Body Slip Angle Observer for Electric Vehicle Stability Control Based on Empirical Tire Model with Fuzzy Logic Approach

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Abstract -For effective implementation of observer for vehicle body slip angle (β) estimation in electric vehicle stability control, an empirical tire model with fuzzy logic approach is proposed, in which a local linearization approximation of the nonlinear tire model is adopted. Tire model parameters of front and rear tires cornering stiffness are identified based on the real vehicle experiments. Based on such modeling method, a hybrid-like observer is developed, in which the local observers are designed as linear observers with Kalman filter theory to overcome the influence of system noise for on-board application. Fuzzy logic approach is adopted to combine the local observer models so as to deal with the nonlinear nature of vehicle dynamics. By choosing the membership functions of weighting factors to be dependent on lateral acceleration and road friction coefficient, the proposed observer is adaptive to different running conditions. The effectiveness is verified by simulation and experimental studies.

I. INTRODUCTION

An important advantage of electric vehicles (EVs) has been recognized is that motor's controllability can provide more flexible and novel ideas for vehicle stability control. Body slip angle (β) is an important value for such control strategies. However, as sensors to measure β value are very expensive, it needs to estimate β from only variables measurable [1] [2] [3]. For the on-board applications, vehicle dynamics are usually described as simplified models in such model-based observers. Then, the states estimations are based on such models and the accuracy of the models determines the effectiveness of their implementations. The most difficult for such β estimation methods is how to deal with the non-linear nature of vehicle dynamics, which makes vehicle characteristics change greatly as vehicle cornering severely compared with that of cornering moderately.

The main nonlinearity of vehicle dynamics comes from the tire force saturation in large tire slip angle region, decided by tires and road adherence conditions. In addition, the nonlinear nature of vehicle dynamics is further complicated for it is influenced by the characteristics of whole chasis elements (tires, suspensions and steering system). It is hard to determine the physical model parameters theoretically. Some empirical tire models with complex form of functions and many concerned parameters are difficult to obtain and also too sophistic for real time running in controllers.

There are many researches about vehicle body slip angle estimation, on linear observers or on nonlinear observers. As for linear observers design, vehicle and tire dynamics are linearized and fixed model parameters are adopted, which can not always get accurate results in different running situations [4] [5] [6]. In the nonlinear observers, the tires characteristics are described as nonlinear functions and with more parameters, which can get more accurate results in different running situations compared with linear observers. However, the nonlinear observers have the disadvantages of implementing complication and theoretical immaturity [7] [8] [9] [10].

Therefore, to make use of linear observer's advantages of simple design and implement, as well as to overcome the above problem, local linearization approximation of the nonlinear tire model and fuzzy logic method is introduced in this paper to construct an empirical tire model. The local tire models for small and large tire slips are described as the same linear form. The nonlinear characteristics of tire lateral force are represented by the different cornering stiffness values for the local tire models, which can be identified by experiments which is apt to perform.

Based on such empirical modeling approach, the proposed observer is a combination of local linear observers with fuzzy logic. The local observers are designed as linear observers with Kalman filter theory to overcome the influence of noise for on-board application.

In the first step for the observer design, the derivation of linear observer equations is developed, based on vehicle dynamics analysis and local linearization approximation of nonlinear tire model. The local vehicle dynamic models are empirical, with the tire cornering stiffness values from experimental identification described in the paper. In the next step, fuzzy logics are introduced to get a fuzzy model of tire lateral force and vehicle dynamics, which is calculated as a weighted sum of the outputs of two local linear models, one for lateral acceleration is small and the other for that is large. Adaptation mechanism for road friction changing is designed by choosing the membership functions of weighting factors dependent on road friction coefficient. Then, the hybrid-like observer mechanism is developed, which is a combination of two local observers based on local vehicle dynamic models and Kalman filter theory [11]. Finally, simulation and experimental studies are carried out to verify the observer.

