Robust Yaw Stability Control for Electric Vehicles Based on Active Steering Control

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***Abstract* — In this paper, robust yaw stability control based on active steering control is proposed for electric vehicles (EVs). A two degree of freedom control method by a disturbance observer is applied to control system for yaw stabilization. Moreover, the feed-forward disturbance compensator is designed to compensate unexpected yaw moment caused by torque differences between left and right driving motors. Since the vehicle control system has a large model variation due to road conditions, the disturbance observer is designed based on a robust control method. The proposed control system is verified by computer simulations using CarSim.**

***Keywords* — Yaw stability control, Disturbance observer, Feed-forward disturbance compensator, Robust control**

I. INTRODUCTION

Due to the increasing concerns in environmental-friendly vehicles and electrification of vehicle systems, researches on electric vehicles have been carried out [1],[2]. Especially, in the motion control field of electric vehicles, the longitudinal motion control methods including an anti-slip control [3], a model following control (MFC) based slip control [4] and slip ratio control based on slip estimation [5] were proposed and applied in actual electric vehicles. These novel slip control methods are based on the advantages of electric vehicles equipped with in-wheel motors. Moreover, in order to improve yaw stability of electric vehicles, the various direct yaw moment control methods utilizing independent torque control were proposed by Hiroshi Fujimoto *et al.*[6],[7]. The advantages of electric vehicles in terms of motion control were summarized as follows [1]:

1) Quick torque generation
2) Easy torque measurement
3) Independent wheel torque control

In this paper, a robust yaw stability control for electric vehicles equipped with active steering system (e.g., a steer by wire system) is proposed. The purpose of this paper is to present a control strategy of active steering control system to improve yaw stability. This paper focuses mainly on the yaw disturbance rejection using feed-forward compensator and a disturbance observer (DOB) [8],[9]. Since the vehicle yaw model is a time varying model dependent on vehicle velocity and road friction, the nominal vehicle yaw model is updated based on measured vehicle velocity [10]. In order to consider model variations in control system, a robust control method is applied to design a Q-Filter for guaranteeing robust performance and stability.

II. VEHICLE DYNAMICS FOR CONTROL DESIGN

The vehicle model used in this study is yaw plane representation with yaw moment as shown in Fig. 1.

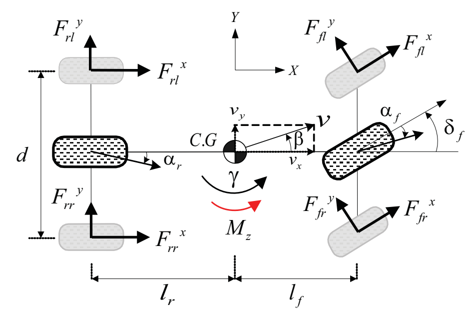


Fig. 1. Planar vehicle model

The governing equations for longitudinal and lateral motion are given by

$$m(\dot{v}_x - \gamma v_y) = F_r^{\,x} + F_f^{\,x}\cos\delta_f - F_f^{\,y}\sin\delta_f \tag{1}$$

$$m(\dot{v}_y + \gamma v_x) = F_r^{\,y} + F_f^{\,x}\sin\delta_f + F_f^{\,y}\cos\delta_f \tag{2}$$

The equation of yaw motion is

$$I_z\dot{\gamma} = l_f F_f^{\,y} - l_r F_r^{\,y} + M_z \tag{3}$$

For small tire slip angle, the lateral tire forces can be linearized as follows:

$$F_f^{\,y} = -2C_f(\beta + \frac{\gamma l_f}{v_x} - \delta_f)\;,\;\; F_r^{\,y} = -2C_r(\beta - \frac{\gamma l_r}{v_x}) \tag{4}$$

where $\beta\ (\approx v_y / v_x)$ is the vehicle side slip angle, γ is the yaw rate, $F_f^{\,y}, F_r^{\,y}$ are the cornering forces of front and rear tires, l_f (=0.73m) is the distance from front axle to center of gravity(C.G), l_r (=0.57m) is the distance from rear axle to C.G, I_z (=150kgm²) is the yaw moment of inertia, m (=360kg) is the vehicle mass, M_z is the yaw moment. C_f, C_r are the cornering stiffness of tires.

