# A Hybrid-like Observer of Body Slip Angle for Electric Vehicle Stability Control: Fuzzy Logic and Kalman Filter Approach

Cong Geng\*, Lotfi Mostefai\*\* and Yoichi Hori\*\*\*

\* Department of Electric Engineering, The University of Tokyo, Tokyo, Japan. Email: geng@horilab.iis.u-tokyo.ac.jp

\*\* Institute of Industrial and Science, The University of Tokyo, Tokyo, Japan. Email: lotfi@horilab.iis.u-tokyo.ac.jp

\*\*\* Institute of Industrial and Science, The University of Tokyo, Tokyo, Japan. Email: hori@ iis.u-tokyo.ac.jp

Abstract—A hybrid-like observer for vehicle body slip angle  $(\beta)$  is proposed, in which a local approximation of the nonlinear tire model is adopted. Local equivalent vehicle models for observer design are built by linear approximation of vehicle dynamics respectively for low and high lateral acceleration operating regimes. Fuzzy logic approach is adopted to combine the local observer models so as to deal with the nonlinear nature of vehicle dynamics. The local observers are designed as linear observers with Kalman filter theory to overcome the influence of system noise. The derivation of this hybrid-like observer's state equations and the estimation mechanism is developed. The fuzzy rules can establish the qualitative relationships among the variables concerning the nonlinear and uncertain nature of vehicle dynamics, such as the saturation of tire force and the influence of road adherence. By adaptation mechanism designing of membership functions in the fuzzy rules, the quantitative accuracy and adaptive performance of the system can be satisfied, which is verified by simulations and experiments.

Keywords — Vehicle body slip angle, Observer, Vehicle stability control, Fuzzy logic, Kalman filter

# I. INTRODUCTION

An important advantage of electric vehicles (EVs) has been recognized is that motor's controllability can provide more flexible and novel ideas for vehicle stability control. Body slip angle ( $\beta$ ) is an important value for such control strategies. However, as sensors to measure  $\beta$  value are very expensive, it needs to estimate  $\beta$  from only variables measurable [1] [2] [3]. The most difficult for  $\beta$  estimation is the non-linear nature of vehicle dynamics.

There are many researches about vehicle body slip angle estimation, on linear observers or on nonlinear observers. As for linear observers design, vehicle and tire dynamics are linearized and fixed model parameters are adopted, which can not always get accurate results in different running situations [4]. In the nonlinear observers, the tires characteristics are described as nonlinear functions and with more parameters, which can get more accurate results in different running situations compared with linear observers. However, the nonlinear observers

have the disadvantages of implementing complication and theoretical immaturity [5] [6].

The main nonlinearity of vehicle dynamics comes from the tire force saturation decided by the tire adherence limits, which makes  $\beta$  response change greatly if the vehicle is cornering severely compared to the case of moderated cornering. This means the modeling structures or model parameters should be vary according to the different operating regimes for practical controller design. In addition, the nonlinear nature of vehicle dynamics is further complicated for it is influenced by the characteristics of whole chasis elements (tires, suspensions and steering system). It is hard to determine the physical model parameters theoretically. Therefore, the effective method for modeling is the key for  $\beta$  observer design.

In this paper, to deal with the difficulties in nonlinearity modeling, as well as to make use of the linear observer and controller advantages of simple design and implement, nonlinear vehicle dynamics is represented by Takagi-Sugeno fuzzy models (know as T-S fuzzy model) [14]. Local approximation of the nonlinear vehicle model and a dynamical interpolation method is introduced in this paper to construct a fuzzy-model-based control system for  $\beta$  estimating.  $\beta$  observer are designed for each local model using kalman filter theory. The proposed system is a combination of local linear observers and controllers with varying switching partition.

