Stiffness Direction Stabilization and Inertia Matrix Diagonalization of Robot Manipulator by Biarticular Muscle

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Abstract—This paper suggests two advantages of the biarticular muscle which has been said to play an important role in human’s motion control; one is stiffness direction stabilization of an endeffector which results in stabilization of the posture, and the other is inertia matrix diagonalization in the dynamics of a robot manipulator.

In order to analyze these characteristics, we adopt three-pair six-muscle model of human arm including biarticular muscles and derive simple relationship between an endpoint force/position and three muscle torques based on the absolute angle of two joints. Based on the derived equation, the relationship between the stiffness in the workspace and in the muscle torque space is calculated and the direction of stiffness that can be enhanced by the biarticular muscle is revealed.

Finally, we suggest a new dynamics of the planar robot manipulator with three muscle torque input where the inertia matrix is diagonalized.

I. INTRODUCTION

Development of a robot manipulator that mimics human musculo-skeletal system and analysis of human muscle system have been researched for more than several years. Some researches focus on the measurement of human impedance/stiffness characteristic [1],[2], others focus on the relationship between the stiffness and actual muscle [3].

However, there has been a big distance between these analysis of human muscle system and its application to the control of robot manipulators [4],[5],[6]. We have developed an analysis methodology and control algorithm that connect these two systems [7].

In this paper, we focus on biarticular muscle and propose the advantages of incorporation of biarticular muscle into a robot manipulator; enhancement of the stiffness direction at the endeffector and inertia matrix diagonalization in manipulator dynamics.

In Section II, the characteristic of the biarticular muscle is explained and included in a planar robot manipulator. In Section III, the absolute angle Jacobian matrix is shown to be efficient to define the relationship between position/force at the endpoint and three muscle torques which includes the biarticular muscle. Then, the relationship between the muscle stiffness and endeffector stiffness is revealed. In Section IV, the dynamics for three muscles are derived based on the manipulator dynamics diagonalizing the inertia matrix. Taking consideration of these two analyses and the characteristic of the pair of muscles, two-degree-of-freedom control is designed for a manipulator with biarticular muscle. Simulation result in Section IV-B verifies the effectiveness of the proposal.

II. INTRODUCTION OF BIARTICULAR MUSCLE IN A PLANAR MANIPULATOR

A. Analysis of Muscle Structure in Human Arm

Structure and dynamics of the human arm is analyzed and reflected to a robot manipulator control, in this section. Figure 1 shows the 3 pairs of muscles: flexors and extensors of two monoarticular muscles and one biarticular muscle in an arm. Tension of each flexor and extensor is described as $f$ in this figure. The tensions of flexors and extensors are widely modeled as [8]

$$f^j = u^j - K r^j \theta, f^e = u^e + K u^e r \theta.$$ (1)

The sign of the second term changes with regard to the direction a muscle is attached. This tension is reflected to the joint torques as

$$T^j_1 = r_1(f^j_1 - f^j_2) + r_1 (f^j_2 - f^j_3)$$
$$= r_1(u^j_1 - u^j_2) + r_1 (u^j_2 - u^j_3) - K_1 r_1 (u^j_1 + u^j_3) \theta_1$$
$$- K_3 r_1 (u^j_3 + u^j_2) \theta_1$$
$$T^j_2 = r_2(f^j_2 - f^j_3) + r_2 (f^j_3 - f^j_1)$$
$$= r_2(u^j_2 - u^j_3) + r_2 (u^j_3 - u^j_1) - K_2 r_2 (u^j_2 + u^j_1) \theta_2$$
$$- K_3 r_2 (u^j_3 + u^j_2) \theta_2,$$ (2) (3)
where \( r_i \) is the radius of the joint \( i \). In Equations (2) and (3), the first two terms are the difference mode which generates torques to rotate the joints, while the last two terms are the sum mode that is related to the stiffness around the joints.

The relationship between the joint torque \( \tau \) and the force \( F \) in Figure 1 are defined as the following based on the muscle dynamics:

\[
\tau = K \theta
\]

Figure 2 is the illustration of these roles of two modes in muscle torque; the difference mode working as a torque and the sum mode which adjusts the stiffness.

