

# Position Error Signal based Control Designs for Control of Self-servo Track Writer

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**Abstract:** Among the many problems in the Self-servo Track Writer (SSTW), the error propagation problem is discussed in this paper. To deal with the error propagation problem, we take two approaches: estimation of the absolute head position and feedforward filter design using the Position Error Signal (PES) in the previous track.

To improve the estimation of absolute position of the head with regard to the whole disc, Kalman filter is designed and removes the estimation error caused by the sensor noise in PES. This Kalman filter estimation can make the relative error in a simulation.

The transition function which can describe the error propagation characteristics of SSTW is derived. Based on it, a feedforward filter which compensates the error propagation using the error signal in the previous track is proposed. Simulations results verify the effectiveness; the feedforward filter design can suppress the propagation of both the absolute error and relative error.

Keywords: Hard Disk Drive, Self Servo Track Writer, Error propagation, Kalman filter, Loop shaping

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## 1. INTRODUCTION

### 1.1 Necessity of Self Servo Track Writer

Hard-disk Drives are, recently, used not only for the computer system but also for consumer electronics such as hard-disk recorder and car navigation system. This requests higher track density which leads to larger time to write the servo tracks. This increase in the number of servo tracks also cost an enormous investment to secure sufficient facilities such as clean room spaces. This situation forces manufactures to develop novel servo track writing (STW) technologies such as non-contact STW, media STW, self STW, and pattern printing STW.

Among these technologies, self servo track writer (SSTW) uses the read and write elements which is already installed in the read/write head of the hard disk drives. This makes the servo track writing process be achieved without an external servowriting machine and consequently reduces time and cost (Szita [2003], Kang et al. [2005]).

The SSTW uses the offset between the read and write elements of the head. At first, guide servo track patterns are written on a disc drive, then a new servo track is written with the write elements of the head while the read elements reads the previously-written servo track. This is self-propagating of servo track writing.

This self-propagation, however, has several drawbacks. Among them, the most important problem is error propagation. If there is some error in servo tracks, for example, a written servo track cannot be written in a perfect circular form; the error also will be propagated through the self servo track writing process.

The causes of this problem can be listed as follows (Nakamura et al. [2004]):

- (1) High gain greater than unity in the sensitivity function especially in high frequency band
- (2) Disturbances and noises in hard disk drive system
- (3) Limited measurable output signal

### 1.2 Main purpose of this research

It is true that there are several other problems in SSTW other than this error propagation problem, only the error propagation problem, however, is focused on in this paper.

This paper is organized as follows; at first, the dynamics of the SSTW is modeled and based on that dynamics, the error propagation characteristic is analyzed in Session 2. Then in Session 3, a method to depress the error propagation is proposed using estimation of the absolute head position. Kalman filter is adopted and shows some improvements comparing with the conventional method. As another approach to the problem, Session 4 analyzes the propagation characteristics in frequency domain, and reflects it in design of the feedforward control. Lastly, simulation results in Session 5 verify our analysis and design method.

## 2. MODELING OF SELF SERVO TRACK WRITER AND FORMULIZATION OF THE PROBLEM

As explained in the previous session, the SSTW uses the writing element and read element in the head; it writes new servo track, tracking the previous servo track signals and repeats the same process. Since the SSTW tracks previous servo tracks when it writes new track, the control for the

SSTW can be the same with the conventional following control.

### 2.1 Modeling of SSTW

Figure 1 shows the propagation characteristics in the SSTW as a block diagram.

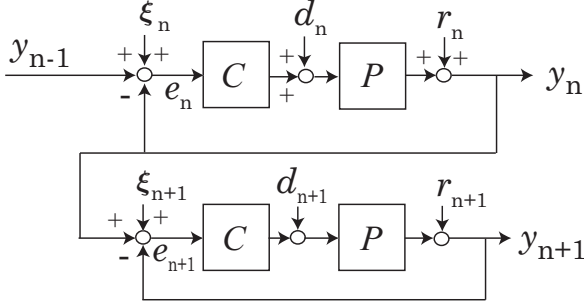


Fig. 1. Propagation in Self-servo Track Writer

One loop consisting of a plant  $P$  and a controller  $C$  represents the following controlled system loop. All the symbols used in figures and remainder of this paper are explained in Table 1.

