, the motion of \(\varphi\) is restricted in the way of equation (6). \(\varphi_0\) is the minimum value of \(\varphi\) which is restricted by front wheels. But this restriction is not taken into consideration here for simplicity.

\[
\begin{aligned}
\{&\ddot{\varphi} = 0 \quad \text{if} \; \varphi \leq \varphi_0 \quad \text{and} \; \ddot{\varphi} \leq 0

&\ddot{\varphi} \text{ moves according to equation (1),(3) if} \; \varphi > \varphi_0 \quad \ldots \ldots \ldots \ldots \ldots (6)
\end{aligned}
\]

Simple Two Bodies Model  
This model uses the same 2 as a model of a wheelchair, but does not consider the detail relationship between a wheelchair and a rider.

The rotating angle of wheelchair wheel \((\theta)\) is propelled by external torque \(\tau\) and disturbance \(d_\theta\). The inclination angle of a rider \((\varphi)\) is propelled by disturbance \(d_\varphi\) and not related to the inertia force of the wheel \((\theta)\) motion. It is certain that the motions of two variables are related but it is the strategy of this model to sum those relationships into the disturbance and make the observer simple. The dynamics of each variable are described in equation (7).

\[
J_\theta \ddot{\theta} = -B_\theta \dot{\theta} + \tau + d_\theta, \quad J_\varphi \ddot{\varphi} = -B_\varphi \dot{\varphi} + d_\varphi, \quad x = r \theta \quad \ldots \ldots \ldots \ldots \ldots (7)
\]

Ignorance of the connection between wheels and a rider will make the observation incorrect; nevertheless the disturbance inputs will compensate this ignorance.

3. Observer Design Using LQG Method  
3.1 Output Equation of Multisensor  
As said in the introduction, we need to obtain correct information on driving condition of a wheelchair in real-time. To this end, here three sensors are used for measurements: encoder, gyroscope and accelerometer. Each sensor has weak point when it is used alone. To overcome these weak points, an observer that will estimate the inclination angle \((\varphi)\) and the driving speed \((\dot{x})\) is designed. This can be a kind of sensor fusion.

In order to design this observer, states in equation (8) are adopted.

\[
x = \begin{pmatrix} \dot{\theta} & \dot{\varphi} & \theta & \varphi & d_\theta & d_\varphi \end{pmatrix}^T \quad \ldots \ldots \ldots \ldots \ldots (8)
\]

Velocity of a wheelchair \(\dot{x}\) is calculated by \(\dot{x} = r \dot{\theta}\) ignoring slips on wheels. Based on these states, output equations of three sensors are described in Table (1).

<table>
<thead>
<tr>
<th>Encoder</th>
<th>(\omega_{\text{enc}} = \theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gyroscope</td>
<td>(\omega_{\text{gyro}} = \varphi)</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>(\omega_{\text{acc}}(\omega_x) = r \theta \cos \varphi + g \sin \varphi)</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>(\omega_{\text{acc}}(\omega_y) = g \cos \varphi - r \theta \sin \varphi)</td>
</tr>
</tbody>
</table>

The measurements \(\omega_x, \omega_y\) in the accelerometer are analyzed in figure (3). \(a_x\) is linearized on the assumption of equation (5). This results in the output equation for the observer shown in equation (9) and (10).

![Diagram](image)

\[
y = \begin{pmatrix} \phi \\ \omega_x \\ a_x \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{B_{\text{acc}}}{J_m} & g & 0 & 0 & \frac{1}{J_m} \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{J_m} \end{pmatrix} u(9)
\]

Equation (9) is the output equation based on the simple two rotating bodies model, and equation (10) is the output equation based on the inverted pendulum model.