The virtual trajectory control which uses the equilibrium point to control the joint angle, also can be represented in this figure. \( \theta_{ref} \) is the equilibrium point in the virtual trajectory algorithm to control the angle \( \theta \). With this designed equilibrium angle \( \theta_{ref} \) and the designed stiffness \( K \), the angle will converge to \( \theta_{ref} \); this is the virtual mode algorithm [9].

Even though the actual muscle model has pairs of flexor and extensor muscles working as an agonistic/antagonistic system, we only concentrate on the difference mode to generate torque; the sum mode which works as adjustment of the stiffness can be replaced by the position feedback control. From this consideration, the antagonistic structure of the muscle will be replaced by the motor in the proposed biarticular muscle robot manipulator.

B. Novel Robot Manipulator Model with the Biarticular Muscle

Three muscle torques generated by three pairs of muscles in Figure 1 are defined as the following based on the muscle force equation (1)

\[
\tau_m^1 = \tau_m^2 = \tau_m^3 = \tau_m^1 + \tau_m^2 + \tau_m^3
\]

if \( r_1 = r_2 = r \) is assumed.

These muscle torque equations and Equation (2), (3) leads to the relationship between the joint torque \( T_j^1, T_j^2 \) and the muscle torque \( \tau_m^1, \tau_m^2, \tau_m^3 \) as the following:

\[
\begin{pmatrix}
T_j^1 \\
T_j^2
\end{pmatrix} =
\begin{pmatrix}
\tau_m^1 + \tau_m^3 \\
\tau_m^2 + \tau_m^3
\end{pmatrix}
\]

Now, this analysis allows us to introduce a configuration of a novel manipulator illustrated in Figure 3, where biarticular muscle is modeled as a linear motor producing the force \( F_m \). This biarticular linear force \( F_m \) leads to a torque \( \tau_m^3 \) in two joints at the same time.

\[
\tau_m^1, \tau_m^2, \tau_m^3 \text{ are the torques generated by monoarticular muscles of two joints, and } \tau_m^3 \text{ is the torque generated by a biarticular muscle tension } F_m. \text{ The torques generated by these two monoarticular muscles and one biarticular muscle can be projected into the joint torques as Equation (7).}

Note again that the torque by the biarticular muscle is added to two joints at the same time.

III. Modification of Jacobian Matrix Based on Biarticular Muscle Model

Figure 4 is the configuration of a two degree-of-freedom planar robot manipulator. In order to control the position of end-effector, the relation between small changes in the position of end-effector and joint angles needs to be defined using the Jacobian matrix described in Equation (8).

Using this Jacobian, the balance between the forces applied on the end-effector and the joint torques also can be described based on the virtual work principle. Equation (9) is the relationship between the force \( F_e \) on the end-effector in Figure 4 and the joint torques \( (T_j^1, T_j^2) \); the force \( F_e \) in Figure 4 is described as \( F_e = (f_x, f_y) \).

\[
J =
\begin{pmatrix}
-l_1 \sin \theta_1 - l_2 \sin(\theta_1 + \theta_2) \\
l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2)
\end{pmatrix}
\begin{pmatrix}
\frac{T_j^1}{T_j^2}
\end{pmatrix}
= J^T \begin{pmatrix}
f_x \\
f_y
\end{pmatrix}
\]

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A. Jacobian Matrix based on Biarticular Muscle Model

The joint torques related to the endeffector force \( F_e \) is distributed to \( \tau_{1}^m, \tau_{2}^m, \tau_{3}^m \) in this section. In order to develop the relationship between the force on the endeffector and the muscle torques, the Jacobian needs to be modified. To this end, we use the relationship between \( \tau_{1}^m \) and the absolute angle \( \theta_1 \); the absolute angle \( \theta_{12} = \theta_1 + \theta_2 \) in Figure 3 can be defined as the output of the biarticular muscle. The point that \( \tau_{1}^m \) affects both joints supports this definition, and the dynamics of \( \tau_{1}^m \) derived in the following sections also shows this output definition is right.