Table 1. Definition of Symbols

$y$	Absolute position of the head
$\Delta y$	Position error from the ideal circle track
$\xi$	Sensor noise in Position Error Signal
$u^{ff}$	Feedforward control input
$d$	Torque disturbance
$r$	Repeatable runout (RRO) w/ flutter noise
$N$	Equivalent disturbance
$e$	Position Error Signal (PES)
$E^a$	Absolute track squeeze error
$E^r$	Relative track squeeze error
$n(\text{subscript})$	$n$ th track
$KF(\text{superscript})$	related with Kalman Filter

Note that the offset between write element and read element in the head in Figure 1 is assumed to be zero in the remainder of this paper to simplify the analysis.

The absolute head position  $y_n$  in  $n$ th track is given to  $(n+1)$ th track as a position reference; this is the propagation characteristic is the SSTW.

### 2.2 Error Propagation in SSTW

Due to the propagation characteristics in the SSTW, errors caused by mechanical disturbances in one track are reproduced from one track to the next. Although the feedback controller ( $C$  in Figure 1) suppresses those disturbances it cannot eliminate the disturbances perfectly. The output error caused by the unsuppressed disturbances will be propagated or even amplified as the track number increases.

For example, the disturbance  $d_n$  in the input terminal is attenuated in the output  $y_n$  with the characteristic of  $\frac{P}{1+PC}$  in  $n$ th track.  $d_n$  remaining in  $y_n$  will be propagated to  $y_{n+1}$  with the characteristic of  $\frac{PC}{1+PC}$ , which means

if the remaining  $d_n$  is in the bandwidth of  $\frac{PC}{1+PC}$ , it will not be deleted forever. The disturbance will be even amplified in the frequency range where the amplitude of the complementary sensitivity function becomes greater than 1 due to the waterbed characteristics.

The SSTW cannot be realized without solving this error propagation problem. For this end, this paper suggests control designs which suppress these propagating errors using the measurable position error signals.

### 2.3 Simulation using Benchmark Problem

This paper adopts a benchmark problem software (MSS [2005]) to analyze and simulate the proposed control algorithm. This benchmark software has been developed by a working group in the Institute of Electrical Engineers of Japan -Technical Committee for Mass Storage Servo Control.

Figure 2 is the block diagram which simulates motion of the head in track writing for one track. With this simulation for one track, the absolute head position  $Y$  and the position error signal  $PES$  are stored and the stored values are used as the reference and the source for the feedforward controller in the next track simulation.

Figure 3 to Figure 5 are the frequency characteristics of the VCM, two feedback controllers and the disturbances which are used in the benchmark problem software. Feedback controller consists of two multirate digital controllers: a discrete PID control and a notch filter which has twice sampling time compared with the PES sampling time ( $37.8\mu s$ ). All the disturbance are produced based on random numbers. RRO noise is given in the same time pattern through all track writings.

The FF block in Figure 2 is a feedforward controller which uses the PES accumulated to the last track as its input. Proper design of this FF can suppress the error propagation. To discuss this design is the main purpose of this paper.

Evaluation of the error propagation is done using  $3\sigma$  (three times of the standard deviation) of  $Y$  and the relative error, as 0 in  $Y$  is assumed to be the center of each track.  $3\sigma$  of  $Y$  evaluates the absolute track squeeze error and  $3\sigma$  of the relative error before adding the measurement noise evaluates the relative track squeeze error.

For the comparison with the error propagation suppression control which will be suggested in the following sessions,  $3\sigma$  of  $Y$  and the relative error without control are illustrated in Figure 6 based on simulations.

This simulation reveals that both  $3\sigma$  increases exponentially with reference to the track numbers without any feedforward control.

## 3. KALMAN FILTER ESTIMATION OF ABSOLUTE HEAD POSITION

In self servo track writing process, the only measurable signal in each track is position error signal which is the error between the current head position and the previously-written servo track. This PES is relative error

