Abstract—The purpose of this paper is to evaluate the effectiveness of the dynamic force distribution methods, which are used for perfect traction force control of in-wheel motored EVs. Integrated with maneuverability improvement control system, the proposed method dynamically distributes the total driving force among the available in-wheel driving motors by according to the working conditions of tires. Due to such precise distribution, the motors can be purposely controlled with different force commands. Therefore, the wheels can be driven in a special way to realize the traction forces and moments, which are required for the motion control. It helps improve turning maneuverability and keep stability of EV, especially in the marginal turning conditions. Experiment results indicate that the control system with the proposed dynamic force distribution method much more greatly improve EV turning characteristics than the one which uses the equal force distribution method.

Index Terms—Electric vehicle, motion control, four wheel drive, dynamic force distribution, dynamics improvement

I. INTRODUCTION

When properties, such as maximum acceleration, climbing capability, high maneuverability and reliability, are required or more driving pleasures are pursued, more EVs are often designed to have more motors [1]. It is possible to design new arrangements of propulsion systems by taking full advantages of those kinds of smart electric device [6]. For example, in-wheel motored EV, which has its driving motors installed into multi-wheels. The power is transferred from energy source to those motors via electrical wires and reaches those motored wheels simultaneously. Thus, the rigid power train might be no longer needed. The typical EVs which are equipped with independently controllable in-wheel-motors are shown in Fig.1.

In this paper, we mention the EVs equipped with no less than four driving motors, which are often referred to as an over-actuated system. It has more actuators (independently controllable in-wheel motors) than what is needed for driving. Those distributed motors make it possible to generate driving and braking torque independently at each wheel without complex mechanical components.

Accordingly, the relevant control technologies are also required. It is meaningful to use redundant motors for realizing a reliable and adaptable control. For example, the redundant driving motors are usually used for improving drivability and keep reliability of control, especially when it drives on a rough terrain. Besides, it also helps obtain multi-optimal objectives and faulty avoidance. For example, optimal tire load ratio and tire slip ratio. If one wheel fails to brake, the redundant one may afford the function of braking.

Dynamic force distribution control is looked on as one of essential technologies for those kinds advanced EVs. According to the total driving force, the proposed dynamic force distribution control optimally distributes the total force and controls for all driving wheels. Not only for the front and rear wheels, but also for the left and right wheels at the same time. And then the motors drive the wheels by differently distributed driving forces and therefore generate the traction forces and moment which are necessary for EV motion control. Due to that generation, the dynamics of EV can be controlled to be what it is wanted. Meanwhile several other optimal objectives, such as minimum tire load ratio can be obtained simultaneously since the suitable utilization of the actuator redundancy.
The mentioned control method for all driving motors is integrated with the whole motion control system to realize some control objectives. For example, force distribution control integrated with direct yaw moment control to improve the maneuverability of vehicle.

This method is evaluated by the experiments on an advanced four-wheel-driven EV, which is shown in Fig.2. Experiment results indicate that the proposed dynamic force distribution method much more greatly improves EV turning maneuverability and stability than the one which uses the equal force distribution method, especially in a marginal turning condition.

The outline of this paper will be organized as follows: section 2 mentions modeling and dynamics analysis of EV when it drives in the constant radius cornering; section 3 mentions design of yaw rate control which is used for maneuverability improvement; section 4 mentions dynamic force distribution method; section 5 mentions experimental evaluation of the proposed method; section 6 mentions preliminary conclusions and future works.

II. MODELING AND DYNAMICS ANALYSIS

In this paper, the planar motions are considered. The pitch and roll motion are assumed to be omitted. The dynamics of EV in the motion of constant radius turning are to be modeled. For this analysis, the steering angle, and side slip are all assumed to have the linear effect on the EV dynamics.

A. Mathematical Model

The four-wheel-driven EV is used to verify the proposed control methods. The free body diagram is shown in Fig.2. We assume that the front steering angles are equal and controlled by driver. Four motored wheels can be controlled independently.