II. VEHICLE DYNAMICS AND OBSERVER MODEL

The observer is based on in-wheel-motored electric vehicle dynamics model (Fig.1). The dynamics of vehicle is described by two degree-of-freedom (2DOF) vehicle model as following equations:

$$\begin{cases} ma_y = F_{xf} \sin \delta_f + F_{yf} \cos \delta_f + F_{yf} \\ I_z \dot{\gamma} = -l_f F_{yf} \sin \delta_f l_f + l_f F_{yf} \cos \delta_f - l_r F_{yr} + N \end{cases}$$
(1)

where a_y denotes vehicle lateral acceleration, γ is yaw rate, δ_f is steering angle of front wheel, N is direct yaw moment, m is mass of vehicle, I_z is yaw inertia moment, l_f is distance between mass center and front axle, l_r is distance between mass center and rear axle, F_{xf} is longitudinal forces of front tires, F_{yf} and F_{yr} are lateral forces of front and rear tires.



Fig.1 Two-freedom vehicle model

Taking vehicle slip angle β and yaw rate γ as state variables, and considering that the kinematics relationship is as $a_{\gamma} = v(\dot{\beta} + \gamma)$ and that $\delta_{\rm f}$ value is relatively small in the vehicle's high speed situations, vehicle's states equations are derived as:

$$\begin{vmatrix}
\dot{\hat{\beta}} = \frac{1}{mv}(F_{yf} + F_{yr}) - \hat{\gamma} \\
\dot{\hat{\gamma}} = \frac{1}{I_z}(l_f F_{yf} - l_r F_{yr} + N)
\end{cases}$$
(2)

For the nonlinearity of tire lateral force characteristics, (2) are the state equations with nonlinear form. To adopt linear observer approach, the observer model is changed into the form of an equivalent linear two freedom model by adopting the value of tire cornering stiffness C_n , which is defined as:

$$C_p = \frac{F_y}{\alpha} \tag{3}$$

Where F_y is the tire lateral force and slip angle α is the tire slip angle at its operating point.

By adopting tire cornering stiffness C_p , the nonlinear vehicle dynamic state equations (2) can be described as an equivalent linear state equation (4):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
(4)
In which,

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{bmatrix} = \begin{bmatrix} \frac{-(2C_f + 2C_r)}{mv} & \frac{-2l_fC_f + 2l_rC_r}{mv^2} - I \\ \frac{-2l_fC_f + 2l_rC_r}{I_z} & \frac{-2l_f^2C_f - 2l_r^2C_r}{I_zv} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{21} \end{bmatrix} = \begin{bmatrix} \frac{2C_f}{mv} & 0 \\ \frac{2l_fC_f}{I_z} & \frac{1}{I_z} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} \delta_f \\ N \end{bmatrix}$$

Where, $C_l \sim C_r$ are the cornering power values of front and rear tires.

For local dynamic models, the equivalent tire cornering stiffness, $C_l \sim C_r$, are difficult to determine theoretically because their values are influenced by the suspension dynamics, the tire characteristics and the steering system. In this paper, the identification method of tire cornering stiffness based on experimental tests performed on the electric vehicle is proposed.

According to (2), the steady state cornering relationship with steering angle input can be formulated by the following equation:

$$\begin{cases} ma_y = F_{yf} + F_{yr} \\ 0 = l_f F_{yf} - l_r F_{yr} \end{cases}$$
(5)

From (5), the expression of the side force applied to the front and rear tires can be deduced as:

$$\hat{F}_{yf} = \frac{l_r}{l} ma_y$$

$$\hat{F}_{yr} = \frac{l_f}{l} ma_y$$
(6)

And the side slip angle of front tires and rear tires can be calculated as:

$$\begin{cases} \hat{\alpha}_{f} = \beta + \frac{\mathcal{H}_{f}}{V} - \delta_{f} \\ \hat{\alpha}_{r} = \beta - \frac{\mathcal{H}_{r}}{V} \end{cases}$$
(7)

Then, based on above equations, if conducting steady state cornering experiments and a_y , β , γ values are measured, the tire cornering stiffness can be identified as:

$$\begin{cases} \hat{C}_f = \frac{F_{yf}}{-2\alpha_f} \\ \hat{C}_r = \frac{F_{yr}}{-2\alpha_r} \end{cases}$$
(8)

For the nonlinearity of vehicle dynamics, cornering experiments with low and high a_y should be conducted respectively to identify the different cornering stiffness values in different operating regimes.

