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VIBRATION SUPPRESSION AND DISTURBANCE REJECTION CONTROL IN SEMI-NYQUIST FREQUENCY REGION BASED ON A NOVEL MULTIRATE SAMPLING CONTROL WITH INITIAL VALUE COMPENSATION

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Abstract. In this paper, novel multirate feedback controllers are proposed for digital control systems, where it is restricted that the speed of the A/D converters are slower than that of the D/A converters. The proposed controllers achieve vibration suppression and disturbance rejection even in the semi-Nyquist frequency region. First, the continuous-time vibration suppression controller is exactly discretized by the multirate sampling control based on the closed-loop characteristics. Second, the multirate repetitive controllers are proposed both by the feedback and feedforward approaches. Moreover, the initial value compensations for observer are proposed in order to prevent the overshoot. The proposed controllers are applied to the settling and following modes of hard disk drive, and the advantages of these approaches are demonstrated by simulations.

Keywords: Multirate sampling control, Vibration suppression, Discretization of controller, Disturbance rejection, Repetitive control, Initial value compensation, Hard disk drive.

1. INTRODUCTION

A digital control system usually has two samplers for the reference signal $r(t)$ and the output $y(t)$, and one holder on the input $u(t)$ as shown in Fig. 1. Therefore, there exist three time periods T_r ; T_y , and T_u which represent the period of $r(t)$; $y(t)$, and $u(t)$, respectively. The input period T_u is generally decided by the speed of the actuator, D/A converter, or the calculation on the CPU. Moreover, the output period T_y is also determined by the speed of the sensor or the A/D converter. Practical control systems usually hold the restrictions on T_u and/or T_y . Thus, the conventional digital control systems make these three periods equal to the longer period between T_u and T_y .

In this paper, the digital control systems with longer sampling period ($T_u < T_y$) are considered. This restriction may be general because D/A converters are usually faster than the A/D converters. Especially, head positioning systems of the hard disk drive (HDD) or the visual servo systems of robot manipulator belong to this category, because the sampling rates of the measurement are relatively slow [1]-[5].

For these systems, it is difficult to suppress vibration and to reject disturbance in high frequency region because the Nyquist frequency is relatively low. In this paper, multirate sampling control is introduced, in which the plant input is changed N times during one sampling period. This scheme is also called the multirate input control. Using this scheme, novel multirate feedback controllers are proposed, which achieve vibration suppression and disturbance rejection even in the semi Nyquist frequency region. Moreover, the

proposed methods are applied to the head-positioning system of hard disk drive.

A vibration suppression controller is generally designed in the continuous-time system. To implement it in the digital control system, the designed analog controller is discretized by the Tustin transformation or other methods. Because these transformations are based only on the open-loop characteristics of the controller, the closed-loop becomes low performance or unstable when the control bandwidth is close to the Nyquist frequency.

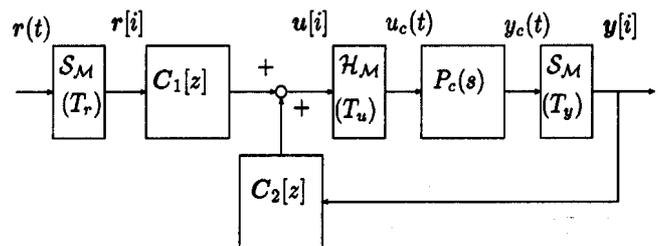


Fig. 1. Two-degree-of-freedom control system

On the other hand, introducing multirate sampling control, the first author proposed a novel discretization method of controllers based on the closed-loop characteristics [6]. In this paper, this approach is extended to the hardware restriction of ($T_u < T_y$) and applied to the vibration suppression controller. The advantages of the proposed method are that the controller is discretized based on the closed-loop characteristics, and the plant state of the digitally controlled system completely matches that of the original continuous-time system at M inter-sample points during T_y .

In the repetitive control system [7], conventional single-rate controllers do not have enough inter-sample performance to reject disturbance in the semi-Nyquist frequency region. On the other hand, authors proposed a novel multirate feedback controller, which achieves the perfect disturbance rejection at M inter-sample points [4]. In this paper, the proposed approach is modified to repetitive control, and applied to reject high order repeatable run-out of hard disk drive.

Repetitive feedback controllers based on the internal model principle have disadvantages that the closed-loop characteristics become worse and difficult to assure stability robustness [8]. Therefore, this paper proposes novel approach which never has these problems, based on the open-loop estimation and disturbance rejection by feedforward approach.