Assuming that vehicle has a constant velocity, the state space equations are represented as follows:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \delta_f \\ M_z \end{bmatrix} \tag{5}$$

where

$$A = \begin{bmatrix} -\frac{2(C_f + C_r)}{m v_x} & -\frac{2(l_f C_f - l_r C_r)}{m v_x^2} \\ -\frac{2(l_f C_f - l_r C_r)}{I_z} & -\frac{2(l_f^2 C_f + l_r^2 C_r)}{I_z v_x} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2C_f}{m v_x} & 0 \\ \frac{2 l_f C_f}{I_z} & \frac{1}{I_z} \end{bmatrix}$$

From dynamic equations (5), the transfer functions from steering angle and yaw moment to yaw rate are given by

$$\begin{aligned} \gamma &= \frac{G_{\delta_f}^{\gamma}(0)(1+T_\gamma s)}{1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2}\delta_f + \frac{G_{M_z}^{\gamma}(0)(1+T_M s)}{1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2}M_z \\ &= P(s)\delta_f + H(s)M_z \end{aligned} \tag{6}$$

where $G_{\delta_f}^{\gamma}(0)$ and $G_{M_z}^{\gamma}(0)$ are the steady state DC gains of the chosen vehicle velocity. T_γ and T_{M_z} are time constants. K is the vehicle stability factor.

$$G_{\delta_F}^{\gamma}(0) = \frac{v_x}{(l_f + l_r)(1 + K v_x^2)}, \quad G_{M_z}^{\gamma}(0) = \frac{v_x (C_f + C_r)}{2(l_f + l_r)^2 C_f C_r (1 + K v_x^2)}$$

$$T_\gamma = \frac{m l_f v_x}{2 C_r (l_f + l_r)}, \quad T_{M_z} = \frac{m v_x}{2(C_f + C_r)}, \quad K = -\frac{m(l_f C_f - l_r C_r)}{2 C_f C_r (l_f + l_r)^2}$$

ζ and ω_n are damping coefficient and natural frequency of electric vehicle control system, respectively.

$$\begin{aligned} \zeta &= \frac{m(l_f^2 C_f + l_r^2 C_r) + I_z (C_f + C_r)}{2(l_f + l_r)\sqrt{m I_z C_f C_r (1 + K v_x^2)}} \\ \omega_n &= \frac{2(l_f + l_r)\sqrt{C_f C_r (1 + K v_x)}}{v_x \sqrt{m I_z}} \end{aligned} \tag{7}$$

III. Active Front Steering Control Based On A Disturbance Observer

In order to satisfy the control objectives (i.e., the reference yaw rate model following control robust to disturbances), the two degree-of-freedom control [5] method based on a disturbance observer is applied. Moreover, a feed-forward disturbance compensator is designed to reject yaw moment disturbance caused by torque difference between in-wheel driving motors. The vehicle yaw dynamics model is time varying due to vehicle velocity and large variation in cornering stiffness, which depends on the road friction. Considering that the vehicle velocity can be measurable based on driven wheel velocity, the vehicle velocity can be used for updating a vehicle model. In order to consider variation in parameters (i.e., cornering stiffness), a multiplicative model uncertainty is introduced to the nominal vehicle yaw model. The control structure of proposed control system is depicted in Fig. 2. As shown in fig. 2, the control system consists of a feed-forward compensator and a disturbance observer for output yaw disturbance rejection.

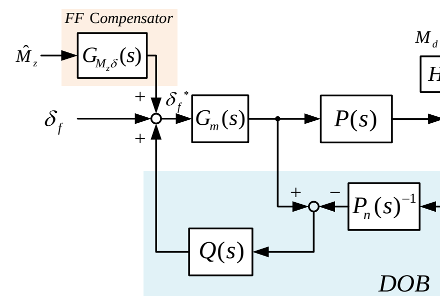


Fig. 2. Block diagram of proposed yaw stability control system

A. Feed-forward Disturbance Compensation

A feed-forward disturbance compensator is designed to achieve yaw stability when anti-slip control [3] is working on split-μ road, where in-wheel motors are independently controlled to avoid wheel slip. This independent motor torque control induces the yaw moment, which can be effective control input to yaw stability control systems due to relatively fast wheel dynamics. However, if an anti-slip controller and yaw stability controller are activated at the same time, (i.e., when cornering with acceleration on split-μ or straight acceleration on split-μ), the yaw moment control based on active steering control can be conditionally effective without deterioration of acceleration performances. In this paper, the steering control system is only used for disturbance rejection. The yaw moment disturbance by motor torque differences is estimated based on a familiar driving force observer (DFO) [11], as shown in Fig. 3(B). The estimated driving force obtained the from wheel dynamic equation (see Fig. 3) is given as

$$\hat{F}_d = \frac{T_m - I_\omega \dot{\omega}}{r} \tag{8}$$

where T_m is the motor torque, I_ω (=0.5kgm²) is the wheel inertia, ω is the wheel angular velocity, r (=0.22m) is the wheel rolling radius.