The absolute angle Jacobian can be written as the following equation based on the Jacobian in Equation (8).

\[
J_{abs} = \begin{pmatrix} -l_1 \sin \theta_1 & -l_2 \sin \theta_{12} \\ l_1 \cos \theta_1 & l_2 \cos \theta_{12} \end{pmatrix} = J \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \tag{10}
\]

With this absolute angle Jacobian the following relationship is satisfied.

\[
\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} = J_{abs} \begin{pmatrix} \theta_1 \\ \theta_{12} \end{pmatrix} \tag{11}
\]

Equation (9) can be divided into two parts like the following equation.

\[
J^T \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} -l_1 \sin \theta_1 f_x + l_1 \cos \theta_1 f_y \\ 0 \end{pmatrix} + \begin{pmatrix} -l_2 \sin \theta_{12} f_x + l_2 \cos \theta_{12} f_y \\ -l_2 \sin \theta_{12} f_x + l_2 \cos \theta_{12} f_y \end{pmatrix} \tag{12}
\]

Considering this, three muscle torques \( \tau_1^m, \tau_2^m, \tau_3^m \) which cope with the external force \( F_e \) can be defined as follows.

\[
\begin{align*}
\tau_1^m &= -l_1 \sin \theta_1 f_x + l_1 \cos \theta_1 f_y, & \tau_2^m &= 0 & \tag{13} \\
\tau_3^m &= -l_2 \sin \theta_{12} f_x + l_2 \cos \theta_{12} f_y & \end{align*}
\]

The muscle torques \( \tau_1^m, \tau_2^m, \tau_3^m \) cannot be decided uniquely from the joint torques \( T_1^j, T_2^j \). If, however, we remove \( \tau_2^m \) intentionally, the relationship can be simplified and it will provide a new relationship between \( F_e \) and muscle torques as the following equation.

\[
\begin{pmatrix} \tau_1^m \\ \tau_3^m \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} T_1^j \\ T_2^j \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} J^T \begin{pmatrix} f_x \\ f_y \end{pmatrix} = (J_{abs})^T \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{15}
\]

The absolute angle Jacobian matches well with the biarticular muscle manipulator; using \( J_{abs} \), the relationship between \( F_e \) and \( \tau_1^m, \tau_3^m \) can be written in a simple way as Equation (15).

When the magnitude and direction of \( F_e \) are given as \( F_e = (F \cos \theta_f, F \sin \theta_f) \), the relationship with the muscle torque can be more simplified using \( J_{abs} \).

\[
\begin{pmatrix} \tau_1^m \\ \tau_3^m \end{pmatrix} = (J_{abs})^T \begin{pmatrix} F \cos \theta_f \\ F \sin \theta_f \end{pmatrix} = \begin{pmatrix} F_l \sin (\theta_f - \theta_1) \\ F_l \sin (\theta_f - \theta_{12}) \end{pmatrix} \tag{16}
\]

Equation (16) is the proposed new kinematic equation which relates \( \tau_1^m, \tau_3^m \) to the characteristics of the external forces: \( F \) and \( \theta_f \). With the biarticular muscle torque coordinate, the endpoint force can be designed in a more simple way; two muscle torques are just two functions of \( \theta_f, \theta_1, \theta_{12} \) and \( F \).

B. Endeffector Force Direction Described by Link Direction

The derived equation (16) reveals to which direction the endpoint force that is generated by each muscle, is directed; the dominant force direction generated by three muscles. Three directions \( A, B, C \) described in Figure 5 are the three major directions. In the following all the angles \( \theta_1, \theta_{12}, \theta_{in} \) are assumed to be positive. Although the sign of magnitude of the force may become opposite with the negative \( \theta_{12} \), the direction relationship derived here hold regardless of the sign of the angles.

\[ \begin{pmatrix} \tau_1^m \\ \tau_3^m \end{pmatrix} = \begin{pmatrix} 0 \\ -F_l \sin \theta_2 \end{pmatrix}. \tag{17} \]

When the endpoint force is directed in the direction of \( C \) where \( \theta_f = \theta_{12} \), the muscle force will be

\[ \begin{pmatrix} \tau_1^m \\ \tau_3^m \end{pmatrix} = \begin{pmatrix} F_l \sin \theta_1 \\ 0 \end{pmatrix}. \tag{18} \]