In the first step for the system design, the derivation of system state equations is developed, based on vehicle dynamics analysis and local approximation of nonlinear tire model, in which the modeling structures are considered theoretically and appropriate for control system design (linear 2-DOF vehicle model as [11]). In the next step, fuzzy modeling approach is used to get a hybrid-like vehicle model which is calculated as a weighted sum of the outputs of two local linear models, one for lateral acceleration ( $a_y$ ) is low and the other for  $a_y$  is high. For practical applications, parameter

identifications are conducted through experiments. To be adaptive to different running conditions and road friction changing, an adaptation mechanism is developed, in which the membership functions of the weighting factors are chosen to be dependent on lateral acceleration value and road friction coefficient. By mean of fuzzy rules, the overall observer is the combination of two local observers based on local linear tire models which by nature leads to a relative simpler design. Furthermore, the nonlinear global system results show high  $\beta$  estimation capabilities and good adaptation to road friction changing. This is verified by simulation and experimental studies.

#### II. VEHICLE DYNAMICS AND FUZZY MODELING

# A. Local Approximation and Linearization of Vehicle Dynamics

The system design is based on in-wheel-motored electric vehicle dynamics model (figure 2).

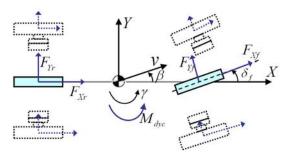


Fig.2 Two-freedom vehicle model
The dynamics of vehicle is approximately described by
2 DOF vehicle model as following equations:

$$\begin{cases} ma_y = F_{xf} \sin \delta_f + F_{yf} \cos \delta_f + F_{yr} \\ I_z \dot{\gamma} = I_f F_{xf} \sin \delta_f + I_f F_{yf} \cos \delta_f - I_r F_{yr} + N \end{cases}$$
(1)

where  $a_y$  denotes vehicle lateral acceleration,  $\gamma$  is yaw rate,  $\delta_f$  is steering angle of front wheel, N is direct yaw moment, m is mass of vehicle,  $I_z$  is yaw inertia moment,  $l_f$  is distance between mass center and front axle,  $l_r$  is distance between mass center and rear axle,  $F_{xf}$  is longitudinal forces of front tires,  $F_{yf}$  and  $F_{yr}$  are lateral forces of front and rear tires.

Taking vehicle slip angle  $\beta$  and yaw rate  $\gamma$  as state variables, and considering that the kinematics relationship is as  $a_y = v(\dot{\beta} + \gamma)$  and that  $\delta_{\bf f}$  value is relatively small in the vehicle's high speed situations, vehicle's states equations are derived as:

$$\begin{cases} \dot{\hat{\beta}} = \frac{1}{mv} (F_{yf} + F_{yr}) - \hat{\gamma} \\ \dot{\hat{\gamma}} = \frac{1}{I_z} (I_f F_{yf} - I_r F_{yr} + N) \end{cases}$$
 (2)

For the nonlinearity of tire lateral force characteristics, (2) are the state equations with nonlinear form. By approximating for local operating regime, the model is changed into the form of an equivalent linear 2DOF model by adopting the value of equivalent tire cornering stiffness  $C_n$ , which is defined as:

$$c_p = \frac{F_y}{a} \tag{3}$$

where  $F_y$  is the tire lateral force and slip angle  $\alpha$  is the tire slip angle at its operating point.

By adopting  $C_p$  value, the nonlinear vehicle dynamic state equations (2) can be described as equivalent linear state equations as (4) at the local operating point:

$$\dot{x} = Ax + Bu \tag{4}$$

In which,

$$\begin{split} \mathbf{A} = & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{bmatrix} = \begin{bmatrix} \frac{-(2C_f + 2C_r)}{mv} & \frac{-2I_fC_f + 2I_rC_r}{mv^2} - I \\ \frac{-2I_fC_f + 2I_rC_r}{I_z} & \frac{-2I_f^2C_f - 2I_r^2C_r}{I_zv} \end{bmatrix} \\ B = & \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{21} \end{bmatrix} = \begin{bmatrix} \frac{2C_f}{mv} & 0 \\ \frac{2I_fC_f}{I_z} & \frac{1}{I_z} \end{bmatrix} \\ x = & \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, u = \begin{bmatrix} \delta_f \\ N \end{bmatrix} \end{split}$$

where,  $C_f \sim C_r$  are the cornering stiffness values of the front and rear tires.