In this research, the observer gain is decided using the LQG (Linear Quadratic Gaussian) method, for this observer uses the multisensor. The noise covariance data for the determination of the observer gain are like below.

\[
Q_{\varphi} = \text{diag}(Q_{\dot{\varphi}}, Q_{\varphi}, Q_{\dot{\varphi}}, Q_{\varphi}, Q_{\varphi}, Q_{\varphi}) \quad \ldots \ldots \ldots \ldots \ldots (11)
\]

\[
R_{\varphi} = \text{diag}(R_{\text{gyro}}, R_{\text{acc}}, R_{\text{acc}}) \quad \ldots \ldots \ldots \ldots \ldots (12)
\]

As \(Q_{\dot{\varphi}}, Q_{\varphi}\) become smaller, the noise in the estimated
\[ y = \left( \begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{2} m r B_m & 0 & 1 & 0 & 0 & 0 \\ \frac{1}{2} B_m & 0 & 0 & 1 & 0 & 0 \\ \end{array} \right) \left( \begin{array}{c} J_m + M l^2 \\ M m l^2 + g \end{array} \right) + \left( \begin{array}{c} 0 \\ 0 \\ g \end{array} \right) x + \left( \begin{array}{c} 0 \\ 0 \\ \frac{1}{2} m r l^2 \end{array} \right) u \] (10)

\( \dot{\theta} \) becomes smaller. The same is for \( Q_\varphi, Q_{\varphi^2} \), and the smaller they are, the less noise will \( \dot{\varphi} \) have. For good control performance, \( \dot{\varphi} \) should have small noise. Large \( R_{\text{acc}} \) will make the estimated states noiseless.

4. Experimental Results

Due to the limitation of sensors, the measurements of \( \dot{x} \) and \( \varphi \) can be incorrect. In order to get \( v_f \) information, we should differentiate the encoder output discretely. If the resolution of the encoder is too low or the angular velocity of the wheel is too low, the discretely differentiated velocity will be very noisy and incorrect. To overcome this, a low pass filter is utilized, but it will make the estimation slow.

And for the measurement of \( \varphi \), the value of a gyroscope is integrated. If there is some noise in the output of the gyroscope, the drift phenomenon will occur.

Proposed observer can overcome these two problems. Experimental results shown in figure 4 explain this point. Each figure shows \( \dot{\theta} \) and \( \varphi \) respectively. In (a), the wheelchair runs straightly without a wheelie, while in (b) there is a wheelie around 4 sec. Red line in the upper figure of each experiment, which shows \( \dot{\theta} \), is the differential of the encoder output. This value is very noisy and delayed, while the estimation of the proposed observer (blue line) is fast and not noisy. To investigate the robustness of proposed observer, we added a noise to the gyroscope output at 10 sec. Compared to the integration of the gyroscope output (red line in the below figure of each experiment), the proposed observer estimation (blue line) shows robust observation results.

![Fig. 4. Estimated States : angular velocity (\( \dot{\theta} \)), inclination angle(\( \dot{\varphi} \)) (Using Two Isolated Rotating Bodies Model)](image)

Figure 4 shows the experimental results done based
on the inverted pendulum model with the same covariance data \( Q_x \) and \( R_y \). This shows similar results with the simple model observer, which ascertains that this simple model is useful enough in the observer design of a wheelchair. From this experimental result, we can conclude that using the proposed observer, the drift phenomenon can be avoided and better velocity information will be obtained.

Figure 4 shows different observation results when choosing different \( Q_x \) and \( R_y \). This suggests that the selection of covariance matrix be adjusted according to the specification of each sensor.

![Fig. 6. Different Observation Results by Different Observer Gain](image)

5. Application for the Advanced Control of Power Assisting Wheelchair

In this section, control designs that are related with the proposed observer are introduced. Variable assist ratio controller \(^{(3)}\), flexible gravity compensation controller \(^{(6)}\), and force(torque) sensor-less power assisting control are the control designs.

