Nonlinear Body Slip Angle Observer for Electric Vehicle Stability Control

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Abstract

This paper proposes a nonlinear observer for Body Slip Angle ($\beta$) estimation, in which a nonlinear tire model is adopted for the observer design. A newly developed method to identify parameters of road surface friction coefficient ($\mu$) is introduced into this observer, which makes the observer adaptive to road condition changing. Simulations and field test are conducted, where the feasibility of $\mu$ identification and effectiveness of the observer are checked, especially for nonlinear cornering situations.

Keywords: electric vehicle, stability control, body slip angle estimation, nonlinear observer, friction coefficient identification.

1. Introduction

An important advantage of electric vehicles (EVs) has been recognized is that motor’s controllability can provide more flexible and novel ideas for vehicle stability control [1] [2] [3]. Body slip angle ($\beta$) is an important value for such control strategies. However, as sensors to measure $\beta$ value are very expensive, it needs to estimate $\beta$ from only variables measurable. The most difficult for $\beta$ estimation is the non-linear characteristics of vehicle. The main nonlinearity comes from the tire force saturation decided by tire vertical load and road friction. Non-linear force characteristic of tire and the uncertainty of road condition make vehicle characteristics change greatly as vehicle cornering in nonlinear area compared with that in linear area. So the effective observers must consider tire nonlinearity and must be adaptive to road friction changing [4] [5]. This gives the great challenge to the design of $\beta$ observer.

To solve the problems, this paper proposes a nonlinear observer for $\beta$ estimation in which a nonlinear tire model is adopted. This nonlinear tire model has higher accuracy over linear tire model to describe the tire...
characteristics as tire slip angle becomes large in vehicle’s nonlinear cornering conditions. In addition, by making use of electric vehicle’s another important merit that EV’s motor torque can be estimated accurately, method to identify road surface friction coefficient is introduced to the observer. The observer structure is shown as Figure 1. Simulations and field tests are conducted to check the observer in different cornering conditions. Analysis of simulation results and field tests data demonstrate the performance of the proposed nonlinear observer.

![Figure 1: Structure of proposed nonlinear \( \beta \) angle observer](image)

2. **Nonlinear \( \beta \) Observer Design**

2.1 **\( \beta \) Observer Structure and Description**

The nonlinear observer structure is shown as Figure 2. The estimate of \( \beta \) is computed as predicted value from states equation corrected by output feedback.

![Figure 2: Proposed \( \beta \) observer with nonlinear tire model](image)

The observer’s state equation is:

\[
\dot{x} = f(x, \dot{x}) - K(\hat{y} - y) \tag{1}
\]

In the observer, \( f(x, \dot{x}) \) describes the state equation with nonlinear tire model. The state variables, input variables and output variables are:

\[
x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ N \end{bmatrix}, \quad y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}
\]
where $\delta_i$ denotes steering angle of front wheel, $a_y$ denotes vehicle lateral acceleration, $N$ is direct yaw moment caused by differential longitudinal forces among tires, $K$ is the feedback matrix of observer.

The observer’s output equation is:

$$
\begin{align*}
\dot{y} &= \hat{y} \\
\dot{a}_y &= v(\mathbf{\beta} + \hat{\gamma}) \\
\end{align*}
$$

(2)

where $v$ denotes the velocity of vehicle.

The following tire model is adopted in the observer to describe the nonlinear characteristics of tire lateral force [6]:

$$
F_{yi} = k_{xi} \frac{2}{\pi} \mu F_{zi} \tan^{-1}\left(\frac{\pi}{2\mu F_{zi}} C_i \alpha_i\right)
$$

(3)

Where $F_{yi}$ denotes tire lateral force, $\alpha_i$ side slip angle of tires, $C_i$ tire cornering stiffness, $F_{zi}$ wheel vertical load, $\mu$ road friction coefficient, $k_{xi}$ is the influence coefficient of tire longitudinal force, $i$ index of tires.

Compared with the linear tire model adopted in the previous $\beta$ observers studies[7], with the nonlinear function of $\tan^{-1}$ and the additional parameter $\mu$, this nonlinear model can describe the saturation characteristics of tire lateral force as tire slip angle gets large.