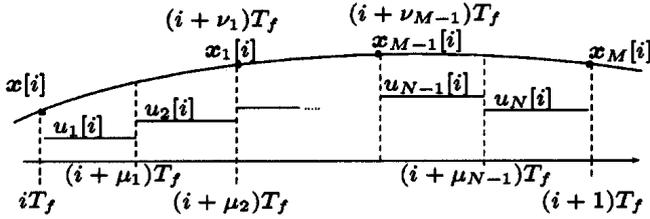


Fig. 2. Multirate Sampling control

2. DISCRETIZATION OF CONTROLLER BASED ON MULTIRATE INPUT CONTROL

In this section, novel discretization method of an analog controller is proposed for the system with longer sampling period ($T_u < T_y$) base on the multirate input control. The proposed method is applied to vibration suppression controller in section 4.1.. For the restriction of $T_u < T_y$, the flame period T_f is defined as $T_f = T_y$ [9], and the dynamics of the controller is described by T_f .

In the proposed multirate scheme, the plant input is changed N times during $T_y (= T_f)$ as shown in Fig. 2. An integer M is selected so as to $M \equiv N/n$ becomes an integer, where N is the input multiplicity and n is the plant order. For simplification, the continuous-time plant is assumed to be SISO system in this paper. The proposed methods, however, can be extended to deal with the MIMO system by the same way as [6].

Plant Discretization by Multirate Sampling

Consider the continuous-time plant described by

$$\dot{x}(t) = A_c x(t) + b_c u(t) \quad , \quad y(t) = c_c x(t). \quad (1)$$

The discrete-time plant discretized by the multirate sampling control (Fig. 2.) becomes

$$x[i+1] = Ax[i] + Bu[i] \quad , \quad y[i] = Cx[i] \quad (2)$$

where $\mathbf{x}[i] = \mathbf{x}(iT)$, and matrices \mathbf{A} ; \mathbf{B} ; \mathbf{C} and vectors \mathbf{u} are given by

$$\begin{bmatrix} A & B \\ C & O \end{bmatrix} \equiv \begin{bmatrix} e^{A_c T_f} & b_1 & \cdots & b_N \\ c_c & 0 & \cdots & 0 \end{bmatrix} \quad (3)$$

$$b_j \equiv \int_{(1-\mu_j)T_f}^{(1-\mu_{(j-1)})T_f} e^{A_c \tau} b_c d\tau \quad , \quad u \equiv [u_1, \dots, u_N]^T \quad (4)$$

$$0 = \mu_0 < \mu_1 < \mu_2 < \dots < \mu_N = 1 \quad (5)$$

The inter-sample plant state at $t = (i + v_k)T_f$ is represented by

$$\tilde{x}[i] = \tilde{A}x[i] + \tilde{B}u[i], \quad (6)$$

$$\tilde{A} | \tilde{B} \equiv \begin{bmatrix} \tilde{A}_1 & \tilde{b}_{11} & \cdots & \tilde{b}_{1N} \\ \vdots & \vdots & & \vdots \\ \tilde{A}_M & \tilde{b}_{M1} & \cdots & \tilde{b}_{MN} \end{bmatrix}, \quad (7)$$

$$\tilde{A}_k \equiv e^{A_c v_k T_f} \quad , \quad \tilde{x} \equiv [x_1, \dots, x_M]^T, \quad (8)$$

$$x_k[i] = x[i + v_k] = x((i + v_k)T_f), \quad (9)$$

$$\tilde{b}_{kj} \equiv \begin{cases} \mu_j < v_k : & \int_{(v_k - \mu_j)T_f}^{(v_k - \mu_{(j-1)})T_f} e^{A_c \tau} b_c d\tau \\ \mu_{(j-1)} < v_k \leq \mu_j : & \int_0^{(v_k - \mu_{(j-1)})T_f} e^{A_c \tau} b_c d\tau \\ v_k \leq \mu_{(j-1)} : & 0 \end{cases},$$

$$0 < v_1 < v_2 < \dots < v_M = 1 \quad (10)$$

where $\mu_j (j = 0, 1, \dots, N)$ and $v_k (k = 1, \dots, M)$ are the parameters for multirate sampling as shown in Fig. 2. If T_f is divided at same intervals, $\mu_j = j/N$, $v_k = k/M$.

Design of continuous-time controller

In this section, the continuous-time controller is designed based on the regulator and the disturbance observer.