With the direction \( B \) where the direction is in parallel with the line from the first joint to the endpoint and \( \theta_f = \theta_1 + \theta_{in} \), the muscle force will be

\[ \begin{pmatrix} \tau_1^m \\ \tau_3^m \end{pmatrix} = \begin{pmatrix} -F_l \sin \theta_{in} \\ -F_l \sin (\theta_2 - \theta_{in}) \end{pmatrix}. \tag{19} \]

Using the sine law, we can derive \( \tau_3^m = -\tau_3^m \), which means that for this direction \( B \) the torque on the second joint only plays the dominant role. We can produce this \( B \) direction force using only \( \tau_2^m \) with \( \tau_1^m = \tau_3^m = 0 \).

\[ \text{Fig. 5. Three Major Directions of Endpoint Force} \]

The derived force equation (16) revealed that \( A, B, C \) are the major three directions that are generated by three muscle torques \( \tau_3^m, \tau_2^m, \tau_1^m \) respectively.

C. Design of Stiffness Ellipse at the Endeffector based on the Proposed Kinematics

Based on the proposed new kinematics, the stiffness at the endeffector can be easily designed and the contribution of the biarticular muscle to posture stabilization can be explained.
Firstly, the stiffness ellipse at the endpoint is described in Equation (20).

\[
\begin{pmatrix}
    f_x^e \\
    f_y^e
\end{pmatrix} = \begin{pmatrix}
    k_1 \cos \theta_e & -k_1 \sin \theta_e \\
    k_2 \sin \theta_e & k_2 \cos \theta_e
\end{pmatrix} \begin{pmatrix}
    \Delta x \\
    \Delta y
\end{pmatrix}
\]  

When the force \( F^e = (f_x^e, f_y^e) \) is applied to the endpoint, the position of the endpoint will change as much as \((\Delta x, \Delta y)\) with this stiffness ellipse design. The tuning parameters in this stiffness ellipse design are \( k_1, k_2 \), the major axis and the minor axis, and \( \theta_e \), the tilt angle of the ellipse.

In order to apply the proposed kinematics, the torque \( \tau_{1m}^{fb} \) is set to 0 simplifying the relationship between the stiffness ellipse matrix in Equation (20) and the gain matrix in Equation (21).

\[
\begin{pmatrix}
    \tau_{1m}^{fb} \\
    \tau_{3m}^{fb}
\end{pmatrix} = \begin{pmatrix}
    K_{1bia}^{bia} & K_{2bia}^{bia} \\
    K_{3bia}^{bia} & K_{4bia}^{bia}
\end{pmatrix} \begin{pmatrix}
    \Delta \theta_1 \\
    \Delta \theta_{12}
\end{pmatrix}
\]  

In order to derive this gain, \( F^e = (f_x^e, f_y^e) \) and \((\Delta x, \Delta y)\) are converted to \( \tau_1^{1m}, \tau_2^{1m}, \tau_3^{1m} \) and \( \Delta \theta_1, \Delta \theta_{12} \) using the absolute angle Jacobian \( J_{abs} \). Putting the stiffness ellipse in the workspace in Equation (20) as a matrix \( K_{ws} \) and using the relationship of Equation (11) and (15), the relationship in Equation (21) is represented as follows.

\[
\begin{pmatrix}
    \tau_{1m}^{fb} \\
    \tau_{3m}^{fb}
\end{pmatrix} = J_{abs}^T K_{ws} J_{abs} \begin{pmatrix}
    \Delta \theta_1 \\
    \Delta \theta_{12}
\end{pmatrix}
\]  

Here we assumed the deviation in the position \((\Delta x, \Delta y)^T\) can be approximated as \( J_{abs}(\Delta \theta_1, \Delta \theta_{12})^T \), which means the amount of deviation is small.