Fig.3 shows the dynamics of that EV on the horizontal plane. We here only consider three degrees of freedom: the motion in the longitudinal direction; the motion in the lateral direction; yaw motion around the vertical axis.

The vehicle dynamics can be expressed as longitudinal, lateral and yaw motion.

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where

\[ \dot{x} = Ax + Bu \]

where

\[ x = \begin{pmatrix} \beta \\ \gamma \end{pmatrix} \]

\[ u = \begin{pmatrix} \delta_f \\ \delta_r \end{pmatrix} \]

\[ A = \begin{bmatrix} -2(C_f + C_r)M^2 \eta & -1 - \frac{2(t_f C_f - L_r C_r)}{L^2 V} \\ \frac{2C_f}{L^2 V} & \frac{1}{L} \end{bmatrix} \]

\[ B = \begin{bmatrix} 2C_f \eta \\ -2C_f \eta \end{bmatrix} \]

where \( C_f \) and \( C_r \) are cornering stiffness of the front and rear tires.

When EV drives in a turn, acceleration or deceleration acts just as a kind of disturbance, which may cause handling instability. Acceleration may enhance under-steer attitude while deceleration increase over-steer tendency. The higher the turning velocity of EV is, the more significantly this trend happens. At last when the limit of lateral acceleration is reached, accelerating turning might change the under-steer characteristics into an over-steer one.

In order to depress that disturbance and improve maneuverability, we design a yaw rate controller. By using this controller, the EV is controlled to follow a reference yaw response, which is almost like a neutral steer characteristic. Active yaw moment control method is used for that design. The mentioned reference yaw response is shown in the Fig.4.

As for that yaw rate controller, active yaw moment can be generated according to the yaw tracking error. However, the generated yaw moment can not be used as the control inputs for the four driving motors directly. In this case, the dynamic force distribution control is needed. It is expected to optimum determines the control inputs for four driving motors according to the required active yaw moment.

III. MANEUVERABILITY IMPROVEMENT CONTROL

The main goal of yaw rate control is to improve maneuverability or maintain stability of the EV in critical driving situations. In those cases, drivers may have difficulties or unable to drive normally. At that time, yaw rate control is expected to work as a drive assistant [2].

In this paper we control the yaw rate according to a desired yaw motion. Direct yaw moment control strategy is used. It integrates yaw rate feedback control with yaw moment feed forward control.

Fig.5 shows the block diagram of the mentioned control method.

A. Reference Mode

As Fig.4 shows, the reference model is determined as a neutral steer characteristic. The yaw rate gain of this model is obtained from the experiments.

![Graph showing maneuverability of the mentioned EV](Image)

**Fig. 4. Maneuverability of the mentioned EV**

### B. Maneuverability Analysis

Handling is of the important of the lateral motion dynamics. It also expresses the maneuverability of an EV. Suppose the EV turns a constant radius circle at a constant small speed. According to Eq. 4, the static gain of steering angle to yaw rate can be defined as

\[ \gamma = \frac{1}{1 - \frac{M^2 L_f C_f - L_r C_r}{C_f C_r} V^2 \delta} \]

According to Eq.5, the mentioned EV has an understeer handling characteristic. Fig.4 shows that kind of maneuverability.

When EV drives in the acceleration or deceleration cornering, the acceleration affects turning performance of EV significantly, especially when near the limit of lateral acceleration.

### Reference Mode

Fig. 5. Yaw rate control for maneuverability improvement
B. Active Yaw Moment Control

As we mentioned above, the active yaw moment control is designed by using the yaw rate feedback control and yaw moment feed forward control. The control law is shown in Fig. 6 in detail [4][7].

IV. DYNAMIC FORCE DISTRIBUTION

There are many motivations and different purposes for traction force distribution. In this paper, we try to improve maneuverability of EV by using optimal force distribution.

