Vibration suppression and disturbance rejection control in torsional system is an important problem in the future motion control. As the newly required speed response is very close to the primary resonant frequency of such systems, conventional techniques based on P&I controller is not effective enough. To overcome the problems, various control strategies have been proposed mainly for controlling 2-inertia system, the simplest model of the flexible system.\(^{[1][2][3][7][12][13][14]}\)

In this paper, we will focus our discussion on the disturbance observer-based techniques. We proposed "resonance ratio control" several years ago and showed its excellent performance by simulation.\(^{[4][9]}\) By feeding back the torsional torque estimated by the disturbance observer, the virtual motor inertia moment can be changed to an arbitrarily value. This means that we can change the resonance frequency and then the resonance ratio.

However, the estimation speed of the disturbance observer used in the resonance ratio control was assumed fast enough compared with the resonant frequency of the controlled object.\(^{[9]}\) Too fast disturbance observer causes implementation problem. We then investigated the effect of estimation speed on various control performances.

In this paper, we propose a novel technique, "slow resonance ratio control", whose advantages are as follows:

1. The optimal speed of the disturbance estimation is given by an explicit formula. It can be relatively slow.
2. The speed controller can be independently designed from the vibration suppression control.

Finally, we explain the specially ordered experimental setup using two motors and adjustable flywheels connected by a flexible shaft. We can adjust not only the inertia moments and the stiffness of the shaft but also the backlash and friction. We confirm the effectiveness of the proposed method by experiments.

**STEEL ROLLING MILL AND 2-INERTIA MODEL**

Figure 1 illustrates the typical configuration of steel rolling mill system. This system is basically a distributed parameter system. By using the modal analysis, it can be modeled as a system having several inertia moments and springs.\(^{[12]}\) For example, 12 inertia moments are needed. 2-inertia system given by Figure 2 is its simplest model. Figure 3 gives its block diagram representation.

In Figure 2, we assume:

\[
J_M + J_L = 1, \quad K_s = 1
\]  

\[1, 2\]

For comparative analysis. These equations mean that the total inertia moment of the motor and the load, and the spring coefficient are fixed to 1, respectively. Various 2-inertia systems with different inertia ratios will be investigated under these relations.

The transfer function from \(T_M\) to \(\omega_L\), which is most important in the closed loop design, is given by

\[
G_{11}(s) = \frac{1}{s} \frac{j_s^2 + K_s}{J_M s^2 + J_M + K_s (J_M + J_L)} = \frac{1}{J_M s^3} \frac{s^2 + \omega_0^2}{s^2 + \omega_0^2}
\]  

\[3\]

This transfer function has two particular points: the resonant and anti-resonant frequencies given by

\[
\omega_0 = \sqrt{\frac{J_M}{J_L}} \left(1 + \frac{J_L}{J_M} \right)
\]

\[
\omega_m = \sqrt{\frac{J_M}{J_L}}
\]  

\[4, 5\]

where \(R_0\) is the inertia ratio given by \(R_0 = J_L/J_M\). At these frequencies, the phase characteristics change drastically.

**SLOW RESONANCE RATIO CONTROL AND ITS DESIGN METHOD**

Figure 4 depicts our proposing new technique: the slow resonance ratio control. Using this configuration, I will explain our idea.

**Ideal fast resonance ratio control**\(^{[9][11]}\)

When \(T_M = 0\) in Figure 4, it gives the ideal "resonance ratio control" based on the fast disturbance observer. In usual disturbance observer applications, 100% of the estimated disturbance is fed back to the motor torque, but in this case, \(1 - K\) of the estimated disturbance is fed back. We can change the virtual motor inertia moment to any value as
The inertia ratio can be changed to

\[ R = \frac{J_L}{J_M} = \frac{J_L}{J_M} = KR_0 \]  

(7)

The resonant frequency is then changed to

\[ \omega_0 = \gamma \left( \frac{K}{L} \right) \left( \frac{1}{J_M} \right) \]  

(8)

The anti-resonant frequency does not change. In the resonance ratio control, by setting the new resonance ratio \( R = \omega_0/\omega_q \) to be \( 2 \sim 5 \), effective vibration control is achieved. Generally by using \( R \), the estimation speed of the disturbance observer is finite in actuality. From some simulations, it is known that the estimation should be done faster than the resonant frequency of the controlled object.

### Slow resonance ratio control

When the estimation speed of the observer is finite, i.e., \( T_q > 0 \), the ideal resonance ratio control is impossible. Generally by using \( Q \) as the low-pass filter part: \( 1/(T_q \omega + 1) \) in Figure 4, the following two important transfer functions can be obtained.

\[ \frac{\omega_M}{T_M} = \frac{1+R}{s^2+\omega_0^2} \frac{s^2+\omega_0^2}{(1+Q)\omega_0^2} \]  

(9)

\[ \frac{\omega_M}{T_M} = \frac{1+R}{s^2+\omega_0^2} \frac{s^2+\omega_0^2}{(1+Q)\omega_0^2} \]  

(10)

We pay more attention to \( \omega_M/T_q \). Its characteristics are as follows.