Consider the continuous-time plant model described by

$$\dot{x}_p(t) = A_{cp} x_p(t) + b_{cp} (u(t) - d(t)) \quad (10)$$

$$y(t) = c_{cp} x_p(t) \quad (12)$$

where $d(t)$ is the disturbance input. Let the disturbance model be

$$\dot{x}_d(t) = A_{cd} x_d(t) \quad , \quad d(t) = c_{cd} x_d(t) \quad (13)$$

For example, the step type disturbance can be modeled by $A_{cd} = 0$, $c_{cd} = 1$. The continuous-time augmented system consisting of (11) and (13) is represented by

$$\dot{x}(t) = A_c x(t) + b_c u(t) \quad (14)$$

$$y(t) = c_c x(t) \quad (15)$$

$$A_c \equiv \begin{bmatrix} A_{cp} & -b_{cp} c_{cd} \\ O & A_{cd} \end{bmatrix}, b_c \equiv \begin{bmatrix} b_{cp} \\ 0 \end{bmatrix}, x \equiv \begin{bmatrix} x_p \\ x_d \end{bmatrix}, c_c \equiv [c_{cp}, 0]$$

For the plant (14), the continuous-time observer is designed from the Gopinath's method by

$$\dot{\hat{v}}(t) = \hat{A}_c \hat{v}(t) + \hat{b}_c y(t) + \hat{J}_c u(t) \quad (16)$$

$$\hat{x}(t) = \hat{C}_c \hat{v}(t) + \hat{d}_c y(t) \quad (17)$$

In order to regulate the plant state and reject the disturbance, the continuous-time regulator is designed by

$$u(t) = f_{cp} \hat{x}_p(t) + c_{cd} \hat{x}_d(t) = f_c \hat{x}(t) \quad (11)$$

$$f_c \equiv [f_{cp}, c_{cd}] \quad (19)$$

Letting e_v be the estimation errors of the observer $e_v = (\hat{v} - v)$, the following equation is obtained.

$$\hat{x}(t) = x(t) + \hat{C}e_v(t) \quad (20)$$

From the above equations, the closed-loop system is represented by

$$\begin{bmatrix} \dot{x}_p(t) \\ \dot{x}_d(t) \\ \dot{e}_v(t) \end{bmatrix} = \begin{bmatrix} A_{Fcp} & O & b_{cp}f_c\hat{C}_c \\ O & A_d & O \\ O & O & \hat{A}_c \end{bmatrix} \begin{bmatrix} x_p(t) \\ x_d(t) \\ e_v(t) \end{bmatrix} \quad (21)$$

where $A_{Fcp} \equiv A_{cp} + b_{cp}f_c$. The transitions of the states x_p ; x_d from $t = iT_f$ to $t = (i + v_k)T_f$ are represented by

$$\begin{bmatrix} x_p[i+v_k] \\ x_d[i+v_k] \\ e_v[i+1] \end{bmatrix} = \begin{bmatrix} e^{A_{Fcp}v_kT_f} & O & * \\ O & e^{A_d v_k T_f} & O \\ O & O & e^{\hat{A}_c T_f} \end{bmatrix} \begin{bmatrix} x_p[i] \\ x_d[i] \\ e_v[i] \end{bmatrix} \quad (22)$$

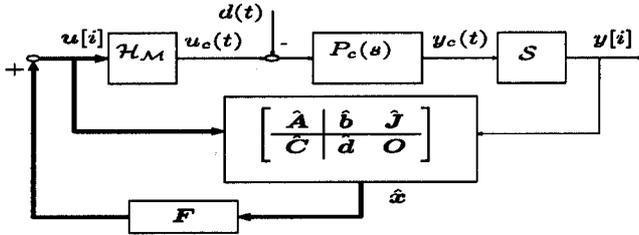


Fig. 3. Multirate control with disturbance observer

Discretization of the controller by multirate input control

In this section, the digital controller is obtained from the continuous-time controller designed in section 2.2. using multirate input control.

Discretizing (14) by the multirate sampling control, the inter-sample plant state at $t = (i + v_k)T_f$ can be calculated from the k th row of (6) by

$$x[i+v_k] = \tilde{A}_k x[i] + \tilde{B}_k u[i] \quad (23)$$

$$\tilde{A}_k = \begin{bmatrix} \tilde{A}_{pk} & \tilde{A}_{pdk} \\ O & \tilde{A}_{dk} \end{bmatrix}, \tilde{B}_k = \begin{bmatrix} \tilde{B}_{pk} \\ O \end{bmatrix}$$