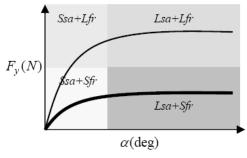


Fig. 3 Tire lateral force characteristics partitioned roughly into 4 different local dynamics (Lsa: large side slip angle, Ssa: small side slip angle, Lfr: large friction, Sfr: small friction)

Since the main contribution to the nonlinearity of the model is from the tires, the cornering stiffness of the tires will play the main role in the model employed in this paper. According to fig.3, these coefficients are large for the tire slip angles are small which means the vehicle

running at low lateral acceleration regimes and are small for the tire slip angles are large which also means the vehicle running at high lateral acceleration regimes. Hence, to describe the vehicle dynamics by equivalent linear 2 DOF model, local models with different  $C_p$  value should be constructed, for low lateral accelerations and for high lateral accelerations.

#### B. Model Parameters Identification

For local dynamic models, the equivalent tire cornering stiffness,  $C_l \sim C_r$ , are difficult to determine theoretically because their values are influenced by the suspension dynamics, the tire characteristics and the steering system. In this paper, the identification method of tire cornering stiffness based on experimental tests performed on the electric vehicle is proposed.

According to (2), the steady state cornering relationship with steering angle input can be formulated by the following equation:

$$\begin{cases}
ma_y = F_{yf} + F_{yr} \\
0 = l_f F_{yf} - l_r F_{yr}
\end{cases}$$
(5)

From (5), the expression of the side force applied to the front and rear tires can be deduced as:

$$\begin{cases} \hat{F}_{yf} = \frac{l_r}{l} ma_y \\ \hat{F}_{yr} = \frac{l_f}{l} ma_y \end{cases}$$
 (6)

And the side slip angle of front tires and rear tires can be calculated as:

$$\begin{cases} \hat{\alpha}_{f} = \beta + \frac{\mathcal{N}_{f}}{V} - \delta_{f} \\ \hat{\alpha}_{r} = \beta - \frac{\mathcal{N}_{r}}{V} \end{cases}$$
 (7)

Then, based on above equations, if conducting steady state cornering experiments and  $a_y$ ,  $\beta$ ,  $\gamma$  values are measured, the tire cornering stiffness can be identified as:

$$\begin{cases}
\hat{C}_f = \frac{F_{yf}}{-2\alpha_f} \\
\hat{C}_r = \frac{F_{yr}}{-2\alpha_r}
\end{cases}$$
(8)

For the nonlinearity of vehicle dynamics, cornering experiments with low and high  $a_y$  should be conducted respectively to identify the different cornering stiffness values in different operating regimes.

## C. Fuzzy Modeling and Local Dynamics

To simplify the fuzzy modelling, the lateral acceleration  $a_y$  is considered as linguistic variable with two fuzzy sets (large and small), whose membership functions are shown in Fig.4. Then, using these fuzzy sets, the fuzzy IF-THEN rules for the vehicle dynamics modelling can be defines as follows:

Rule *i*: (local model *i*)  
IF 
$$|a_y|$$
 is  $F_i$ , THEN  $\dot{x} = A_i x + B_i u$ 

Two models are chosen to describe the overall vehicle dynamics that take the form of equation (4) and the model parameters namely the equivalent tire cornering stiffness are identified according to the steady state regime given by (8).

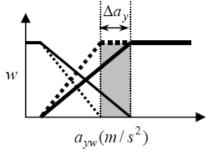


Fig.4 Varying membership functions vs lateral acceleration.

For the local model 1, tire works at its small slip region,  $A_1$  and  $B_1$  are calculated according to the cornering stiffness  $C_p$  has largest value  $C_{p1}$ . For the local model 1, tire works at its large slip region,  $A_2$  and  $B_2$  are calculated according to the cornering stiffness  $C_p$  has quite smaller value  $C_{p2}$ .

Then, the whole nonlinear dynamics of vehicle is described with the proposed varying switching partition by interpolating the two models with fuzzy logic according to a membership function. By appropriate choice of the membership function, the calculation of vehicle dynamics for different operating regimes (from low  $a_v$  value to high  $a_v$  value) can be done.

