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Fractional Order Impedance Control by Particle Swarm Optimization

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Abstract: This paper suggests a methodology to realize fractional order impedance using feedback control of a motor, which can be called fractional impedance control. First, a novel discretization method of a fractional order integrator is proposed based on the coefficient fitting by particle swarm optimization. Based on this fractional order integrator, fractional order impedance control is realized using feedback control design. The characteristics of fractional impedance are analyzed by simulations, and the algorithm is implemented in a motor and the characteristics are made clear by experiments

Keywords: fractional order impedance, fractional order calculus, particle swarm optimization, human-friendly motion control, impedance control

1. INTRODUCTION

Impedance concept has been a quite important issue in robot control, and now its significance becomes clearer when robot applications are oriented toward assisting human. There are a number of researches on impedance control and impedance estimation. Many of the researches are on force control of robots, which generates position references for robots using force sensors.

In human neuromuscular system, this impedance is also important concept. [1] has explored human's ability to perceive mechanical impedance, which means human perceives impedance as information of his external environment. Paying attention to this characteristic, controllers of robots which co-work with human are usually designed focusing on impedance design[2], [3].

There are other researches that indicate the impedance model used in general impedance control is insufficient to describe the impedance to which people are accustomed. Some researches adopt the neural networks to describe more complicated impedance [4], [5]. They point out the time invariant linear impedance model which is generally describe as $\frac{1}{Ms^2+Bs+K}$, is so limited that it can hardly represent complex task strategies.

In this paper, as a promising impedance control method, fractional order impedance control is proposed. Fractional order calculus which realizes integration/differentiation in fractional order is an old topic. It, however, recently gains increasing attentions as a promising tools in mechatronics and biological systems[6], [7], [8]. Especially [6] introduces the fractional order calculus in biomimetic actuator control and reveals the possibility of fractional order calculus to model the dynamics of biomimetic materials. Also [9] applied this fractional order calculus concept to the theory of electrical impedance of botanical elements. Although they focus on the possibility of fractional calculus as a new dynamic model, none of them have used it as a desired impedance of actuator.

This paper utilizes the fractional order calculus in impedance design of a motor. Concluding from the references, this fractional order impedance has more possibility to describe human-friendly impedance, since it can provide more tunable elements and realize various impedance: for instance, the middle phase of stiffness and damping.

This paper is organized as follows.

In Section 2 a novel discretization method of fractional integrator based on the particle swarm optimization algorithm is suggested and compared with other realization methods to verify the effectiveness of the suggested algorithm . Then using the proposed discretization of fractional order integrator, force sensor-less fractional order impedance controller is designed in Section 3. Some experimental results using one-armed robot show the proposed algorithm can achieve fractional order impedance characteristics in wide frequency range.

2. DESIGN OF FRACTIONAL ORDER IMPEDANCE BY POSITION FEEDBACK CONTROL

The concept of fraction order calculus is not new. However recent developments of this fractional order calculus has pointed that the fractional order calculus can improve performance, especially robustness of feedback control when it is applied to control design. In these researches, the fractional order calculus is realized using certain approximation schemes[6]. Theoretically, the fractional order differentiation/integration needs an infinite dimensional linear filter, however, in practical point of view, a band-limit implementation is sufficient to use and this can be realized using a finite dimensional approximation.

2.1 Realization Methods of Fractional Order Calculus

Among existing implementation methods of fractional order calculus, the CRONE(Contrôle Robuste d'Ordre Non Entier) method [7] and CFE(Continued Fraction Expansion) approach[10] are most well known methods.

The CRONE control achieves a band-limited fractional differentiator/integrator based on broken-line approximation approach. Figure 1 illustrates an example of the approximation of band-limit fractional order integration.