For the plant (14) discretized by (2), the discrete-time observer on the sampling points is obtained by

$$\hat{v}[i+1] = \hat{A}\hat{v}[i] + \hat{b}y[i] + \hat{J}u[i] \quad (24)$$

$$\hat{x}[i] = \hat{C}\hat{v}[i] + \hat{d}y[i] \quad (25)$$

As shown in Fig. 3., let the feedback control law be

$$u[i] = F_p \hat{x}_p[i] + F_d \hat{x}_d[i] = F \hat{x}[i] \quad (26)$$

where $F \equiv [F_p; F_d]$. From (23) to (20), the closed-loop system is represented by

$$\begin{bmatrix} x_p[i+v_k] \\ x_d[i+v_k] \\ e_v[i+1] \end{bmatrix} = \begin{bmatrix} \tilde{A}_{pk} + \tilde{B}_{pk}F_p & \tilde{A}_{pdk} + \tilde{B}_{pk}F_d & \tilde{B}_{pk}FC \\ O & \tilde{A}_{dk} & O \\ O & O & \hat{A} \end{bmatrix} \begin{bmatrix} x_p[i] \\ x_d[i] \\ e_v[i] \end{bmatrix} \quad (27)$$

Comparing (22) and (27), if the following conditions are satisfied, the plant state (x_p) of the digitally controlled system completely matches that of the original continuous-time system at M inter-sample points on $t = (i + v_k)T_f$.

$$\tilde{A}_{pk} + \tilde{B}_{pk}F_p = e^{A_{Fcp}v_kT_f} \quad (28)$$

$$\tilde{A}_{pdk} + \tilde{B}_{pk}F_d = O \quad (29)$$

$$e_v[i] = O \quad (30)$$

The simultaneous equations of (28) and (29) for all $k=(1, \dots, M)$ become

$$\tilde{A}_p + \tilde{B}_p F_p = E, \tilde{A}_{pd} + \tilde{B}_p F_d = O \quad (31)$$

$$\left[\tilde{A}_p \mid \tilde{A}_{pd} \mid \tilde{B}_p \mid E \right] \equiv \begin{bmatrix} \tilde{A}_{p1} & \tilde{A}_{pd1} & \tilde{B}_{p1} & e^{A_{Fcp}v_1T_f} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{A}_{pM} & \tilde{A}_{pdM} & \tilde{B}_{pM} & e^{A_{Fcp}v_MT_f} \end{bmatrix}$$

Because non-singularity of the matrix B_p can be assured [4, 10], F_p and F_d are obtained by

$$F_p = \tilde{B}_p^{-1}(E - \tilde{A}_{pd}), F_d = -\tilde{B}_p^{-1}\tilde{A}_{pd} \quad (32)$$

Moreover, [6] proposed the discretization for observer based on multirate output control, where the plant output is detected more frequently than the control period ($T_y < T_u$). However, in this paper, discrete-time observer (24) is simply obtained, so that the eigenvalues of \hat{A} become identical to those of $\hat{A}cT_f$, because the plant is assumed to have longer sampling period ($T_y > T_u$). Substituting (24) for (26), the feedback type controller is obtained by

$$\begin{bmatrix} \hat{v}[i+1] \\ u[i] \end{bmatrix} = \begin{bmatrix} \hat{A} + \hat{J}F\hat{C} & \hat{b} + \hat{J}D\hat{d} \\ F\hat{C} & F\hat{d} \end{bmatrix} \begin{bmatrix} \hat{v}[i] \\ y[i] \end{bmatrix} \quad (33)$$

Initial value compensation

In this section, the initial value of the controller (33) is considered in order to eliminate the estimation error of the observer and satisfy (30). From (25), if $x[0]$ is known, the initial value of controller should be set by

$$\hat{C}\hat{v}[0] = x[0] - \hat{d}y[0] \quad (34)$$

By this compensation, it is possible to prevent the overshoot of the step (or initial value) response because the plant state converges only by the mode of the regulator. Therefore, f_{cp} should be designed to assign the eigenvalues of A_{Fcp} to the small (or zero) overshoot region.

3. REPETITIVE CONTROL BASED ON MULTIRATE INPUT CONTROL

In this section, two multirate repetitive controllers are proposed, which are 1) feedback approach based on internal model principle and 2) feedforward disturbance rejection approach based on the open-loop estimation.

Feedback repetitive control

The periodic disturbance of $T_0 \equiv 2\pi/\omega_0$ can be represented by

$$d(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos k\omega_0 t + b_k \sin k\omega_0 t \quad (35)$$

Letting the disturbance model (13) be (35), the repetitive feedback controller is obtained by (33), which has internal model $s^2 + (k\omega_0)^2$ in discrete-time domain.