Therefore, the following equation can represent the fuzzy models of the vehicle:

$$x = \sum_{i=1}^{2} w_i (A_i x + B_i u)$$
 (9)

where  $w_1$  and  $w_2$  are the membership functions for local model 1 and local model 2. For simplification, straight line function is chosen for the membership function design as shown in figure 4. The formulation  $w_1(a_y)$  and  $w_2(a_y)$  are as follows:

$$w_{1}(a_{y}) = \begin{cases} 1 - \frac{1}{a_{yw}} & |a_{y}| \leq a_{yw} \\ 0 & |a_{y}| > a_{yw} \end{cases}$$
 (10)

$$w_{2}(a_{y}) = \begin{cases} \frac{1}{a_{yw}} a_{y} & |a_{y}| \leq a_{yw} \\ 1 & |a_{y}| > a_{yw} \end{cases}$$
 (11)

where the coefficient  $a_{yw}$  describes the value of  $a_y$  value at tire/road adherence limit (road friction coefficient  $\mu$ ) when the tire force is saturation.

Road condition is one of the most important factors that must be considered in vehicle dynamic stability control, since the road friction coefficient  $\mu$  is uncertain and may change according to the road condition, the fuzzy partition describing the vehicle model must be adaptive to such variations fig.4.

 $\mu$  value can be identified with different methods. In the EVs stability control, one way the authors adopted previously is to identify  $\mu$  value by analyzing wheel rotation dynamics, which takes advantage of accurate knowledge of the EVs motor torque values [12]. With the identified  $\mu$  value,  $a_{yw}$  is used an adjustment parameter of the weighting functions partition to form an adaptation mechanism to cope with the variation of tire/road adherence conditions (see as  $\Delta a_v$  fig.4). In this work,

 $a_{yw}$  is set to be a linear function to  $\mu$  with a low pass filter to remove the noise of  $\mu$  identification as follows:

$$a_{yw} = k_{\mu} \frac{1}{1 + T_f s} \mu \tag{12}$$

where  $k_{\mu}$  is the adaptation gain,  $T_f$  is the constant of  $1^{\text{st}}$  order low-pass filter.

# III. $\beta$ Observer Design Based on Fuzzy Models

#### A. Kalman Filter for Local β Observer Design

Based on the local linear models, the  $\beta$  observer is designed with Kalman filter theory [15]. For the on-board application in real time, the continuous-time model of (4) is converted into discrete time model considering process noise and measurement noise as follows:

$$x[n+1] = G_i x[n] + H_i u[n] + \omega[n]$$
  

$$y_{ij}[n] = C_i x[n] + D_i u[n] + \upsilon[n]$$
(13)

where the covariance vector of process noise and measurement noise are assumed to be the same for all dynamics:

$$E(\omega[n]\omega[n]^T) = Q, E(\upsilon[n]\upsilon[n]^T) = R$$

Zero-order hold method is used for the discretization, then:

$$G_{i} = \begin{bmatrix} 1 + T_{s}a_{11} & T_{s}a_{12} \\ T_{s}a_{21} & 1 + T_{s}a_{21} \end{bmatrix}, H_{i} = \begin{bmatrix} T_{s}b_{11} & T_{s}b_{12} \\ T_{s}b_{21} & T_{s}b_{21} \end{bmatrix}$$

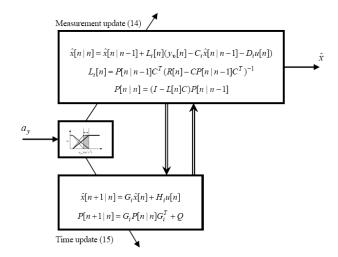
where  $T_s$  is sampling time

Based on the discrete state equations (4), discrete Kalman filter theory can be applied for a linear observer design. Vehicle lateral acceleration  $a_y$  and yaw rate  $\gamma$  are 2 measurable variables in vehicle and are chosen as

output variables of the observer.

$$y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}$$
,  $C = \begin{bmatrix} 0 & I \\ va_{11} & v(a_{12} + I) \end{bmatrix}$ ,  $D = \begin{bmatrix} 0 & 0 \\ vb_{11} & 0 \end{bmatrix}$ 

The recursive algorithm of discrete Kalman Filter is then applied separately to estimate local dynamics that can be stated by the following diagram,



where  $\hat{x}$  and  $\hat{y}$  are the estimation values of x and y, respectively.  $L_i$  is the feedback gain of local observer which is derived using Kalman filter theory.