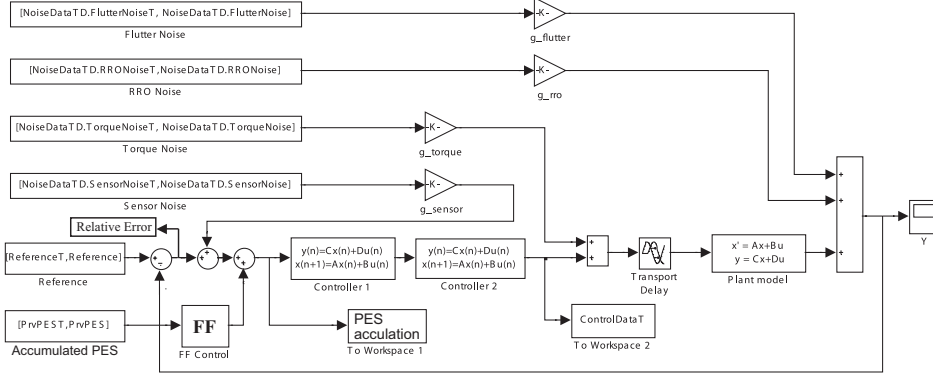


Fig. 2. Block-diagram of Benchmark Simulation

between two servo tracks but not an absolute error from the track that should be written - a concentric circle with the same distances from other circles; this is one cause of the error propagation characteristics.

If the absolute error or the absolute head position is available, the error propagation can be corrected based on the absolute error. To this end, the absolute head position needs to be observed. This is another interpretation of the feedforward control which sets FF in Figure 2 as 1. In this session, an improved absolute head estimation method is suggested using Kalman filter.

### 3.1 Estimation by Linear Addition of PES

Equation (2) is an algorithm to estimate the absolute position head suggested in Nakamura et al. [2004].

$$e_n[k] = y_{n-1}[k] - y_n[k] + \frac{1}{1+PC}\xi_n[k] \quad (1)$$

$$\begin{aligned} \sum_{i=1}^n e_i[k] &= y_0[k] - y_n[k] + \sum_{i=1}^n \left( \frac{1}{1+PC} \right)^i \xi_i[k] \\ &= \Delta \hat{y}_n[k] = u_n^{ff}[k] \end{aligned} \quad (2)$$

$\Delta \hat{y}_n[k]$  is the accumulation of the PES to the last track and also it will be the absolute error of current head position. Then, this  $\Delta \hat{y}_n[k]$  is used as a feedforward control input to the next servo track writing, which will suppress the absolute error.

This estimation, however, also accumulates the measurement noise which increases  $3\sigma$  of  $Y$  and relative error, which is illustrated in Figure 7

### 3.2 Effect of Measurement Noise in Head Position Estimation

With the feedforward control of Equation (2), the error to be attenuated will be  $e_n[k] + u_n^{ff}[k]$  in Equation (3)

$$e_n[k] + u_n^{ff}[k] = y_0[k] - y_n[k] + \sum_{i=1}^n \left( \frac{1}{1+PC} \right)^i \xi_i[k] \quad (3)$$

As this is the estimated absolute track squeeze error, the head position of  $(n+1)$ th track can be positioned on the

absolute correct circle like Equation (4) in the bandwidth of feedback controller.

$$y_n[k] \rightarrow y_0[k] + \sum_{i=1}^n \left( \frac{1}{1+PC} \right)^i \xi_i[k] \quad (4)$$

The problem is the second term; the accumulation of the measurement noise. Even though the mean value of the measurement noise is zero, the variance increases in this accumulation. This is the propagation of measurement noise. Simulation indicates this characteristic.

Figure 7 is the result.  $3\sigma$  of  $Y$  and  $PES$  are increasing with regard to the track number although the slope becomes smaller than Figure 6. This result reveals that the head position estimation by accumulating the PES can suppress to some extent but cannot eliminate it.

### 3.3 Accurate Head Position Estimation by Kalman Filter Design

To remove the effect of the measurement noise, a Kalman filter is designed for correct estimation of the absolute head position. The dynamics from  $y_n$  to  $y_{n+1}$  is used for the dynamics of the Kalman filter design. VCM is modeled as a second order nominal model and the notch filters in the feedback controller are not included in this dynamics for simplification.