5.1 Prevention of Overturn The position of the inclination angle \( \varphi \) in the phase plane can denote the stability of the wheelchair in the pitch direction. Figure 7 shows the phase plane of \( \varphi \). It is divided into three regions depending on the level of danger: A) proper safety zone \((\dot{\varphi} < 0 \text{ and below the negative slope asymptote})\), B) semi-safety zone \((\varphi < 0, \dot{\varphi} > 0 \text{ and below the negative slope asymptote})\), and C) dangerous zone \((\text{above the negative slope asymptote})\), and the location of \( \varphi \) in these three zones indicates the stability of the wheelchair.

Power assistance with fixed assist ratio (which means the extent of increase) sometimes provides excessive assisting power, and the excessive power results in the overturn of the wheelchair especially on slopes. The ratio should be changed according to \( \varphi \).

Using the estimation of \( \dot{\varphi} \), the assist ratio is changed as shown in the equation (13) and the figure 8, and this change stabilizes the wheelchair and prevent it from overturning on hills.

\[
\alpha = \alpha_{\text{max}} \exp(\beta \frac{\dot{\varphi}_p}{\theta_p}) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \cdot (13)
\]

![Fig. 7. Man-wheelchair Phase Plane](image)

5.2 Gravity Compensation Control We suggested the effect of gravity on a wheelchair be removed using two degree of freedom controller. This controller uses the velocity\((\theta)\) of a wheelchair, and feedbacks velocity errors. Figure 9 shows the structure of this gravity compensation controller. If the velocity information is obtained by the differential calculation of encoder readings, it will be very noisy especially at low speed. This noise in the velocity information in the feedback loop worsens comfortability. Using the proposed observer, this noise problem can be solved.

![Fig. 9. Structure of Gravity Attenuation Controller in a Power Assisting Wheelchair](image)

Two kinds experiment were done. One is done on level ground and the other is done on a hill. The result is shown in figure 10. In contrast to the drive on level ground, on a hill the controller produces a certain amount of motor torque while there is no human torque input. Almost same torque is produced even when the wheelchair descend the hill.

The amount of produced torque can be adjusted by the \( B_d \) parameter, and the parameter \( J_d \) will adjust the
response time against gravity.

5.3 Force Sensor-less Power Assisting Control

Power assisting controller usually employs force sensors to measure the force to be assisted. This limits the assistance and only the force applied to the certain region is assisted. The proposed observer also estimates the disturbance($d_0$) which is working on the wheelchair. This observed disturbance can indicate the exerted human torque roughly. Using this observed disturbance $d_0$, torque sensor-less power assisting controller for a wheelchair is designed like figure 11.

The purpose of this controller is not the increase in the power, but the decrease in the inertia of the wheelchair. With this control system, the transfer function from human force to velocity is:

$$T_{C2}(s) = \frac{1}{Js + B + A} \left( \frac{J_M s + B_M + A}{J_M s + B_M} \right) \cdots (14)$$

$J_M, B_M$ are parameters of model dynamics and can be chosen arbitrarily. $A$ is a feedback gain for velocity tracking. Appropriately chosen $J_M, B_M$ (smaller than $J, B$) will make a system sensitive to a proper extent and provide good assistance. The experimental results are presented in figure 12.

In this experiment, two types of wheelchairs are provided to a user, and the user rides wheelchairs and propels the wheelchair with almost the same torque in both cases. Figure (a) shows the observed disturbances including human force. These disturbances almost correspond to the imposed human torque, and the ranges of the observed disturbances in (a) are similar in both cases, especially in the first stroke. Figure (b) shows the velocities of the wheelchair. Velocity with proposed control is bigger than the one without control. The proposed control works as power assisting control.

6. Conclusion

In this paper, we proposed an observer design which is useful in wheelchair controls, and showed that the LQG designed observer using multisensor is very robust to sensor noise and provides good estimations. Three controllers that are related with this observer are introduced. The selection of the covariance matrix for the LQG observer design are left for further discussion.

This observer design is a necessary technology in the human-friendly motion control (4).

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