From (27) and (29), the influence from disturbance $x_d[i]$ to the inter-sample state $x_p[i + v_k]$ becomes zero at $t = (i + v_k)T_f$

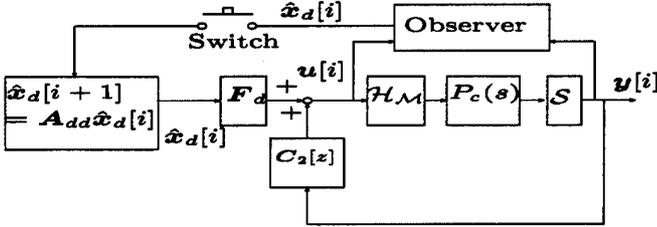


Fig. 4. Feedforward repetitive control

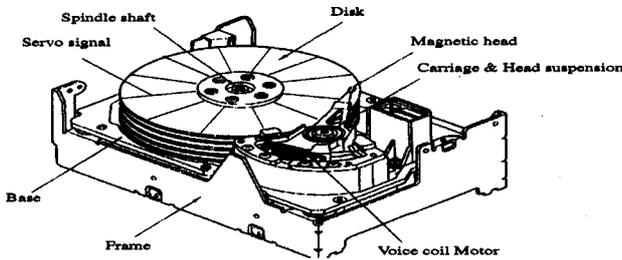


Fig. 5. Hard disk drive

Moreover, $x_p[i]$ and $e_v[i]$ converge to zero at the rate of the eigenvalues of $!! \sim A \text{ pM} + \sim B \text{ pM F p}$ and $\wedge A$ (the poles of the regulator and observer). Therefore, the repetitive disturbance is perfectly rejected ($x_p[i + v_k] = 0$) at M inter-sample points in the steady state.

Feedforward repetitive control

The repetitive feedback control based on the internal model principle has disadvantages that the closed-loop characteristics become worse and difficult to assure stability robustness [8]. Therefore, in this section, novel repetitive controller based on the open-loop estimation and feedforward disturbance rejection are proposed as shown in Fig. 4.

The repetitive disturbance is estimated by the open-loop disturbance observer. When the estimation converges to the steady state, the switch turns on at $t = t_0$. After that, the switch turns off immediately. The repetitive disturbance is calculated from the initial value $!! \wedge \times d[t_0]$ by

$$\hat{x}_d[i + 1] = A_{dd} \hat{x}_d[i] \quad (36)$$

where $A_{dd} = e^{A_{cd}T_f}$. Because the disturbance feedforward F_d is obtained by (32), the perfect disturbance rejection is achieved at M inter-sample points. The advantage of this approach is that the stability robustness can be guaranteed easily only by the conventional feedback controller $C_2[z]$. Moreover, by introducing the initial value compensation of the feedback controller $C_2[z]$ at $t = t_0$, the transient response can be improved after this switching

action. It is possible to prevent the overshoot by setting the initial state $\hat{v}[t_0]$ to be (34) using the estimated value of open-loop observer $\hat{x}[t_0]$

4. APPLICATIONS TO HDD

In the head-positioning control of hard disk drives (Fig. 5), the control strategy is divided into three modes; seeking mode, settling mode, and following mode. In the seeking mode, the head is moved to the desired track as fast as possible. Next, the head is settled to the track without overshoot in the settling mode. After that, the head needs to be positioned on the desired track while the information is read or written. In the following mode, the head is positioned finely on the desired track under the vibrations generated by the disk rotation and disturbance [11].

Amplifier gain	K_a	1.996	A/V
Force constant	K_f	2.95	N/A
Mass	M_p	6.983	g
Track pitch	T_p	3.608	$\mu\text{m}/\text{trk}$
Sampling time	T_s	138.54	μsec
Input multiplicity	N	4	
Mechanical resonance	ω_{1n}	$2\pi \times 2.7 \times 10^3$	rad/sec
Damping	ζ_{1n}	0.1	

Tab. 1. Parameters of the plant

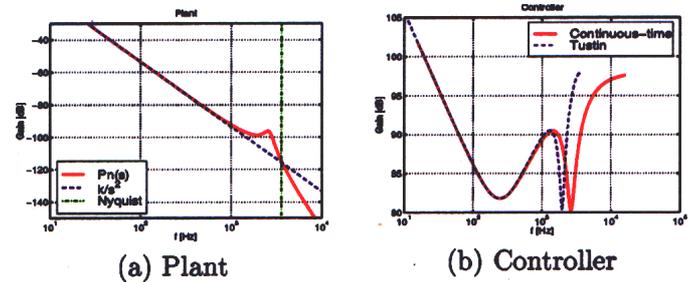


Fig. 6. Frequency responses

In this section, the proposed feedback controllers are applied to the settling and following modes. While servosignals are detected at a constant period about 100 $[\mu\text{s}]$, the control input can be changed 2~4 times between one sampling period in the recent hardware [3, 4, 5]. Therefore, the proposed approaches are applicable.