III. FUZZY MODELING OF TIRE AND VEHICLE DYNAMICS

Since the main contribution to the nonlinearity of the model is from the tires, the cornering stiffness of the tires will play the main role in the model employed in the above observer. The fuzzy modeling of tire and vehicle nonlinearity is based on the linguistic description as follows:

a. In mild steering maneuver, lateral acceleration and tires side slip angle are small, the tires work in the linear region and the cornering stiffness values are large.

b. In severe steering maneuver, lateral acceleration and tires side slip angle are large, the tires work in the nonlinear region and the cornering stiffness values are small.

Such tire nonlinearity can also be affected by tire-road adhesion conditions, which is shown as Fig.2. Hence, to describe the vehicle dynamics by equivalent linear 2DOF model, local models with different cornering stiffness value should be constructed, for low lateral accelerations and for high lateral accelerations.



Fig.2 Tire lateral force characteristics: partitioned roughly into 4 different local dynamics (Lsa: large side slip angle, Ssa: small side slip angle, Lfr: large friction, Sfr: small friction)

To simplify the fuzzy modeling, the lateral acceleration a_y is considered as linguistic variable with two fuzzy sets (large and small), whose membership functions are shown in Fig.3. Then, using these fuzzy sets, the fuzzy IF-THEN rules for the vehicle dynamics modeling can be defines as follows: Rule *i*: (local model *i*)

IF $|a_v|$ is F_i , THEN $\dot{x} = A_i x + B_i u$

Two models are chosen to describe the overall vehicle dynamics that take the form of equation (4) and the model parameters namely the equivalent tire cornering stiffness are identified according to the steady state regime given by (8).



Fig.3 Varying membership functions vs lateral acceleration.

For the local model 1, tire works at its small slip region, A_1 and B_1 are calculated according to the cornering

stiffness has larger value. For the local model 1, tire works at its large slip region, A_2 and B_2 are calculated according to the cornering stiffness has quite smaller value.

Then, the whole nonlinear dynamics of vehicle is described with the proposed varying switching partition by interpolating the two models with fuzzy logic according to a membership function. By appropriate choice of the membership function, the calculation of vehicle dynamics for different operating regimes (from low a_y value to high a_y value) can be done.

Therefore, the following equation can represent the fuzzy models of the vehicle:

$$x = \sum_{i=1}^{2} w_i (A_i x + B_i u)$$
(9)

where w_1 and w_2 are the membership functions for local model 1 and local model 2. For simplification, straight line function is chosen for the membership function design as shown in figure 4. The formulation $w_1(a_y)$ and $w_2(a_y)$ are as follows:

$$w_{1}(a_{y}) = \begin{cases} 1 - \frac{1}{a_{yw}} a_{y} & |a_{y}| \le a_{yw} \\ 0 & |a_{y}| > a_{yw} \end{cases}$$
(10)

$$w_{2}(a_{y}) = \begin{cases} \frac{1}{a_{yw}} a_{y} & |a_{y}| \le a_{yw} \\ 1 & |a_{y}| > a_{yw} \end{cases}$$
(11)

where the coefficient a_{yw} describes the value of a_y value at tire/road adherence limit (road friction coefficient μ) when the tire force is saturation.

Road condition is one of the most important factors that must be considered in vehicle dynamic stability control, since the road friction coefficient μ is uncertain and may change according to the road condition, the fuzzy partition describing the vehicle model must be adaptive to such variations fig.3.

 μ value can be identified with different methods. In the EVs stability control, one way the authors adopted previously is to identify μ value by analyzing wheel rotation dynamics, which takes advantage of accurate knowledge of the EVs motor torque values [9]. With the identified μ value, a_{yw} is used an adjustment parameter of the weighting functions partition to form an adaptation mechanism to cope with the variation of tire/road adherence conditions. In this work,

 a_{yw} is set to be a linear function to μ with a low pass filter to remove the noise of μ identification as follows:

$$a_{yw} = k_{\mu} \frac{1}{1 + T_f s} \mu$$
 (12)

where k_{μ} is the adaptation gain, T_f is the constant of 1st order low-pass filter.

IV. FUZZY HYBRID-LIKE OBSERVER DESIGN

A hybrid-like observer is designed by applying Kalman filter theory for the constructed fuzzy discrete time vehicle models. The proposed observer structure is as Fig. 4.