The yaw moment is calculated as follows:

$$\hat{M}_z = \frac{d}{2}\left(\hat{F}_{d,1} - \hat{F}_{d,2}\right) \tag{9}$$

where d (=0.9m) is a track width, $\hat{F}_{d1}$, $\hat{F}_{d2}$ are estimated left and right driving forces. The feed-forward compensator $G_{M_z}^{\delta}(s)$ is composed of inverse models of vehicle yaw dynamics and a steering motor. From (6), $G_{M_z}^{\delta}(s)$ is

$$\begin{aligned} G_{M_z}^{\delta}(s) &= P^{-1}(s)H(s)G_m^{-1}(s) \\ &= \frac{G_{M_z}^{\gamma}(0)(1+T_M s)}{G_{\delta_f}^{\gamma}(0)(1+T_\gamma s)}\cdot(1+\tau_m s) \end{aligned} \tag{10}$$

where $G_m(s)$ is the steering motor dynamics, which is simplified to first order low pass filter with a cutoff frequency of 15Hz (i.e., time constant τ_m is 0.011sec).

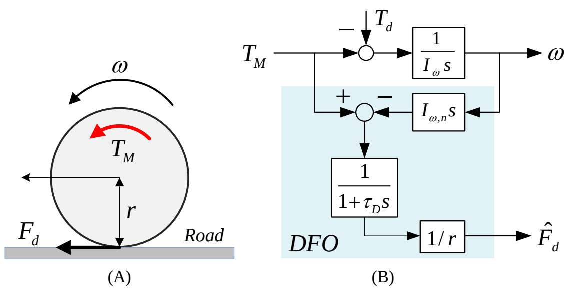


Fig. 3. (A) Wheel dynamics, (B) Driving force observer (DFO)

B. Feedback Disturbance Compensation Based on DOB

The two degree of freedom control algorithm [8],[9] based on DOB is proposed for robustness to external disturbances and model uncertainties. For the sake of design simplicity, the nominal yaw dynamics model is chosen as a first order system as follows [10]:

$$P_n(s) = \frac{G_{\delta_F}^{\gamma}(0)}{1+\tau_p s} \tag{11}$$

where τ_p (=0.08sec) is the time constant of yaw model. Since the vehicle system is subjected to model parameter variations, (i.e., variation in cornering stiffness dependent on road conditions), the vehicle yaw dynamics model can be expressed as a nominal model with a multiplicative model uncertainty, i.e.,

$$P(s) = P_n(s)[1+W(s)\Delta(s)] \tag{12}$$

Where $P_n(s)$ is a nominal vehicle yaw model, $W(s)$ is a proper and stable boundary function of the model uncertainty. $\Delta(s)$ is a random stable transfer function with the bounded magnitude (i.e., $\|\Delta\|_\infty < 1$). $W(s)\Delta(s)$ is easily obtained by (12).

$$W(s)\Delta(s) = \frac{P(s)-P_n(s)}{P_n(s)} \tag{13}$$

where $P(s)$ is an actual vehicle yaw dynamics model, which is obtained from the nominal vehicle yaw model with parametric uncertainty (i.e., cornering stiffness variation range: $C_f = [7000\ 14000]$, $C_f = [9000\ 17000]$). In this paper, nominal cornering stiffness values are $C_f = 12000\ N/rad$, $C_r = 15000\ N/rad$, which are values for high-μ road, respectively. Note that since the magnitude of $\Delta(s)$ is bounded to one, the maximum $|W(s)\Delta(s)|$ is equal to $|W(s)|$ for all frequency ranges.

Generally, a DOB is designed to reject disturbances and compensate for model uncertainties by regarding as equivalent disturbances. In DOB design, it is important to design Q-Filter (i.e., $Q(s)$) so that $Q(s)P_n(s)^{-1}$ must be realizable. The control system including a DOB also must be robust in terms of stability and performance. The robust stability of inner loop formed by the DOB is assured if a following condition is held for all frequencies.

$$|Q(j\omega)| < \frac{1}{|W(j\omega)\Delta(j\omega)|} \tag{14}$$

In this paper, $Q(s)$ is designed as a first order low pass filter

$$Q(s) = \frac{1}{1+\tau s} \tag{15}$$

where τ is a Q-filter design factor which must be chosen to have good control performance up to the frequency bandwidth of vehicle yaw motion. As shown in Fig. 4, the magnitude of Q-filter is less than one of the inverse of model uncertainty boundary function, where τ of the Q-filter is chosen as 0.159sec.