One interesting point is that if we divide the workspace stiffness matrix \( K_{ws} \) into an axis matrix and a rotation matrix in Equation (23), the rotation angle \( \theta_e \) can be included in the Jacobian \( J_{abs} \).

\[
K_{ws} J_{abs} = \begin{pmatrix}
    k_1 & 0 \\
    0 & k_2
\end{pmatrix} \begin{pmatrix}
    \cos \theta_e & -\sin \theta_e \\
    \sin \theta_e & \cos \theta_e
\end{pmatrix} \begin{pmatrix}
    -l_1 \sin \theta_1 - l_2 \sin \theta_{12} \\
    l_1 \cos \theta_1 - l_2 \cos \theta_{12}
\end{pmatrix} = \begin{pmatrix}
    k_1 & 0 \\
    0 & k_2
\end{pmatrix} \begin{pmatrix}
    -l_1 \sin(\theta_1 + \theta_e) - l_2 \sin(\theta_{12} + \theta_e) \\
    l_1 \cos(\theta_1 + \theta_e) + l_2 \cos(\theta_{12} + \theta_e)
\end{pmatrix}
\]  

This rotation in the absolute angle Jacobian results in the following gain decision.

\[
K_{1bia}^{bia} = \frac{1}{2} l_1 g k_1 \sin \theta_1 \sin(\theta_1 + \theta_e) + k_2 \cos \theta_1 \cos(\theta_1 + \theta_e)
\]
\[
K_{2bia}^{bia} = \frac{l_1 d_2}{2} k_1 \sin \theta_1 \sin(\theta_1 + \theta_e) + k_2 \cos \theta_1 \cos(\theta_1 + \theta_e)
\]
\[
K_{3bia}^{bia} = \frac{l_1 d_2}{2} k_1 \sin \theta_1 \sin(\theta_1 + \theta_e) + k_2 \cos \theta_1 \cos(\theta_1 + \theta_e)
\]
\[
K_{4bia}^{bia} = \frac{l_1 d_2}{2} k_1 \sin \theta_1 \sin(\theta_1 + \theta_e) + k_2 \cos \theta_1 \cos(\theta_1 + \theta_e)
\]

This is the proposed gain decision to achieve the stiffness characteristics at the endeffector. If \( k_1, k_2, \theta_e \) are specified the gains to realize the stiffness ellipse are determined as above.

Note that the non-diagonal matrix elements \( K_{2bia}^{bia}, K_{3bia}^{bia} \) are not zero in this gain, which means muscle torques \( \tau_1^{1m}, \tau_3^{1m} \) need the angle information of other torque output in order to make the arbitrary stiffness ellipse at the endpoint.

D. Endeffector Stiffness Direction Stabilization by Biarticular Muscle

The gains described in Equations (24) need to share angle information of all muscles; \( \tau_1^{1m} \) feedbacks the biarticular muscle’s angle \( \theta_1 \) and \( \tau_3^{1m} \) feedbacks the angle of the first joint, as described in Equation (21). However, this kind of feedback of angle information which needs the angle information that is not connected to the muscle, cannot be done in real muscles; their viscoelasticity only feedbacks the angle where the muscles are connected.

Even though this restriction is not applicable to the robot manipulator control, it can show the role of the biarticular muscle viscoelasticity - stiffness direction enhancement at the endeffector. Simulations are done to clarify how the direction of stiffness at the endeffector changes with the biarticular muscle. Equation (28) is the gain design to reflect the restriction of angle information feedback.

\[
\begin{pmatrix}
    \tau_{1m}^{fb} \\
    \tau_{3m}^{fb}
\end{pmatrix} = \begin{pmatrix}
    K_1 & 0 & 0 \\
    0 & K_2 & 0 \\
    0 & 0 & K_3
\end{pmatrix} \begin{pmatrix}
    \Delta \theta_1 \\
    \Delta \theta_2 \\
    \Delta \theta_{12}
\end{pmatrix}
\]

Figure 6 is the illustration of simulation results which shows this endeffector stiffness characteristic; with the biarticular elasticity, the stiffness ellipse at the endeffector is controlled to be in the horizontal direction regardless of the angle of the second joint. This characteristic is very important walking or jumping.