Vibration suppression control based on multirate input control

Let the nominal model of this plant be

$$P_c(s) = \frac{K_f K_a}{M_p} \frac{1}{s^2} \frac{1}{s^2 + 2\zeta_{1n}\omega_{1n}s + \omega_{1n}^2} \quad (37)$$

The parameters of this plant are shown in Table 1. This model is obtained from the experimental setup of 3.5-in hard disk drive [4, 5]. As shown in Fig. 6.(a), the actual plant has the first mechanical resonance mode around 2.7 [kHz], and its variation range is ± 500 [Hz]. The Nyquist frequency

(3.6 [kHz]) is close to this resonance mode. Therefore, it is very difficult to suppress the vibration in the conventional single-rate controller.

Continuous-time controller is designed by regulator and disturbance observer, in which the disturbance is modeled by the step type function $d(s)=1/s$, the poles of the regulator are set to $(s + \omega_c)^4$, and those of the observer are set to $!!(s + \omega_c)^2 (s^2 + 2\zeta_1\omega_{1n}s + \omega_{1n})$. As shown in Fig. 6.(b), this controller has notch characteristic at the resonance frequency. The parameter ω_c is tuned so that the bandwidth of the closed-loop system is set as high as possible, and stabilize the ± 1 [kHz] resonance variation. Fig. 6.(b) also shows that the Tustin transformation has large approximation error because the resonance mode is close to the Nyquist frequency.

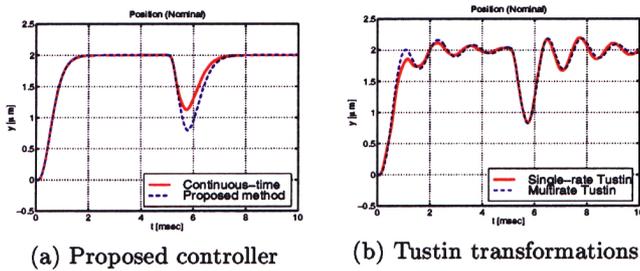


Fig. 7. Step and disturbance responses

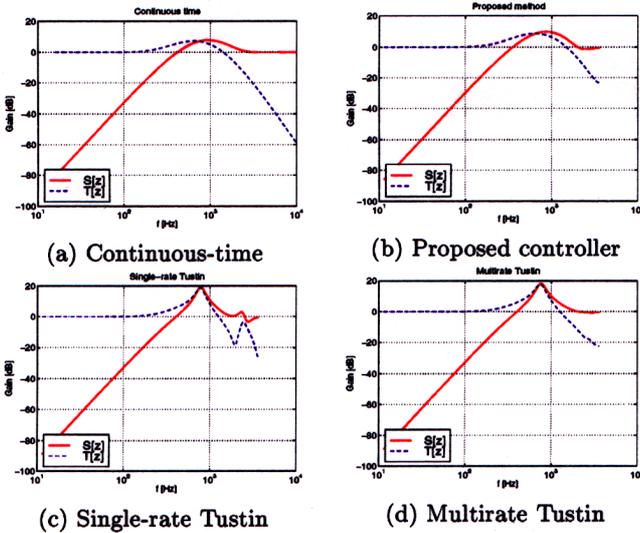


Fig. 8. Frequency responses $S[z]; T[z]$

Simulated results are shown in Fig. 7., which indicates that the proposed method has better performance than the Tustin transformations. While the responses of the Tustin transformations are oscillated, that of the proposed method has no vibration and same response as ideal continuous-time system. "Multirate Tustin" method is composed of the digital controller discretized by Tustin transformation on T_s/N and the interpolator which has an up-sampler and a zero-order-hold [12]. Fig. 8. shows the sensitivity and complementary sensitivity functions $S[z]; T[z]$ of the closed loop systems. As shown in Fig. 8.(b), the proposed method can remain the ideal characteristics of the original continuous-time controller (Fig. 8.(a)), because the

proposed method is based on the closed-loop system. On the other hand, in the conventional Tustin transformations (Fig. 8.(c) and (d)), the closed-systems are quite different from the original analog system, because those controllers are discretized based only on the open-loop characteristics.

Repetitive control based on multirate input control

In this section, the proposed multirate repetitive controllers are applied to the following mode. The block diagram of the following mode is shown in Fig. 9. The disturbance $d_y(t)$ represents the vibration of the track generated by the disk rotation, which is called track runout. The objective of this mode is to make the position error $pe(t)$ zero. $n(t)$ and $d_u(t)$ represent the measurement noise and acceleration disturbance, respectively.