The states  $\mathbf{x}_i^{KF}$  in Equation (5) is defined as the states of the VCM and two controllers.

$$\begin{aligned} \mathbf{x}_i^{KF}[k+1] &= \mathbf{A}\mathbf{x}_i^{KF}[k] + \mathbf{B}\mathbf{u}_i^{KF}[k] + \mathbf{G}\mathbf{w}_i^{KF}[k] \\ y_i^{KF}[k] &= \mathbf{C}\mathbf{x}_i^{KF}[k] + \mathbf{B}\mathbf{u}_i^{KF}[k] + \mathbf{H}\mathbf{w}_i^{KF}[k] + v_i[k] \end{aligned} \quad (5)$$

The input  $u_i^{KF}$  is the signal to the PID controller. Since the reference position can be assumed to be 0 ignoring the offset between the write element and read element of the head, the feedforward control  $u_i^{ff}[k]$  will be this input  $u_i^{KF}$ . The measurable signal PES ( $e[k]$ ) added by the feedforward control  $u_i^{ff}[k]$  is used as the output  $y_i^{KF}[k]$  of the dynamics. The estimated output  $\hat{e}_n[k] + \hat{u}_n^{ff}[k]$  is the estimation of the absolute head position we need. The dynamics  $\mathbf{A}$  in Equation (5) for the Kalman filter is derived from that of  $\frac{CP}{1+CP}$ .

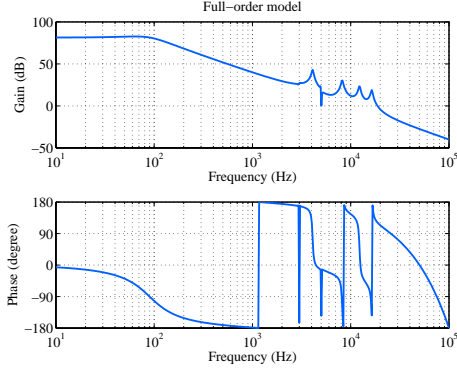


Fig. 3. Frequency Response of VCM Model

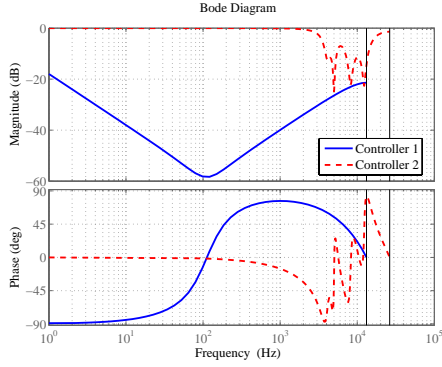


Fig. 4. Frequency Response of Controller

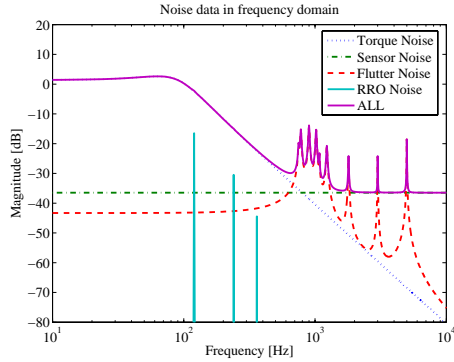


Fig. 5. Frequency Characteristics of Disturbances

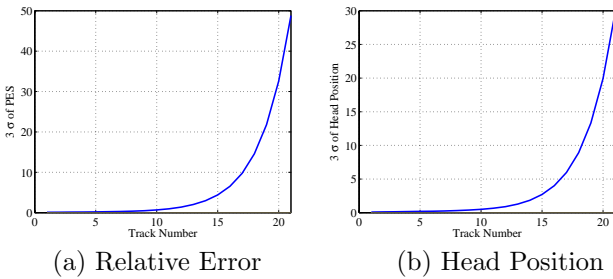


Fig. 6. Transitions of  $3\sigma$  of Relative Error (a) and Head Position (b)

Considering these relationships, the Kalman filter is designed as Equation (6). The estimated relative error  $\hat{e}_n[k]$  is accumulated for the feedforward control in the next track.