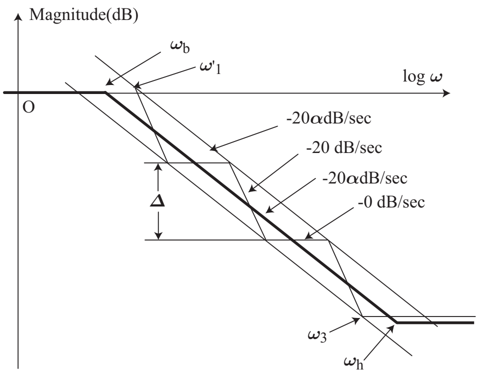


Fig. 1 An Example of Broken-line Approximation ($N = 3, r = 0.5$)

ω_h's and ω_b's are the poles and zeros which determine the interval of fractional integration. The approximation is formulated as Equation (1) to (3).

$$\left(\frac{\frac{s}{\omega_h}+1}{\frac{s}{\omega_b}+1}\right)^r \simeq \prod_{i=0}^{N-1}\frac{\frac{s}{\omega_i'}+1}{\frac{s}{\omega_i}+1}, \tag{1}$$

where

$$\frac{\omega_{i+1}}{\omega_i'}=\eta, \frac{\omega_i'}{\omega_i}=\zeta, \omega_0=\eta^{\frac{1}{2}}\omega_b, \omega_{N-1}'=\eta^{-\frac{1}{2}}\omega_h \tag{2}$$

and

$$\zeta\eta=\left(\frac{\omega_h}{\omega_b}\right)^{\frac{1}{N}}, (\zeta\eta)^r=\zeta. \tag{3}$$

This approach is based on the fractal concept and the greater N is the better the approximation becomes.

The other implementation method is to discretize the fractional order differentiator s^r using continued fraction expansion. The Tustin transformation is one of the most widely used generating function and [6] uses this Tustin rule and the Al-Alaoui operator to discretize s^r, expanding it using the continued fraction expansion(CFE). Equation (4) describes the approximation s^r in the form of the tustin rule.

$$D^r(z)=\left(\frac{2}{T}\right)^r \text{CFE}\left\{\left(\frac{1-z^{-1}}{1+z^{-1}}\right)^r\right\} \tag{4}$$

The CFE of $D^r(z)$ has the following form:

$$D^r(z)=1+\cfrac{z^{-1}}{\frac{1}{2r}+\cfrac{z^{-1}}{-2+\cfrac{z^{-1}}{\frac{3}{2}\frac{r}{r^2-1}+\frac{z^{-1}}{2+\cdots}}}} \tag{5}$$

This expansion can produce a certain order IIR form filter that can discretize s^r to a certain accuracy.

2.2 Fractional Order Integrator Design by Particle Swarm Optimization

Motivated from the CFE of the Tustin transformation in Equation (5), A novel discretization method of $\frac{1}{s^r}$ is proposed; Tustin transformation of a half order integration $\frac{1}{s^{0.5}}$ is approximated by a rational function of z^{-1}.

$$\begin{aligned}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^{-\frac{1}{2}} &\simeq \frac{n_0+n_1z^{-1}+\cdots+n_Kz^{-K}}{1+d_1z^{-1}+\cdots+d_Kz^{-K}}\\ &= f(z)\end{aligned} \tag{6}$$

The only problem is how to decide the coefficients n_0 to d_K. Although expansions or other theoretical approaches can be employed for this, totally different one approach is adopted here: PSO (Particle Swarm Optimization) is that.