Fig.4 Structure of hybrid adaptive observer

In the observer, there are two Kalman filter local observers respectively based on the above local model 1 and local model 2, which get the estimation results of β_{ob1} and β_{ob2} respectively. For the on-board application in real time, the continuous-time model of (4) is converted into discrete time model considering process noise and measurement noise as follows:

$$x[n+1] = G_i x[n] + H_i u[n] + \omega[n]$$

$$y_{\upsilon}[n] = C_i x[n] + D_i u[n] + \upsilon[n]$$
(13)

where the covariance vector of process noise and measurement noise are assumed to be the same for all dynamics:

 $E(\omega[n]\omega[n]^T) = Q, E(\upsilon[n]\upsilon[n]^T) = R$

Zero-order hold method is used for the discretization, then:

$$G_{i} = \begin{bmatrix} 1 + T_{s}a_{11} & T_{s}a_{12} \\ T_{s}a_{21} & 1 + T_{s}a_{21} \end{bmatrix}, H_{i} = \begin{bmatrix} T_{s}b_{11} & T_{s}b_{12} \\ T_{s}b_{21} & T_{s}b_{21} \end{bmatrix}$$

where T_s is sampling time

Based on the discrete state equations (4), discrete Kalman filter theory can be applied for a linear observer design. Vehicle lateral acceleration a_y and yaw rate γ are 2 measurable variables in vehicle and are chosen as output variables of the observer.

$$y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}, \ C = \begin{bmatrix} 0 & l \\ va_{11} & v(a_{12} + l) \end{bmatrix}, \ D = \begin{bmatrix} 0 & 0 \\ vb_{11} & 0 \end{bmatrix}$$

The recursive algorithm of discrete Kalman Filter is then applied separately to estimate local dynamics that can be stated by the following diagram,



where \hat{x} and \hat{y} are the estimation values of x and y, respectively. L_i is the feedback gain of local observer which is derived using Kalman filter theory.

Similar as the fuzzy IF-THEN rules for the vehicle dynamics modelling, the fuzzy rules for β observer fuzzy logic can be defines as follows:

Rule *i*: (local observer *i*)

IF $|a_v|$ is F_i , THEN $\hat{\beta}_{ob} = \hat{\beta}_{obi}$.

By introducing this fuzzy logic, the final value is the weight addition of the two local linear are enough to cover the main nonlinear features of the dynamics and give the proposed observer the ability to overpass linear observer in term of performances. The overall fuzzy observer is given by ,

$$\hat{\beta}_{ob} = \sum_{i=1}^{2} w_i \hat{\beta}_{obi} \tag{16}$$

The advantages of linear observer as simple design and fast running are kept and nonlinear problem can be solved at the same time.

V. SIMULATION AND EXPERIMENTAL STUDIES

The simulation situation is set with a sinusoid steering angle input to simulate consecutive lane change maneuvers of the test vehicle. The amplitude of input steering angle is large enough to make the tire span linear and nonlinear working region. The simulation results in different road friction conditions are as Fig. 5. Both the two sub-observer results can not fit the real value well for the whole running situations, for they are based on local linear model with fixed parameter describing a little segment of vehicle operating regime characteristics. Comparatively, the hybrid observer results can always follow the real ones well and have satisfying ability to adapt with different road friction conditions.



Field tests are conducted in our experimental EV, UOT March II. UOT March II is equipped with acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure real vehicle state values. Fig.6 and Fig.7 are the results of field tests of the observer in moderate and severe cornering situations. The experiments demonstrate the observer effectiveness and suitable for real time application for its high on-board calculating speed.



VI. CONCLUSIONS

In this paper, a hybrid-like vehicle slip angle observer is proposed based on an empirical tire model with the method of fuzzy logic. The key point of the observer is the fuzzy modeling of vehicle dynamic which can describe the nonlinearity of tire lateral force characteristics. Tire cornering stiffness values are identified by experiments. The local models of the observer are still linear ones, and the linear observer with Kalman filter theory is implemented. Therefore, the complication of nonlinear observer problem is avoided and the running time is greatly shortened. The simulation and experimental results verify the effectiveness of the proposed observer.

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