There have been some researches on robotics which introduce the biarticular muscle structure to improve stabilization in jumping robot. The analysis in this paper clarifies the reason of this stabilization; the biarticular muscle elasticity stabilizes the direction of the stiffness ellipse.

IV. DIAGONALIZATION OF INERTIA MATRIX BY BIARTICULAR MUSCLE MODEL

A. Dynamics of Manipulator with Biarticular Muscle Torque Input

Equation (29) shows the dynamics of a robot manipulator, where \( g \) is the acceleration of gravity, \( d_i \) is the distance from the center of a joint \( i \) to the center of the gravity point of the link \( i \), \( m_i \) is the weight of the link \( i \), \( J_i = m_i d_i^2 + I_i \), and \( I_i \)
is the moment of inertia about an axis through the center of mass of link \( i \).

\[
\begin{pmatrix}
J_1 + J_2 + m_2 l_1^2 + 2m_2 l_1 d_2 \cos \theta_2 & J_2 + m_2 l_1 d_2 \cos \theta_2 \\
J_2 + m_2 l_1 d_2 \cos \theta_2 & J_2
\end{pmatrix}
\begin{pmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2
\end{pmatrix}
+ \begin{pmatrix}
-m_2 l_1 d_2 \sin \theta_2 (\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2) \\
m_2 l_1 d_2 \sin \theta_2 \dot{\theta}_1^2 + g(m_1 d_1 + m_2 l_1) \cos \theta_1 \\
g(m_1 d_1 + m_2 l_1) \cos \theta_1 + gm_2 d_2 \cos (\theta_1 + \theta_2)
\end{pmatrix}
+ \begin{pmatrix}
T_1^f \\
T_2^f
\end{pmatrix}
= \begin{pmatrix}
0 \\
0
\end{pmatrix}
\tag{29}
\]

Taking the relationship of Equation (7) into consideration, the dynamics of three muscle torques can be derived. We suggest that the dynamics of the biarticular muscle torque is defined as

\[
\tau_3 = (J_2 + m_2 l_1 d_2 \cos \theta_2) (\ddot{\theta}_1 + \ddot{\theta}_2) - m_2 l_1 d_2 \sin \theta_2 (\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2)
+ gm_2 d_2 \cos (\theta_1 + \theta_2).
\tag{30}
\]

With this dynamics definition, the dynamics for two monarticular muscle torques is made independent with each other as the following equations.

\[
\tau_1 = (J_1 + m_2 l_1^2 + m_2 l_1 d_2 \cos \theta_2) \ddot{\theta}_1 + g(m_1 d_1 + m_2 l_1) \cos \theta_1
\tag{31}
\]

\[
\tau_2 = -m_2 l_1 d_2 \cos \theta_2 \ddot{\theta}_2 + m_2 l_1 d_2 \sin \theta_2 (\dot{\theta}_1 + \dot{\theta}_2)^2
\tag{32}
\]

Focusing only on the inertia force terms, we can find that the inertia for each torque is defined without any co-relation with other torques in this dynamics. The \( \mathbf{M}_{\text{bia}} \) in Equation (33) is the newly-defined inertia matrix which is diagonalized.

\[
\mathbf{M}_{\text{bia}} =
\begin{pmatrix}
J_1 + m_2 l_1^2 + m_2 l_1 d_2 \cos \theta_2 & 0 & 0 \\
0 & J_2 + m_2 l_1 d_2 \cos \theta_2 & 0 \\
0 & 0 & -m_2 l_1 d_2 \cos \theta_2
\end{pmatrix}
\tag{33}
\]

Note that the last line of the matrix corresponds to the relationship between the torque \( \tau_3^m \) and the angle \( \dot{\theta}_{12} \), as we stated previously the angle \( \dot{\theta}_{12} \) is the output of the torque \( \tau_3^m \). Although the dynamics of \( \tau_1^m, \tau_2^m, \tau_3^m \) cannot be determined uniquely, the proposed dynamics can be quite efficient in decoupling the co-relation of joint torques.