This PSO can optimize the approximation performance of the filter (6) producing the optimal coefficients of n_0 to d_K which will decrease the approximation error. This approximation error is defined as the difference between the frequency responses of ideal $\frac{1}{s^{0.5}}$ and the multirate filter $f(z)$ in Equation (6) in the frequency domain. Equation (7) and (8) are the error definition used in the PSO.

$$\sum_{k=1}^{K}\left\{20\log 10\left|\left(\frac{1}{j\omega_k}\right)^{\frac{1}{2}}\right|-20\log 10|f(j\omega_kT)|\right\} \tag{7}$$

$$\sum_{k=1}^{K}\left\{\arg\left(\frac{1}{j\omega_k}\right)^{\frac{1}{2}}-\arg\left(f(j\omega_kT)\right)\right\} \tag{8}$$

Based on this definition, the PSO evaluates the gain and phase error at K numbers of frequencies and finds the optimal coefficients which will minimize these errors. At first, the optimal coefficients for a second order multirate filter($K = 2$ in Equation (6)) are calculated. Three types of frequency bandwidth (50Hz to 500Hz, 0.1Hz to 100Hz, and 0.01Hz to 5Hz) are evaluated and three filters are produced. Figure 2 are the result described in the Bode diagrams.

In Figure 2, the red dotted lines are the ideal frequency responses of $\frac{1}{s^{0.5}}$ and the blue solid lines are the produced frequency responses of optimized fractional order integrator designed by the proposed PSO algorithm.

2.3 Comparison with Other Realization Methods

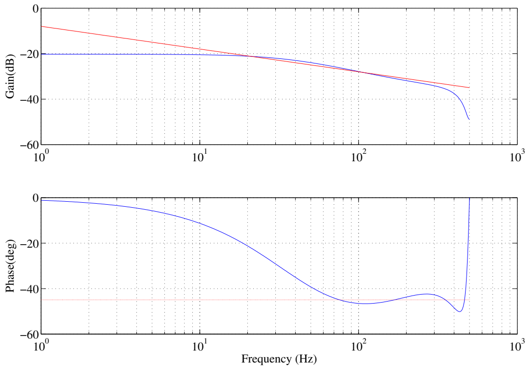
Here, performances of other fractional order calculus implementation methods are compared with our proposed method. First, Bode diagram of $s^{0.5}$ approximated by the CFE in [10] is plotted in Figure 3.

Third order approximation,

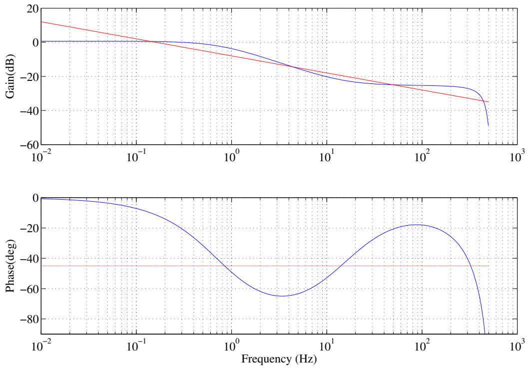
$$44.72\frac{1-0.5z^{-1}-0.5z^{-2}+0.125z^{-3}}{1+0.5z^{-1}-0.5z^{-2}-0.125z^{-3}} \tag{9}$$

is used for this approximation. Compared with Figure 2 (a), the result in Figure 3 is fit with the ideal response with wider range of frequency; since Figure 3 has higher order than our proposed method. However, this CFE algorithm tends to show a peak near nyquist frequency which may cause vibration. Additionally, our proposed method can adjust the frequency ranges where the response needs to be approximated.

Another implementation the CRONE also can adjust this approximated frequency range. Figure 4 shows



(a) Optimized in the range of 50Hz to 500Hz ($n_0 = 0.0223, n_1 = 0.0138, n_2 = -0.0047, d_1 = -0.3612, d_2 = -0.3155$)



(b) Optimized in the range of 0.1Hz to 1000Hz ($n_0 = 0.0091, n_1 = 0.0015, n_2 = -0.0089, d_1 = -0.6799, d_2 = -0.3139$)

Fig. 2 Optimized Fractional Order Integrator

Bode diagram of the CRONE method which approximates $s^{-0.5}$ with a second order filter. w_b is set as 10Hz, and w_h as 500Hz. The result shows the CRONE implementation is good at gain-fitting (a proper gain is multiplied to bias the gain), while its phase is the most distant from the ideal phase among three methods.