### B. Hybrid-like Observer Design Based on Fuzzy Models

A hybrid-like observer is designed by applying Kalman filter theory for the constructed fuzzy discrete time vehicle models [9]. The proposed observer structure is as figure 5.

In the observer, there are two Kalman filter local observers respectively based on the above local model 1 and local model 2, which get the estimation results of  $\beta_{ob1}$  and  $\beta_{ob2}$  respectively.

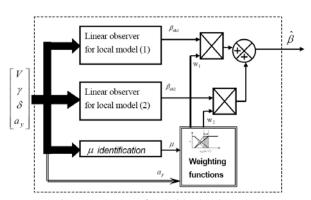


Fig.5 Structure of hybrid adaptive observer

Similar as the fuzzy IF-THEN rules for the vehicle dynamics modelling, the fuzzy rules for  $\beta$  observer fuzzy logic can be defines as follows:

Rule *i*: (local observer *i*)  
IF 
$$|a_y|$$
 is  $F_i$ , THEN  $\hat{\beta}_{ob} = \hat{\beta}_{obi}$ .

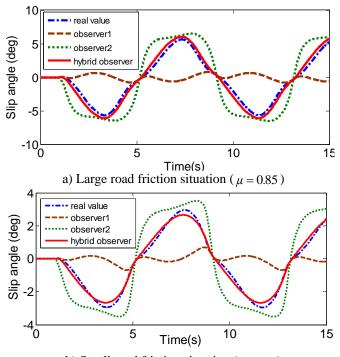
By introducing this fuzzy logic, the final value is the weight addition of the two local linear are enough to cover the main nonlinear features of the dynamics and give the proposed observer the ability to overpass linear observer in term of performances. The overall fuzzy observer is given by ,

$$\hat{\beta}_{ob} = \sum_{i=1}^{2} w_i \hat{\beta}_{obi}$$

The advantages of linear observer as simple design and fast running are kept and nonlinear problem can be solved at the same time.

### IV. SIMULATION AND EXPERIMENTAL STUDIES

The simulation situation is set with a sinusoid steering angle input to simulate consecutive lane change maneuvers of the test vehicle. The amplitude of input steering angle is large enough to make the tire span linear and nonlinear working region. The simulation results in different road friction conditions are as figure 6. Both the two sub-observer results can not fit the real value well for the whole running situations, for they are based on local linear model with fixed parameter describing a little segment of vehicle operating regime characteristics. Comparatively, the hybrid observer results can always follow the real ones well and have satisfying ability to adapt with different road friction conditions.



b) Small road friction situation ( $\mu = 0.4$ ) Fig.6 Simulation results of hybrid observer

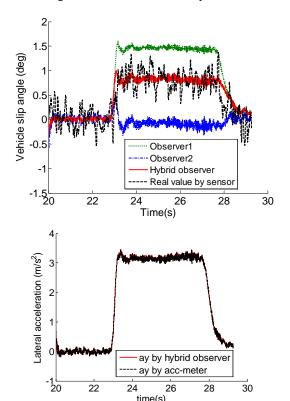


Figure 7: Field test results of  $\beta$  observer

Field tests are conducted in our experimental EV, UOT March II. UOT March II is equipped with acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure real vehicle state values. Figure 7 and Figure 8 are the results of field tests of the observer in

moderate and severe cornering situations. The experiments demonstrate the observer effectiveness and suitable for real time application for its high on-board calculating speed.