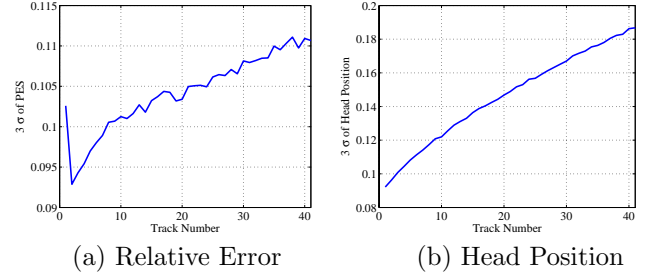


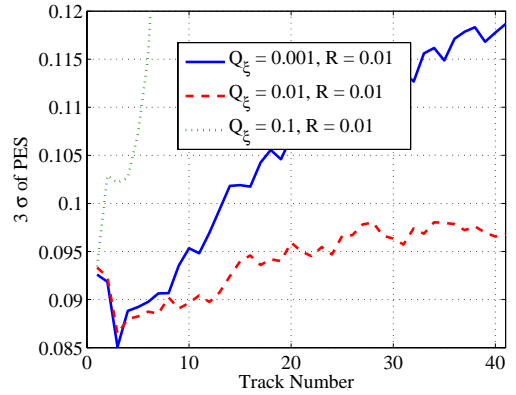
Fig. 7. Transitions of  $3\sigma$  of Errors

$$\begin{aligned} \hat{x}_n^{KF}[k+1] &= \mathbf{A}\hat{x}_n^{KF}[k] + \mathbf{B}u_n^{ff}[k] \\ &\quad + \mathbf{K}\left(\mathbf{PES}_n[k] + u_n^{ff}[k] - \mathbf{C}\hat{x}_n^{KF}[k] - \mathbf{D}u_n^{ff}[k]\right) \\ \hat{e}_n[k] + \hat{u}_n^{ff}[k] &= \hat{y}_n^{KF}[k] = \mathbf{C}\hat{x}_n^{KF}[k] + \mathbf{D}u_n^{ff}[k] \end{aligned} \quad (6)$$

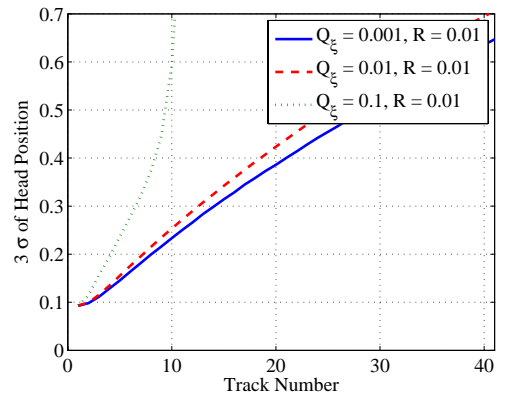
$\mathbf{K}$  is the Kalman filter gain designed based on the covariance of  $\mathbf{w}^{KF}$  and  $\mathbf{v}$  in Equation (5). Note that the measurement noise  $\xi$  works not only as the output noise  $\mathbf{v}$  but also the system noise  $\mathbf{w}^{KF}$ .

In Kalman filter design,  $\mathbf{v}$  does not affects the states but it only affects the measurement.  $\xi$ , however, affects the states  $\mathbf{x}^{KF}$ , since it is fed back to the feedback controller. Considering this point, one of the columns in  $\mathbf{G}$  in Equation(5) is designed as the same with  $\mathbf{B}$ .

Covariance  $\mathbf{Q}_{w_\xi}$  for that system noise  $w_\xi$  and  $\mathbf{R}$  for  $\mathbf{v}$  are chosen as the tuning knobs.



(a) Relative Error



(b) Head Position

Fig. 8.  $3\sigma$  of Errors with Kalman Filter

Figure 8 is simulation results of Kalman filter. Compared with Figure 8, the error propagation is suppressed more.

Although the relative error shows some convergence, the absolute error and the feedforward input with position error signal diverge here. This may be attributed to the poor covariance design. From Figure 8(a), we can say there seems to be some optimal covariance value for position error. By optimizing the covariance,  $K$  for better suppression performance can be obtained.

#### 4. FEEDFORWARD CONTROL DESIGN BASED ON THE FREQUENCY CHARACTERISTICS OF ERROR PROPAGATION

In this session, feedforward control (FF in Figure 2) design is discussed. This approach is different from the estimation of the head position and also can be combined with the head position estimation suggested in the previous session. Du et al. [2005] is one of these approaches.

This approach can also be interpreted as iterative learning control or repetitive control (Chen et al. [2006]) as it uses the signals in the previous execution. Melkote et al. [2006] is one loyal application of ILC to the SSTW propagation problem. However, their filter design was limited to a gain, not a filter which has a dynamics. The design proposed in this section is a dynamic filter based on frequency characteristics of VCM.