This comparison clarifies that our proposed method is effective when designing a digital filter which can discretize fractional order calculus in a certain range of frequencies.

A simulation which shows characteristic of the fractional order integrator in time domain can reveal what reaction force a human can feel when this fractional order integrator is applied to impedance control. As a more fundamental simulation, difference in step response between an integrator and fractional order integrator is conducted. Figure 5 is the simulation result. With a unit step input, three calculation: integration, low pass filtering, and fractional order integration.

The response of fractional order integrator is quite fast at first; almost equal to proportional calculation. After quite short period, however, its response becomes slower, and it finally converges a certain value same with a low

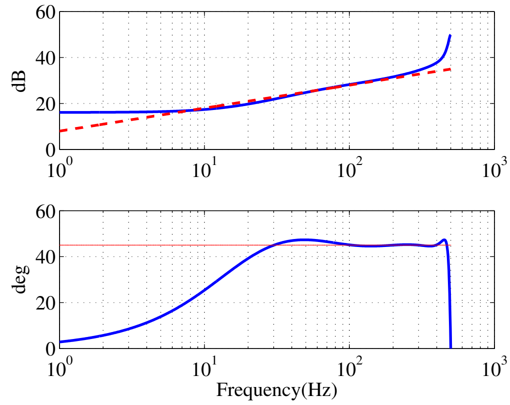


Fig. 3 $s^{0.5}$ Discretized by the CFE Approximation

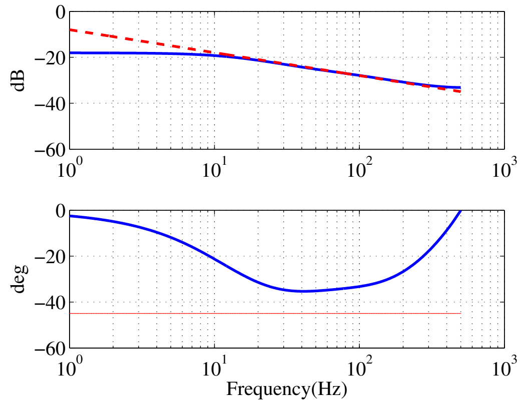


Fig. 4 $s^{-0.5}$ Discretized by the CRONE Approximation

pass filter output. This result explains that the smaller phase delay in fractional order calculus can realize fast reaction in short time range and smooth reaction in long time range, which can not be accomplished by an integrator or a low pass filter.

3. DESIGN OF FRACTIONAL ORDER IMPEDANCE BY POSITION FEEDBACK CONTROL

Using the proposed fractional order calculus discretization, fractional order impedance control design is proposed in this section.

3.1 Design of Impedance Generated by a Motor Force

Impedance that a man will feel when he applies his force to a motor is determined by the reaction force that the motor generates. This reaction force is decided by a feedback controller based on a displacement. The relationship between the reaction force and the displacement from the reference position or velocity by external force is determined by the sensitivity function or disturbance response of a controller.

This fact enables us to design force control without a force sensor; force control can be achieved by design of the sensitivity function which is identified with the impedance. As will be discussed in the next section, the position control gain works as stiffness element

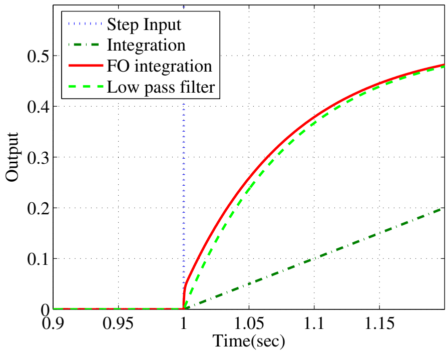


Fig. 5 Step Response of Approximated Fractional Order Integrator

in the impedance concept and the velocity control gain works as viscosity element; by adjusting these gains, the impedance in force control can be designed.