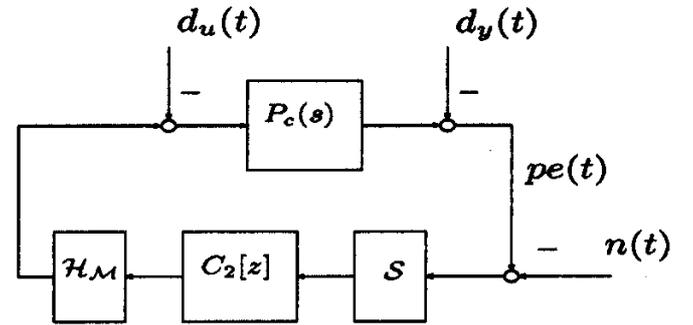


Fig. 9. Following mode

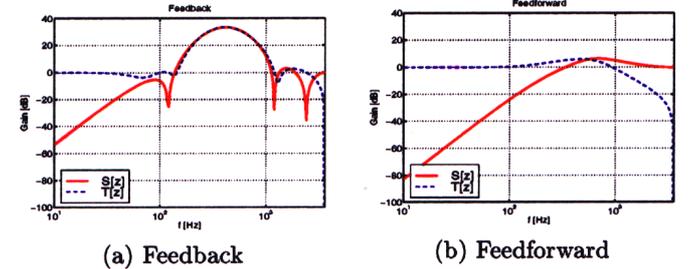


Fig. 10. Frequency responses $S[z]; T[z]$

In the following mode, two kinds of disturbance should be considered; repeatable and non-repeatable runout. Repeatable runout (RRO) is synchronous with the disk rotation, and non-repeatable runout (NRRO) is not synchronous. In this paper, the RRO is perfectly rejected by the proposed repetitive controllers at M inter-sample points.

For simplification, the plant is modeled by

$$P_c = \frac{K_f K_a}{M_p} \frac{1}{s^2} \quad (38)$$

and RRO are considered at 1st, 10th, and 20th order¹.

$$d(t) = \sum_{k=1,10,20} a_k \cos k\omega_0 t + b_k \sin k\omega_0 t \quad (39)$$

¹ In practice, these modes should be selected by the experimental analysis.

where $\omega_0 = 2\pi 120[\text{rad/sec}]$.

Fig. 10. shows the closed-loop characteristics both of the feedback (Fig. 3.) and feedforward (Fig. 4.) repetitive control systems. Fig. 10.(a) indicates the disadvantages of the feedback repetitive controller, where the closed-loop characteristics become worse and difficult to assure stability robustness. On the other hand, in the proposed feedforward repetitive control (Fig. 4.), the closed-loop characteristics depends only on $C_2[z]$ which do not need to have the internal model of (39). Therefore, the feed back characteristics are better than the feedback approach as shown in Fig. 10.(b).

Fig. 11. shows the simulated results of the proposed repetitive feedforward control under the 20th order sinusoidal runout. The switch turns on at just $t_0 = 10[\text{ms}]$. As shown in Fig. 11.(a), the position error converges quickly after the switching action. Moreover, it is shown that the proposed initial value compensation can prevent the large overshoot. Fig. 11.(b) shows that the inter-sample response of the conventional single-rate controller has large error in the steady state. On the other hand, the errors of the plant position and velocity become zero at every $T_y/2$ by the proposed controllers². Moreover, the inter-sample position error of the proposed multirate method is much smaller than that of the single-rate controller.

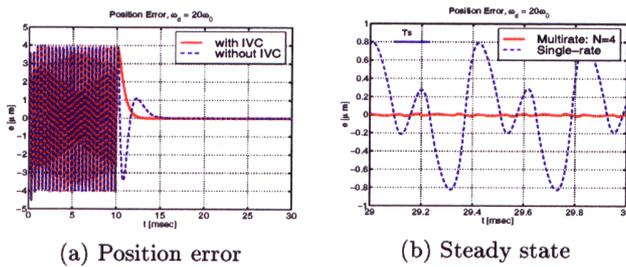


Fig. 11. Feedforward repetitive control. $d_y(t) = T_p \sin k\omega_0 t$; $T_p = 3.6\mu\text{m}$; $k = 20$:

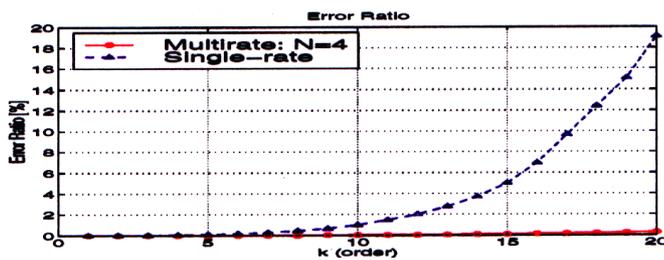


Fig. 12. Error ratio $E_R(k)$. (20th order corresponds to 2.4 [kHz].)