When using four actuators to realize the required forces and moments, some of the actuators are redundant. How to manage the redundant motors and obtain the control of the dynamics is the main topic of the dynamic force distribution. For the redundancy resolution which is used for the control utilizing redundant actuators, the nonlinear optimization methods are often adopted.

In the dynamic force distribution control, the total control effectors, which are forces and moments required for the dynamic control of EV, are previously generated. Then next, those demanded control effectors are distributed among individual actuators. By using those distributed commands, actuators can be suitably activated to drive EV as what it is desired to be. Meanwhile, the per-required control effectors will be certainly realized.

It is well known that the friction force which happens between the tire and road has a maximum value. If the command of traction force control is larger than that limitation, the wheel will slip or lock. Further, style of traction force distribution, for example rear wheel drive or front wheel drive, affects EV characteristics. That is to say that force distribution what we discuss here should consider those limitations. Therefore it is a constrained optimal problem [9].

In Fig. 6, the control effectors are the total driving force, $F_{ac}^\ast$, and yaw moment, $N_z^\ast$. Four actuators are used to generate those two control effectors and control one state of the EV. In fact, only two actuators are enough for that purpose. However, in this case, there are other two redundant actuators can be used.

That kind of force distribution can be summarized in Fig. 7.

As Fig. 7 shows, force distribution control get the optimal control input commands for the next minor force controllers of the motors.

In order to distributing the active yaw moment, $N_z^\ast$ and $F_{ac}^\ast$, the dynamics of EV should be considered. Those are looked on as the constraints for the distribution. The mainly constraints can be written as

$$F_{ac}^\ast = F_{x1}^\ast + F_{x2}^\ast + F_{x3}^\ast + F_{x4}^\ast$$  \hspace{1cm} (6)

where $F_{ac}^\ast$ is the optimum generated driving command for each motor.

This equation means the sum of the generated longitudinal force commands should be equal to the required total accelerating force to meet driver's traction or braking command.

$$N_z^\ast = d_f(F_{x2}^\ast - F_{x1}^\ast) + d_r(F_{x4}^\ast - F_{x2}^\ast)$$  \hspace{1cm} (7)

where $d_f$ and $d_r$ are treads of front wheels and rear wheels respectively.

This equation means longitudinal traction force should be able to realize the required yaw moment to stabilize yaw rate.

Because there are four variables and less condition equations, it is impossible to get unique solutions under such conditions.

On the other hand, as for control design, efficiently utilizing such redundancy might bring advantages. For example, because the force commands to control motored wheels are not determined uniquely, when one or two motored wheels break down, the control system can still drive EV normally.

Therefore, for the purpose of optimization, another condition can be adopted for the controls resolution. We can design that condition by defining some kind of objective functions. As mentioned above, it is necessary to know the tire working conditions when realize the force distribution. Thus, we define one kind of objective function which consists of the tire work load ratio.

It is well known that the tire friction force condition, which is shown in Fig. 8, satisfies

$$F_x^2 + F_y^2 \leq \mu_{max}^2 F_z$$  \hspace{1cm} (8)
where $F_x$ is the longitudinal friction force, $F_y$ is the lateral friction force, $F_z$ is the normal force, $\mu_{\text{max}}$ is the maximum friction coefficient.

We here define tire work load ratio by considering that inequality equation.

\[
\rho_i = \sqrt{\frac{F_{x_i}^2 + F_{y_i}^2}{F_{z_i}^2}}
\]  

(9)

where $F_{z_i}^2$ is the square of the normal force on the wheel. We here assume that the normal force can be estimated by the longitudinal and lateral load transitions. $F_{y_i}^2$ is square of the lateral force on the tire. In this study, we assume that the limitations of the tire maximum friction force do not be exceeded. It is also means that the constraints of the tire limitation do not be activated.