3.2 Fractional Order Impedance Control by Feedback Control

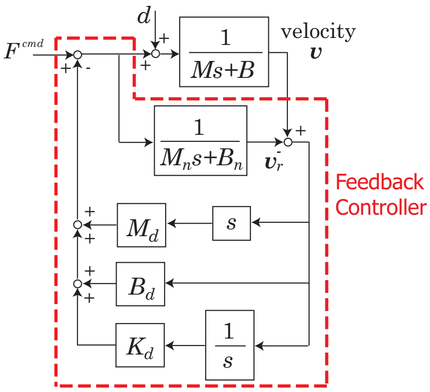


Fig. 6 Impedance Control by Velocity Feedback Control

Figure 6 is the proposed feedback controller structure where M is the true mass of a target plant, B is its true viscosity and J_n and B_n are the nominal model of the mass and the viscosity in the feedback controller. This controller can adjust the disturbance response in terms of impedance. Assuming $J_n = J$, $B_n = B$, the controller generates force F_m, the amount of which can be described like Equation (10)

$$F_m = \left((M_d + M)\Delta\dot{v} + (B_d + B)\,\Delta v + K_d \int \Delta v dt\right), \quad (10)$$

where Δv is the deviation in velocity by the disturbance.

This relationship is the mass-viscosity-elasticity model which is widely-used in the impedance control; the gains M_d, B_d, K_d work as the mass, viscosity and elasticity. By adjusting these gains, the impedance of the motor can be designed.

The feedback control strategy has been used in disturbance attenuation control in human-friendly motion control[12] where the degree of disturbance attenuating torque should be designed in terms of impedance. [12] shows the usefulness of this impedance design approach in disturbance attenuation control design of power assistive devices.

If the integrator or differentiator in Figure 6 is substituted with fractional order calculus, that can realize fractional order impedance. Figure 7 is one example of this frictional order impedance control.

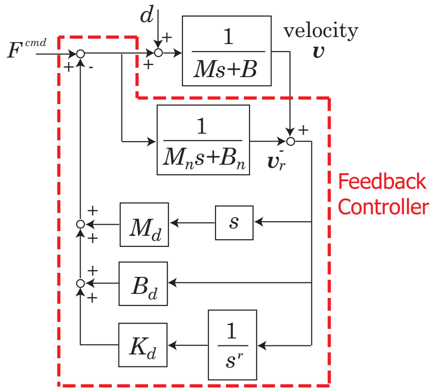


Fig. 7 Structure of Fractional Impedance Control

If r is set as 0.5 at a certain range of frequency, it can realize the middle state of viscosity and elasticity, which can generate a novel reaction force that can be felt by human.

3.3 Experimental Verification of Fractional Order Impedance

The fractional order impedance control is implemented in an one-armed motor, and experiments are conducted. The purpose of the experiment is to verify the

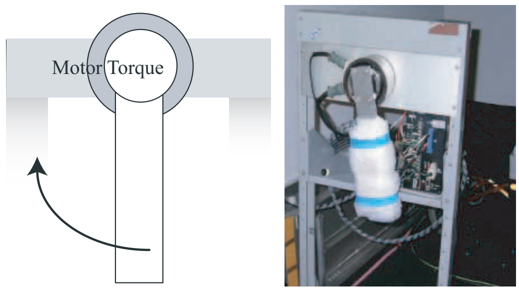


Fig. 8 One-armed Robot

proposed feedback implementation of fractional order impedance can realize the actual fractional order characteristics.

0.5th order intergrator and differentiator are employed in this implementation. Figure 9 and 10 show the frequency characteristics of the pseudo half order integrator and differentiator we use for this experiment.