##### 4.1 Derivation of Propagation Characteristic Function

Figure 9 shows the block diagram of the feedforward control in SSTW. Measured data in the previous track is filtered and provided as a feedforward control in the next track.

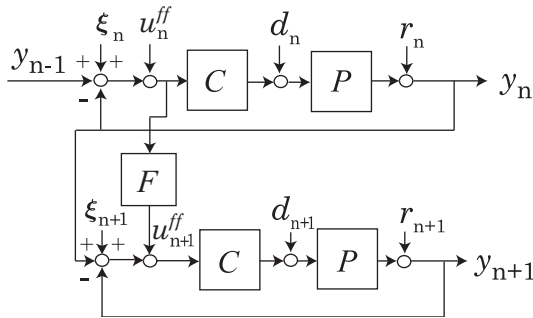


Fig. 9. SSTW Control using PES Feedforward

It is another interesting point that the estimation of head position by accumulating the PES can be categorized as this feedforward control which sets  $F$  as 1 or 0.88.

The transition matrix on how two variables  $u^{ff}$  and  $y$  propagate is calculated as Equation (7).

$$\begin{pmatrix} u_{n+1}^{ff}[k] \\ y_n[k] \end{pmatrix} = \begin{pmatrix} F(1-T)F(1-T) \\ T \end{pmatrix} \begin{pmatrix} u_n^{ff}[k] \\ y_{n-1}[k] \end{pmatrix} \quad (7)$$

From this matrix, the relationship between  $u_n^{ff}$  and  $y_{n-1}$  is given as

$$Tu_n^{ff}[k] = F(1-T)y_{n-1}[k]. \quad (8)$$

This simplifies the propagation characteristics as the below equation.

$$y_{n+1} = (T + F - TF)y_n \quad (9)$$

Error propagation can be suppressed by shaping this propagation characteristics;  $T + F - TF$ . Using the descriptions  $C, P$  in Figure 9, the propagation characteristics can be described as

$$T + F(1-T) = \frac{CP}{1+CP} + \frac{F}{1+CP} = T_{pr}. \quad (10)$$

The feedforward filter  $F$  can be designed based on this equation. If  $F$  is designed to make the infinity norm of  $T_{pr}$  less than unity, error propagation will be converged to zero, which is proved in the iterative learning control theory Bristow et al. [2006].

##### 4.2 Analysis of Propagation Characteristics of Track Errors

In this subsection, propagation characteristics of two track errors are analyzed: the absolute track squeeze error and the relative track squeeze error. The effect of disturbance and noise to these two errors are focused.

The disturbance and noise described in Figure 5 - torque disturbance, flutter noise, RRO noise and sensor noise - are considered in the propagation analysis. In the remainder of this paper, all noises except the sensor noise are dealt with as one equivalent noise which is applied to the output of the plant and described as  $N$  while the sensor noise is described as  $\xi$ .

The absolute track squeeze error,  $E^a$  can be given as  $-y[k]$  since the reference position for each track is assumed to be zero. Its propagation is described as Equation(11). Equation (11) is the case of the relative track squeeze error  $E^r$ .

$$\begin{aligned} E_{n+1}^a[k] &= T_{pr}E_n^a[k] - FSN_n[k] + SN_{n+1}[k] + T\xi_{n+1}[k] \\ E_{n+1}^r[k] &= T_{pr}E_n^r[k] - FS(N_{n-1}[k] - N_n[k]) \\ &\quad + S(N_n[k] - N_{n+1}[k]) + T(\xi_n[k] - \xi_{n+1}[k]) \end{aligned} \quad (11)$$

The absolute squeeze error is excited by  $N$  and  $\xi$  through  $(1-F)$  and  $T$  respectively. Note that  $F$  will affect not only the propagation characteristics  $T_{pr}$  but also the way the noise  $N$  excites the absolute track squeeze error.

Comparing Equation (11), the relative track error is excited by the difference of the noise between the previous track and the current track.