We define the objective function for the implementation of the dynamic force distribution as

\[
J = ||\Theta||^2 \frac{1}{2}
\]  

(10)

where $\Theta = (\rho_1, \rho_2, \rho_3, \rho_4)^T$ is a vector of tire work load ratio. In this study, Eq.10 is expressed in a detail way as

\[
J = \sum_{i=1}^{4} \frac{F_{x_i}^2 + F_{y_i}^2}{F_{z_i}^2}
\]  

(11)

Mathematically speaking, the dynamic force distribution control which is argued as a constrained optimization problem in this study can be resolved by using the equations Eq.11, Eq.6, and Eq.7.

From the viewpoint of control, those equations means the conversion of "virtual" control effectors, which are specified by a higher level control system, to physical control inputs among an available set of actuators.

V. EXPERIMENTS AND EVALUATIONS

The design of experiment is shown in Fig.8. The in wheel motored EV is controlled to drive in acceleration to turn a circle with constant radius, which is 15 meters. The initial velocity of vehicle when control begins is about 10 km/h. The friction coefficient is about 0.3~0.4. During experiment, the steering angle is kept as possible as a same constant value. The driving force is kept constant too [10].

EV is controlled to follow a desired behavior, which is the neutral steering characteristic when EV turns a constant radius circle. In this experiment, we try to realize that neutral steer performance by controlling yaw rate of EV.

By that experiment, we evaluate our proposed dynamic force distribution methods by comparing those with the one who distribute traction force for the left and right motors in an equal way.

Without any control method, as Fig.9 shows, EV can not follow the desired neutral steering characteristic when it drives in an acceleration turning. The error between the reference yaw rate and the real one becomes larger and larger.

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\[
V_0 = 10 \text{ m/s}
\]

Fig. 8. Experiment design for the evaluation of the proposed method

\[
R = 15 \text{ m}
\]

Fig. 9. Description of the experiment. The turning radius is 15 meters; steering angle is 200 degree; acceleration torque of four motors is about 1036 Nm, the longitudinal acceleration is about 0.14g; the gain of yaw rate feedback controller is 200; the limit of lateral acceleration is about 0.35g

\[
\text{Acceleration time [s]}
\]

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On the other hand, in this experiment, Fig.9 shows that one kind of yaw moment control with equal force distribution method can improve the characteristic of EV. This result has been verified by the control of vehicle driven by gasoline engines.

However, if the proposed optimal force distribution methods are used, the EV turning performance can be improved greatly.

The EV trajectories in the three cases which are compared in the Fig.9 are described in the Fig.10. Besides that we also try different control methods, which act as the yaw rate controllers. The 2-DOF controller with the proposed dynamic force distribution method is evaluated. The yaw rate response results are shown in Fig.11.

Further, it is well known that lateral acceleration when EV drives in an acceleration turning is another important ability about handling characteristic.

We also discuss the effect of proposed control method on the lateral acceleration of EV. We compare the equal force distribution method with the proposed methods. The effect of proposed dynamic force distribution methods on the lateral acceleration is shown in Fig.12.

The experiment results indicate that the maximum lateral acceleration is enlarged by the proposed method. Otherwise, in case of equal force distribution, the effect on enlargement of lateral acceleration of EV is not so much as the proposed method.

From other point of view, that also means that the dynamic optimum force distribution methods improve the turn performance of EV.

VI. CONCLUSIONS AND FUTURE WORKS

In this paper, we discuss the dynamic force distribution methods for EV motion control. We evaluate the effect of the proposed methods by the experiments. The results show that the discussed force distribution method can greatly improve the performance of EV when it derives in an acceleration turning.

We also discuss the technologies about dynamic force distribution in this paper. For example, the yaw rate control logic and tire force observation.

In the future, the estimation of tire working conditions should be improved [5],[8]. For example, the lateral friction force can be known easily and accurately. Based on the obtained information of tire forces, the dynamic optimum force distribution method will be realized online more easily.

REFERENCES