Figure 9 is 0.5th order integrator approximated by $\left(\frac{s+1/\tau}{s+1/\tau'}\right)^{-0.5}$ with $\tau = \frac{1}{2\pi\times 0.1}$ and $\tau' = \frac{1}{2\pi\times 20}$, and Figure 10 is 0.5th order differentiator approximated by $\left(\frac{\tau s+1}{\tau' s+1}\right)^{0.5}$ with the same τ and τ'. Both show good fractional order calculus characteristic in the frequency range 0.2Hz to 10Hz.

Using these frequency characteristics, fractional order

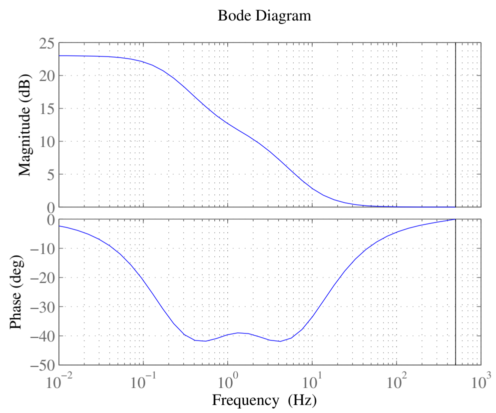


Fig. 9 Half Order Integrator

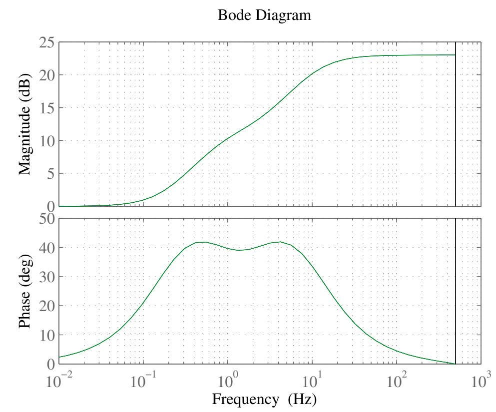


Fig. 10 Half Order Differentiator

impedance is build as Equation (11) and (12).

$$\frac{1}{Ms^2+Bs(1+\frac{1}{\tau}s^{-0.5})} \simeq \frac{1}{Ms^2+Bs\left(\frac{s+1/\tau}{s+1/\tau'}\right)^{-0.5}} \tag{11}$$

$$\frac{1}{Ms^2+K(\tau s^{0.5}+1)} \simeq \frac{1}{Ms^2+K\left(\frac{\tau s+1}{\tau' s+1}\right)^{0.5}} \tag{12}$$

Equation (11) represents 0.5th order stiffness with no other stiffness using 0.5th order integrator. Equation (12) represents 0.5th order damping with no other damping using 0.5th order differentiator.

These impedances are implemented using feedback described in Figure 11 and 12. 0.5th order stiffness is achieved by integrating measured velocity by the filter in Equation (11) with the damping B = 0.1, while 0.5th order damping by differentiating measured position through the filter in Equation (12) with the stiffness K = 1.

Figure 13 and 14 are the experimental results. Four values are drawn in each figure: the applied external torque, the angle and velocity of the arm, and the reaction torque that motor exerts against the applied torque. The reaction torque by the motor represents the impedance to the external force and output displacement.

In order to verify the frequency characteristic, sinusoidal torque with the frequency 1 Hz is applied as external torque. The relationship between the reaction torque

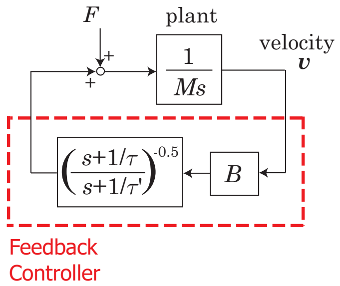


Fig. 11 Half Order Stiffness

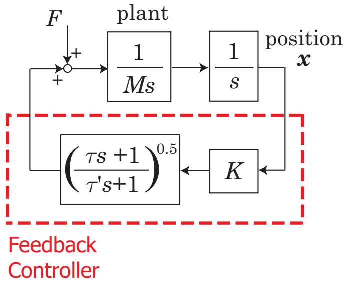


Fig. 12 Half Order Damping

by motor and angle/velocity of the arm shows fractional impedance characteristic. In both experiments, the reaction torque exists in the phase between the angle and velocity. This shows the torque proportional to the states that is 0.5th integration of velocity or 0.5th differentiation of angle is successfully generated.