B. Verification of Proposed Diagonalized Inertia Matrix by Simulation

The effectiveness of the proposed diagonalized inertia matrix is validated by simulations. We propose to use the proposed dynamics as the inverse dynamics for the feedforward control of each muscle torque so that it can be designed independently. Equations (34) to (36) are the designed feedforward control input for three muscle torques.

\[
\tau_1^m = (J_1 + m_2 l_1^2 + m_2 l_1 d_2 \cos \theta_2) \ddot{\theta}_1 + g(m_1 d_1 + m_2 l_1) \cos \theta_1
\tag{34}
\]

\[
\tau_2^m = -m_2 l_1 d_2 \cos \theta_2 \ddot{\theta}_2 + m_2 l_1 d_2 \sin \theta_2 (\dot{\theta}_1 + \dot{\theta}_2)^2
\tag{35}
\]

\[
\tau_3^m = (J_2 + m_2 l_1 d_2 \cos \theta_2) \ddot{\theta}_{12} - m_2 l_1 d_2 \sin \theta_2 (\dot{\theta}_2^2 + 2\dot{\theta}_1 \dot{\theta}_2)
+ gm_2 d_2 \cos \theta_1
\tag{36}
\]

For the disturbance terms, there may be other distribution to \( \tau_1^m, \tau_2^m, \tau_3^m \). The proposed feedforward control focuses on decoupling of disturbance; \( \tau_1^m \) deals with the gravity due to \( \theta_1 \), \( \tau_2^m \) deals with the Coriolis force, and disturbance terms assigned on \( \tau_3^m \) enables this decoupling.

Simulation is performed on a planar manipulator, and the effect of the gravity is ignored. For this reason, the feedforward compensation for the gravity in Equation (34) and (36) are removed.

Figure 7 is the block diagram of simulation. Red line represents the feedforward control, and blue line represents the feedback control. Since we want to verify the proposed dynamics, only feedforward control is adopted and the feedback gain \( K_{\text{bia}} \) is set to 0.

Equation (22) can be used as feedback control design and control stiffness ellipse at the endeffector. The simulation result of this feedback control is explained in our other paper [7]. In this paper, only feedforward control is used to verify the proposed dynamics.

Connection without any sign means the signals are added. Three muscle torques are projected to two joint torque based on Equation (7) and applied to a planar manipulator. Inverse dynamics.1 to 3 use the inverse dynamics in Equations (34) to (36).

In order to validate the fundamental property of the proposed method, no modeling error is considered in this simulation.
C. Tracking Performance by Feedforward Control

First, a sinusoidal wave is added as a reference angle of \( \theta_1 \). The frequency of the wave will show the performance of reference tracking. 1Hz and 5Hz waves are added. Initial angles of \( \theta_1 \) and \( \theta_2 \) is \( \theta_0^1 = 0, \theta_0^2 = \frac{\pi}{4} \). Figure 8,9 are the results of \( \theta_1 \). With high frequency reference, there come some errors due to the low pass filter in differentiation in the feedforward control. Although \( \frac{\pi}{4} \) rad is set as a reference angle of \( \theta_2 \), the actual angle shows the errors in Figures 10 and 11. In actual manipulator control, this kind of time delay is not a problem since the trajectory is designed in more detail way and feedback control will reduce the error.

From these results we can verify the proposed dynamics is effective enough to be used as feedforward control design.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper proposed two advantages of the biarticular muscle: one is stiffness direction stabilization of an endeffector which results in stabilization of the posture, and the other is inertia matrix diagonalization in the dynamics of a robot manipulator.

These characteristics are analyzed based on the three-pair six-muscle model of human arm including biarticular muscles and by associating the muscle force with the joint torques. Simple kinematics is derived that defines the relationship between an endpoint force/position and three muscle torques. By associating the muscle force with the joint torques, the relationship between an endpoint force/position and three muscle torques is calculated.

The proposed kinematics revealed that the elasticity constraints the direction of the stiffness ellipse at the endeffector in the horizontal direction regardless of the joint angles. This stiffness stabilization contributes to the stabilization of the arm/leg posture in robots.

Finally, we suggest a new dynamics of the planar robot manipulator with three muscle torque input where the inertia matrix is diagonalized. Simulation verifies the effectiveness of the proposed dynamics with the diagonalized inertia matrix.

REFERENCES


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