#### 5. SIMULATION OF PROPOSED FEEDFORWARD CONTROL USING BENCHMARK PROBLEM

Based on Equation (10),  $F$  is designed as Equation (12) to decrease the norm of  $T_{pr}$  under unity, with the gain  $K$  between 0 and 1;

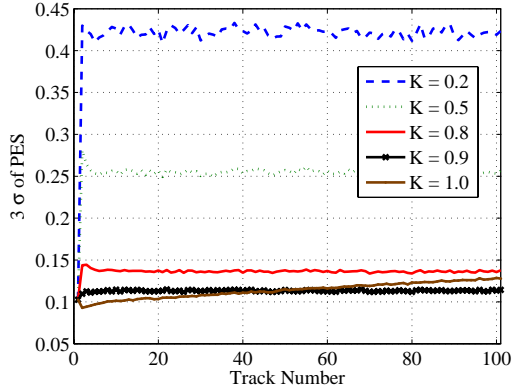
$$F = K + (K-1)CP_n, \quad (12)$$

$$T_{pr} = T + F(1-T) = K \quad (13)$$

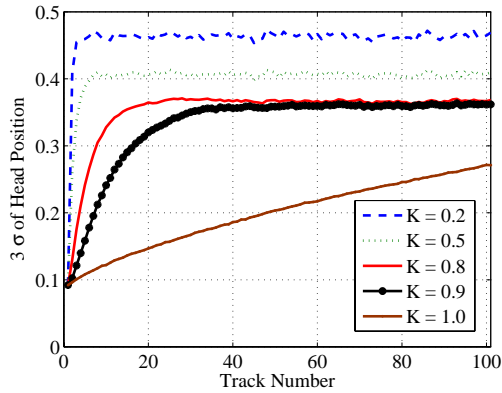
where  $P_n$  is the nominal dynamics of VCM which consists of a second order system. This  $F$  makes the propagation characteristic  $T_{pr}$  in Equation (10)  $K$  if  $P_n = P$ .



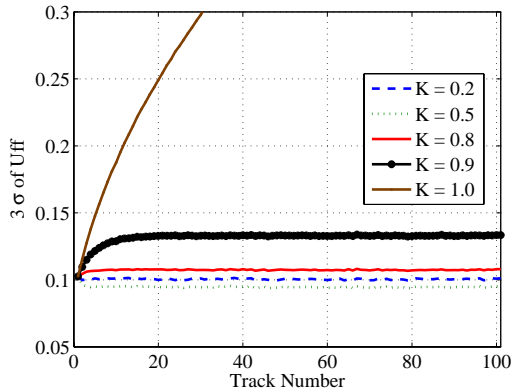
Simulations are done changing  $K$  to 0.2, 0.5, 0.8. The results are illustrated in Figure 10.



(a) Relative Error



(b) Head Position



(c) Feedforward Input added with PES

Fig. 10.  $3\sigma$  of Errors and Feedforward Input

Simulations are conducted until the track reaches 100th track. Both the relative track error and absolute track error do not diverge and converge to particular values for  $K$  values. In ideal case where  $P = P_n$ ,  $K$  will be the propagation gain which means the smaller  $K$  is, the faster the values of PES and Y will converge. This is verified in Figure 10.(b). However, the small  $K$  which will improve the convergence performance leaves large converged errors.

Another noticeable point in this simulation is the difference between converged values of two errors. In the case of Gain 0.9, the relative track error converges around 0.14 while the absolute track error converges around 0.36.

Relative track error is more important in terms of practical use. If the distance between adjacent tracks is constant with small deviation, it can be used as servo track although the absolute shape of the track is not a perfect circle.

The gain  $K$  can decrease these converged error values; as the gain  $K$  approaches near 1, the level of the residual errors decreases. Finally when the gain  $K$  becomes 1, the stability limitation of propagation characteristic in Equation (12), the errors starts to diverge. This is why the simple accumulation of the previous PES cannot suppress the divergence of errors.  $K$  also should be designed considering the robustness against modeling error.

## 6. CONCLUSION

This paper focuses on the propagation characteristics of SSTW and suggests several solutions to deal with the propagation problem: accurate estimation of the absolute head position based on Kalman filter design and a feedforward filter design based on the analysis of the propagation characteristic in the frequency domain.

As for the estimation of the absolute head position design, the simulation results show the effectiveness of Kalman filter design on the measurement noise. To design effective feedforward filter, the propagation characteristics in the SSTW is analyzed in mathematical ways. The proposed feedforward filter based on that propagation characteristic succeeded to eliminate the error propagation. By adjusting the gain  $K$  in Equation (12), the value to which the error converges can be controlled.

The dynamics how this  $K$  can adjust the residual error should be researched more.

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