Fig. 12. shows analyzed results of the error ratio $E_R(k)$ for the disturbance order k . Considering the intersample response, the error ratio is calculated by

$$E_R^2(k) \cong \frac{\int_{t_s}^{t_s+kT_0} pe^2(t) dt}{\int_{t_s}^{t_s+kT_0} d_y^2(t) dt} \quad (40)$$

where $d_y(t) = T_p \sin k\omega_0 t$; $T_0 = 2\pi/\omega_0$, and t_s is selected as 20[s] in order to evaluate the steady state. In the high frequency region close to the Nyquist frequency (3.6[kHz]), the disturbance rejection performance is much improved by the proposed multirate control, compared with the single-rate controller. Therefore, it is found that the proposed method can demonstrate much effective performance for high-order disturbance.

5. CONCLUSION

In this paper, the digital control systems which have hardware restrictions of $T_u < T_y$ are assumed, novel multirate feedback controllers are proposed, which achieve vibration suppression and disturbance rejection in the semi-Nyquist frequency region. Moreover, the novel initial value compensation for observer is proposed in order to prevent the overshoot. Furthermore, the proposed methods are applied to the settling and following modes of the hard disk drive. The advantages of these approaches are demonstrated by the simulations. The proposed methods can be extended to the plants with time delay [5].

The future works are experimental evaluations of the proposed methods.

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6. REFERENCES

- [1] Chiang, W. W.: Multirate state-space digital controller for sector servo systems. Conf. Decision Contr.. 1990, pp. 1902-1907.
- [2] Weaver, P. A. - Ehrlich, R. M.: The use of multirate notch filters in embedded-servo disk drives. Amer. Control Conf.. June 1995, pp. 4156-4160.
- [3] Takakura, S.: Design of the tracking system using N-Delay two-degree-of-freedom control and its application to hard disk drives. IEEE Conf. Control Applications. August 1999, pp.170-175.
- [4] Fujimoto, H. - Hori, Y. - Yamaguchi, T. - Nakagawa, S.: Proposal of perfect tracking and perfect disturbance rejection control by multirate sampling and applications to hard disk drive control. Conf. Decision Contr.. December 1999, pp. 5277-5282.
- [5] Fujimoto, H. - Hori, Y. - Yamaguchi, T. - Nakagawa, S.: Proposal of seeking control of hard disk drives based on perfect tracking control using multirate feedforward control. IEEE Int. Workshop Advanced Motion Control. March 2000, pp. 74-79.
- [6] Fujimoto, H. - Kawamura, A. - Tomizuka, M.: Generalized digital redesign method for linear

² In the proposed method, the perfect disturbance rejection is assured $M(= N/n_p = 4/2 = 2)$ times during T_y .

feedback system base on N-delay control. IEEE/ASME Trans. Mechatronics. 1999, vol. 4, no. 2, pp. 101-109.

- [7] Hara, S. - Yamamoto, Y. - Omata, T. - Nakano, M.: Repetitive control system - a new-type servo system. IEEE Trans. Automat. Contr.. 1988, vol. 33, pp. 659-668.
- [8] Smith, C. - Takeuchi, K. - Tomizuka, M.: Cost effective repetitive controllers for data storage devices. 14th IFAC World Congress. July 1999, vol. B, pp. 407-412.
- [9] Araki, M.: Recent developments in digital control theory. 12th IFAC World Congress. July 1993, vol. 9, pp. 251-260.
- [10] Araki, M. - Hagiwara, T.: Pole assignment by multirate-data output feedback. Int. J. Control. 1986, vol. 44, no. 6, pp. 1661-1673.
- [11] Yamaguchi, T. - Numasato, H. - Hirai, H.: A mode-switching control for motion control and its application to disk drives: Design of optimal mode-switching conditions. IEEE/ASME Trans. Mechatronics. 1998, vol. 3, no. 3, pp. 202-209.
- [12] Gu, Y. - Tomizuka, M. - Tornero, J.: Digital redesign of continuous time controller by multirate sampling and high order holds. Conf. Decision Contr.. Dec. 1999, pp. 3422-3427.

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