Figure 15 and 16 are experimental results with the external torque frequency of 0.7Hz and 4Hz. These results validates that the fractional order impedance characteristic is hold in wide frequency range by the proposed method.

4. CONCLUSION

A novel implementation method of fractional order integrator and how to realize that fractional order impedance using that fractional order integrator are suggested in this paper. Simulation results shows that the fractional order calculus can generate different time response and phase characteristics than integrator or low pass filter.

Fractional order impedance implement by position feedback control is suggested and experiment is done to verify the effectiveness of the suggestion. The experimental results show that the proposed control method provides an one-armed robot fractional order impedance in a wide frequency range.

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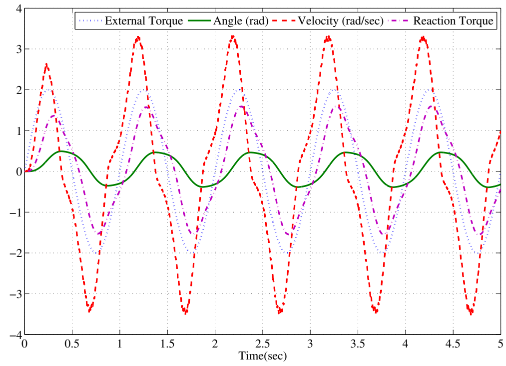

Fig. 13 Half Order Stiffness

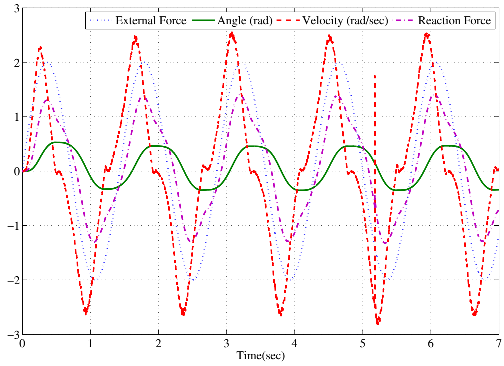

Fig. 15 Half Order Stiffness (0.7Hz external torque)

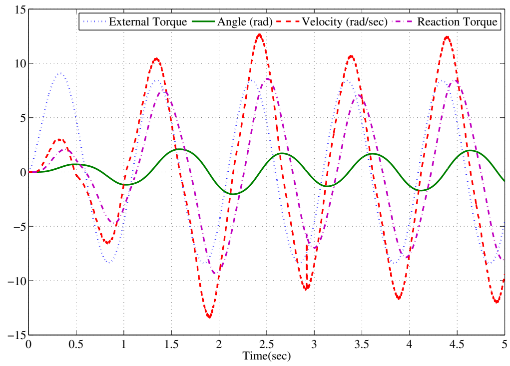

Fig. 14 Half Order Damping

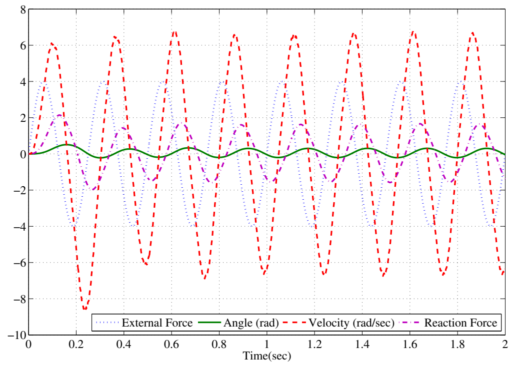

Fig. 16 Half Order Stiffness (4Hz external torque)

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