The dynamics of vehicle is described as:

$$
\begin{align*}
ma_y &= F_{sf} \sin\delta_f + F_{yr} \cos\delta_f + F_{gr} \\
I_z \dot{\gamma} &= -l_f F_{sf} \sin\delta_f + l_f F_{yr} \cos\delta_f - l_r F_{yr} + N \\
\end{align*}
$$

(4)

where $m$ is mass of vehicle, $I_z$ is yaw inertia moment, $l_f$ is distance between mass center and front axle, $l_r$ is distance between mass center and rear axle, $F_{sf}$ is longitudinal forces of front tires, $F_{yr}$ and $F_{gr}$ are lateral forces of front and rear tires which can be calculated according to above nonlinear tire model.

Considering the kinematics relationship as equation 2 and that $\delta_f$ value is relatively small in the vehicle’s high speed situations, the observer’s nonlinear states equations are derived as:

$$
\begin{align*}
\dot{\mathbf{\beta}} &= \frac{l}{mv}(F_{sf} + F_{yr}) - \hat{\gamma} \\
\dot{\gamma} &= \frac{l}{I_z}(l_f F_{sf} - l_r F_{yr} + N) \\
\end{align*}
$$

(5)

### 2.2 Observer Feedback Matrix Design

Till now, an observer model with nonlinear form is derived. However, the nonlinear observer’s design and application are relatively much difficult [7]. So this paper tries applying linear observer design method to solve the nonlinear problem. To do this, the observer model is changed into the form of an equivalent linear two freedom model by adopting the value of extended tire cornering power $c'_p$, which is defined as:
\[ c'_p = \frac{F_y}{\alpha} \]  \hspace{1cm} (6)

Where \( F_y \) is the tire lateral force and \( \alpha \) is the tire slip angle at its operating point. \( c'_p \) is updated according to the calculated \( \alpha \) value and \( F_y \) value from tire lateral force model as equation (3).

The design method of linear \( \beta \) observer refers to paper [8]. By adopting tire cornering power \( c'_p \), the nonlinear observer state equation (5) can be described as an equivalent linear state equation (7) at each operating point:

\[
\dot{x} = Ax + Bu
\]  \hspace{1cm} (7)

In which,

\[
A = \begin{bmatrix}
-\left( C'_\beta + C'_r + C'_w + C'_n \right) & -l_f(C'_r + C'_n) + l_r(C'_n + C'_r) \\
-\frac{mv}{I_z}l_f(C'_r + C'_n) + l_r(C'_n + C'_r) & -\frac{mv^2}{I_z}l_f^2(C'_r + C'_n) - l_r^2(C'_n + C'_r)
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{C'_\beta + C'_r}{l_f(C'_r + C'_n)} & 0 \\
\frac{mv}{l_f(C'_r + C'_n)} & 1
\end{bmatrix}
\]

\[
x = \begin{bmatrix}
\beta \\
\gamma
\end{bmatrix}, \quad u = \begin{bmatrix}
\delta_f \\
N
\end{bmatrix}
\]

Where, \( c'_r \sim C'_w \) are the extended cornering power values of tires.

The above equations have the same structures as the linear observer of reference paper [8]. So the same design method of gain matrix \( K \) can also be adopted. According to paper [8], \( K \) is selected as following, for high response and robustness purposes.

\[
K = \frac{\left[ l_f(C'_r + C'_\beta) - l_r(C'_r + C'_\beta) \right] \lambda_1 \lambda_2 I_z}{(C'_\beta + C'_r)(C'_r + C'_n)(l_f + l_r)^2} - \frac{l}{\nu} \frac{1}{I_z}
\]

\[
-\lambda_1 - \lambda_2 \frac{mv(C'_r + C'_\beta)i_f^2 + (C'_r + C'_n)i_r^2}{[(C'_r + C'_\beta)i_f - (C'_r + C'_n)i_r]I_z}
\]

Where, \( \lambda_1 \) and \( \lambda_2 \) are the assigned pole values of the observer.

### 2.3 Road Surface Friction Coefficient Identification

Evidently, the accuracy of the proposed observer is determined by the tire model. According to equation (3), there are 2 basic parameters in the tire model: tire cornering stiffness \( C \) and road friction coefficient \( \mu \). Cornering stiffness \( C \) is the characteristic of tire and is consistent generally. Friction coefficient \( \mu \) is uncertain and may change largely with the change of road conditions.