#### V. Conclusion

This work pays the main attention to an algorithmic solution of the nonlinear vehicle dynamic state observer problem, as well as the real-time application aspect. The authors present the design of vehicle state observer for inwheel-motored electric vehicle by fuzzy modeling and fuzzy logic. T-S fuzzy models are employed by approximating the nonlinear vehicle dynamics with linear local models. Fuzzy logic is introduced with the membership functions for lateral acceleration and with adaptation mechanism for road friction coefficient changing. A hybrid-like observer according to the fuzzy logic is designed based on the constructed. The local observers are designed by applying Kalman filter theory in the linear observers. The quantitative accuracy and adaptive performance of the system can be satisfied, which is verified by simulations and experiments.

#### REFERENCES

- [1] Yoichi Hori, "Future Vehicle driven by Electricity and Control Research on 4 Wheel Motored "UOT March II" ", in AMC2002 Proc. (7th International Workshop on Advanced Motion Control Proceedings), pp.1-14, 2002.
- [2] Donghyun kim, Sungho Hwang, and Hyunsoo Kim, "Vehicle Stability Enhancement of Four-Wheel-Drive Hybrid Electric Vehicle Using Rear Motor Control", IEEE Trans. On Vehicular Technology, Vol. 57, no. 2, 727-735, March 2008.
- [3] Keiyu Kin, Osamu Yano and Hiroyuki Urabe, "Enhancements in vehicle Stability and steerability with slip Control", JSAE Review 24, pp. 71-79, 2003.
- [4] Christoph Arndt, Johannes Karidas and Rainer Busch, "Design

- and Validation of a Vehicle State Estimator", Proc. Of AVEC'04 (The 7<sup>th</sup> International Symposium on Advanced Vehicle Control), pp. 41-45, 2004.
- [5] L. Imsland, T.A. Johansen, T.I. Fossen, etc, "Vehicle velocity estimation using nonlinear observers", Automatica, Vol.42, pp. 2091-2103, 2006
- [6] F. Cheli, E. Sabbion, M. Pesce, S. Melzi, "A methodology for vehicle sideslip angle identification: comparison with experimental data", Vehicle System Dynamics, Vol.45, No.6, pp. 549-563, June 2007.
- [7] J. Th. Paul, Venhovens, K. Naab, "Vehicle Dynamics Estimation Using Kalman Filters", Vehicle System Dynamics, Vol.32, pp. 171-184, 1999.
- [8] Liu, Ch.Sh. and Peng, H., "Road Friction Coefficient Estimation For Vehicle Path Prediction", Vehicle System Dynamics, Vol.25 Suppl, pp. 413-425, 1996.
- [9] M.C. Best, T.J. Gordon and P.J. Dixon, "An Extended Adaptive Kalman Filter for Real-time State Estimation of Vehicle Handling Dynamics", Vehicle System Dynamics, Vol.34, pp. 57-75, 2000.
- [10] Cong Geng, Toshiyuki Uchida and Yoichi Hori, "Body Slip Angle Estimation and Control for Electric Vehicle with In-Wheel Motors", in IECON2007 Proc. (The 33rd Annual Conf. of the IEEE Industrial Electronics Society Proceedings), pp.351-355, 2007
- [11] Yoshifumi Aoki, Toshiyuki Uchida and Yoichi Hori, "Experimental Demonstration of Body Slip Angle Control based on a Novel Linear Observer for Electric Vehicle", in IECON2005 Proc. (The 31st Annual Conf. of the IEEE Industrial Electronics Society Proceedings), pp.2620-2625,2005.
- [12] Geng Cong and Hori Yoichi, "Nonlinear Body Slip Angle Observer for Electric Vehicle Stability Control", in EVS23 Proc. (23rd International Electric Vehicle Symposium and Exposition), 2007.
- [13] T. A. Wenzel, K. J. Burnham, etc, "Motion Dual Extended Kalman Filter for Vehicle State and Parameter Estimation", Vehicle System Dynamics, Vol.44, No.2, pp. 153-171, February 2006.
- [14] R. Babuska and H. Verbruggen, "An overview of fuzzy modeling for control", Control Engineering Practice, vol. 4, no. 11, pp. 1593 – 1606, 1996.
- [15] Dan Simon, "Kalman filtering for fuzzy discrete time dynamic systems", Applied Soft Computing 3, pp. 191–207, 2003.