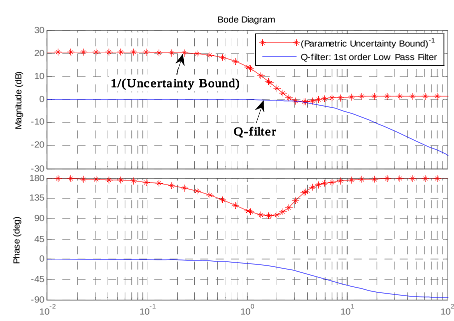
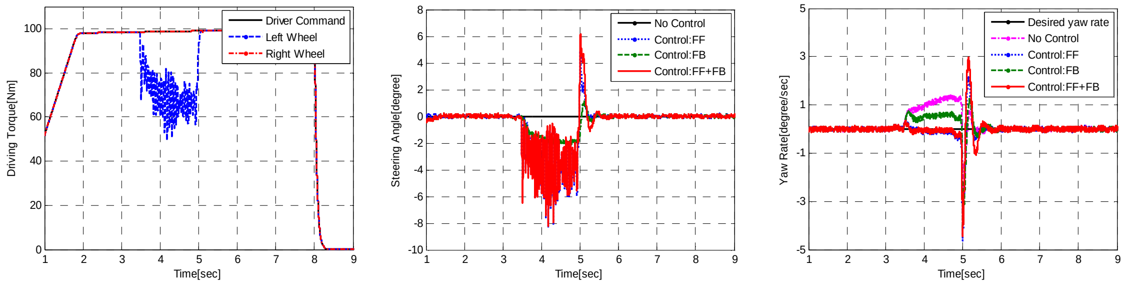


Fig. 4. Bode plot of the designed Q-filter for disturbance rejection (Vx=30 km/h)

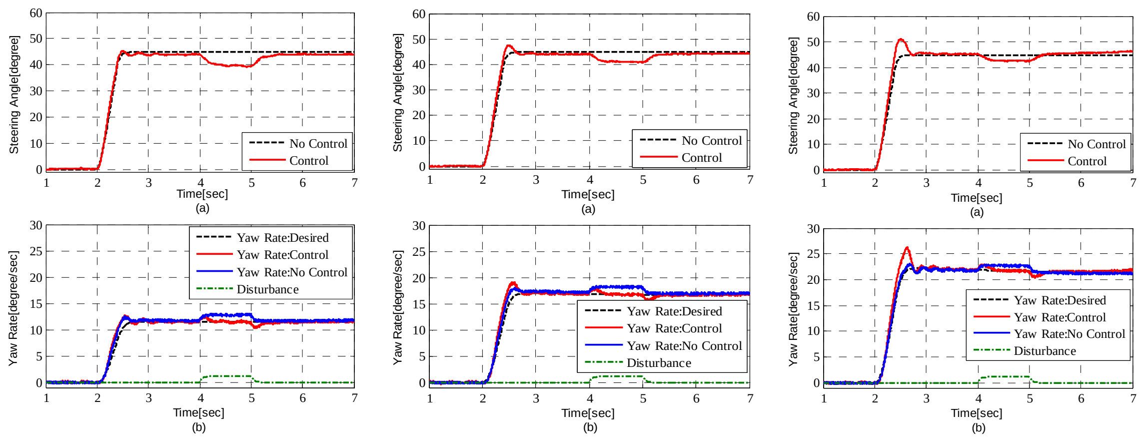
IV. Simulation Results

The proposed yaw stability controller is verified by computer simulations using CarSim. Fig. 5 shows the simulation results for verifying the feed-forward disturbance compensator on split-μ. As shown in Fig. 5(A), motor torque of the left wheel on low-μ surface is controlled to avoid wheel slip, on the other hand, motor torque of right wheel is equal to driver's torque command. This torque difference induces a yaw moment, as shown in Fig. 5(C). Fig. 5 shows that the proposed feed-forward



(A) (B) (C)

Fig. 5. Simulation results for yaw stability control (Driving maneuver: Acceleration with Anti-slip control on split-μ): (A) Driving motor torque, (B) Steering wheel angle, (C) Yaw rate



(A) (B) (C)

Fig. 6. Simulation results for yaw stability control (Driving maneuver: 45 degree step steering): (A) Velocity: 20km/h, (B) Velocity: 30km/h, (C) Velocity: 40km/h

disturbance compensator is effective to quickly reject yaw disturbance caused by motor torque differences. In order to verify the two degree of freedom controller based on a DOB, the electric vehicle simulation with 45 degree step steering is carried out, as shown in Fig. 6. The simulation results show good disturbance rejection at different vehicle velocity (i.e., considering that maximum vehicle velocity of experimental electric vehicles is 50 km/h, vehicle simulations with velocity conditions up to 40km/h are reasonable).

V. Summary and Conclusion

In order to improve yaw stability of electric vehicles, robust yaw stability controller based on a feed-forward disturbance compensator and a DOB is designed. In this paper, the yaw stability control system is realized by only using an active steering system (i.e., conventional yaw stability control is based on independent torque control of in-wheel driving motors). The simulation results show that the proposed yaw stability controller based on a well-designed DOB is expected to improve yaw stability. In future works, the robust yaw motion control based on control system integration of active steering control and independent in-wheel motor control will be introduced.

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