There have been many methods for \( \mu \) identification proposed in previous studies [4] [5] [9]. Paper [9] put forward an effective way which comes from one of excellent features of EVs, i.e., motor torque can be known easily and precisely from motor current. Therefore, the motor’s driving-force can be estimated easily. And then, the road friction coefficient identification based on driving force and wheel slip ratio can be realized. This paper introduces this \( \mu \) identification method into the \( \beta \) observer to adapt the road condition changes (the principal structure is shown as Figure.1). To check the feasibility of the \( \mu \)
identification, off-line calculation is conducted using the data measured by the experimental EV as the vehicle acceleration running in dry asphalt. The results confirm the validity of the introduced method.

![Graph 1: Tire slip ratio vs Time](image1)

![Graph 2: Observed driving force vs Time](image2)

![Graph 3: Identified friction coefficient vs Time](image3)

Figure 3: Results of \( \mu \) identification

3. Simulation Studies

To test the effectiveness of the nonlinear observer, Simulations are conducted with some typical steering angle input modes. The results of the comparison with previous linear observer are shown in Figures 4 and 5. When the lateral acceleration is small, which means tires lateral forces are in their linear working region, \( \beta \) estimate values of both the linear and nonlinear observers have relatively high accuracy. When the lateral acceleration getting larger, which means tires lateral forces are in their nonlinear region, \( \beta \) estimate values of linear observers deviate from the real values. Comparatively, the nonlinear observer still has enough accuracy in such situations.

Figures 6 and 7 show the simulation results when different cornering stiffness and road surface friction coefficient values are applied in \( \beta \) observer. The estimate \( \beta \) value in nonlinear region is insensitive with the deviation of cornering stiffness \( c \), for the tire lateral force tends to be its saturation value when tire slip angle gets large, which is independent with \( c \) value. The robustness over tire cornering stiffness error is one of the advantages of nonlinear observer over the linear ones. The results also show the estimate \( \beta \)
values are more sensitive to the road surface friction coefficient errors. This is for the reason that the tire lateral force saturation values are determined by road friction conditions. As tire forces approach the road friction limitation, the calculated tire forces with incorrect $\mu$ value may be quite different from the real ones. These simulations confirm the importance of $\mu$ identification to ensure the observer’s accuracy.

![Simulation results of $\beta$ observer](image)

**Figure 4:** Lane change maneuver simulation results of $\beta$ observer

![Simulation results of $\beta$ observer](image)

**Figure 5:** Simulation results of $\beta$ observer (Vehicle accelerates with constant $\delta_f = 3^\circ$)

![Simulation results with different stiffness values](image)

**Figure 6:** Simulation results with different stiffness values (Vehicle accelerates with constant $\delta_f = 3^\circ$)

![Simulation results as observer friction coefficient ($\mu_{ob}$) is different from real one ($\mu_v$)](image)

**Figure 7:** Simulation results as observer friction coefficient ($\mu_{ob}$) is different from real one ($\mu_v$)
4. **Experiments of $\beta$ Observer**

To check the proposed $\beta$ observer, field tests are conducted in our experimental EV, UOT March II. UOT March II is equipped with acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure real vehicle state values. Results of field tests shown in Figure 8 and Figure 9 demonstrate the effectiveness of the nonlinear observer in both the linear and nonlinear cornering situations of the experimental vehicle.

![Figure 8: Field test results of $\beta$ observer](image)

**Linear area (driver steering angle=90°, v=40 km/h)**

![Figure 9: Field test results of $\beta$ observer](image)

**Nonlinear area (driver steering angle=90°, v=60 km/h)**

5. **Concluding Remarks**

In this paper, by adopting nonlinear tire model, a nonlinear $\beta$ observer is proposed, which is checked to be effective by simulations and field tests, especially in nonlinear cornering region. This is because that the nonlinear tire model can describe the saturation characteristics of tire lateral force. Another advantage of the proposed observer is its robustness over tire cornering stiffness parameter errors. To make the observer adaptive to road friction condition changing, method to identify parameters of road surface
friction coefficient ($\mu$ value) is introduced into this observer. The test results demonstrate the validity of $\mu$ identification method. The planning future works are the adequate experimental studies on the effectiveness of the observer in different road conditions.

6. References


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