- When the observer is very fast, i.e., \( T_q = 0 \) if \( Q = 1/(T_q \omega + 1) \), by putting \( Q \perp 1 \), eq.(11) is obtained.

\[ \frac{\omega_M}{T_M} = \frac{1+R}{s^2+\omega_0^2} \frac{s^2+\omega_0^2}{(1+Q)\omega_0^2} = \frac{1}{2} \frac{\omega_0^2}{s^2+\omega_0^2} \]  

(11)

- When the observer is very slow, i.e., \( T_q = 0 \) if \( Q = 1/(T_q \omega + 1) \), by putting \( Q \perp 0 \), eq.(12) is obtained.

\[ \frac{\omega_M}{T_M} = \frac{1+R}{s^2+\omega_0^2} \frac{s^2+\omega_0^2}{(1+Q)\omega_0^2} = \frac{1}{2} \frac{\omega_0^2}{s^2+\omega_0^2} \]  

(12)

These two curves have the intersection point at

\[ \omega_0 = \sqrt{1+\frac{R}{2} \omega_0^2} \]  

(13)

and the amplitude there is given by

\[ \frac{\omega_M}{T_M} = \frac{1+R}{\omega_0} \frac{2}{R-R_0} \]  

(14)

Interestingly, all curves having any \( T_q \) pass this point. Hence, if \( T_q \) is selected so that this point is the local maximum, vibration suppression can be realized most effectively. Such \( T_q \) is given by

\[ T_q = \sqrt{\frac{1+\frac{R+5R_0}{4}}{\frac{1+3R+R_0}{2} \frac{1}{\omega_0}}} \]  

(15)

This is the optimal estimation speed (the optimal time constant) of the disturbance observer when we use the first order observer.

For reference, the optimal estimation speed given by Iwata in Umida's slow disturbance observer is given by \([9]\)

\[ T_q = \sqrt{\frac{1+\frac{R}{2}}{\omega_0}} \]  

(16)

This value is close to the value to put \( R = 0 \) in eq.(15). In other word, Umida's slow disturbance observer is the special case to put \( R = 0 \) in the slow resonance ratio control.

### Design of K

\( K \) is the ratio of \( R \) (the new inertia ratio that the resonance ratio control aims to realize) to \( R_0 \) (the original inertia ratio), i.e., \( R = KR_0 \). \( \omega_0 \) is also the function of \( R_0 \). When \( R_0 \) is given as a parameter of the original system, from eqs.(14) and (15), the optimal estimation speed \( T_q \) and the peak amplitude at \( \omega_0 \) are the functions of only \( K \).

Figure 5 draws the peak amplitude at \( \omega_0 \) as the function of \( K \). The peak at \( \omega_0 \) decreases when \( K \) increases. On the other hand, from Figure 6, we can see that \( \omega_0 = (1/T_q) \) becomes bigger when \( K \) increases, which means that faster estimation is required. It must cause implementation problem. Hence we need a compromise.

From Figures 5 and 6, if we select \( K = 5 \sim 10 \), the peak is relatively small while keeping \( \omega_0 \) not so big for a wide range of \( R_0 \). For smaller \( K \), \( \omega_0 \) becomes much smaller, which reduces implementation problem, because the controller has no fast parts.

![Figure 4: Configuration of the slow resonance ratio control.](image)

![Figure 5: The peak amplitude at \( \omega_0 \) v.s. \( K \).](image)

![Figure 6: The optimal estimation speed of the disturbance observer.](image)
**Design of the speed controller**

In eqs. (9) and (10), \( \omega_s/T_M \) converges to 1/\( s \) when \( s \to 0 \), because we designed so as to keep their DC gains same regardless of \( R_s \). It means that, in any \( R_s \) cases, the 2-inertia system can be seen 1-inertia system having \( J_0=M/L=1 \) as the total inertia moment.

It is very convenient if we can use the fixed P&I speed controller designed for the 1-inertia system. Here, we put

\[
K_p = \frac{1}{T_u}, \quad K_i = \frac{K_p}{2.5T_u}
\]

(17), (18)

\( T_u \) is the specified response time of speed control. Here, we put \( T_u = 1/\omega_0 \) hoping to realize the command response as fast as the antiresonant frequency. Eq.(18) means that we selected the integral time constant to be 2.5 times of the speed control response. In simulation, Two-Degree-Of-Freedom P&I controller is used to reduce the overshoot in command response. It can be realized simply by putting \( b=0.5 \) in Figure 4.

**Summary of the design procedure**

The following is the summary of the design procedure of the "slow resonance ratio control" proposed in this paper.

1. Put \( K=5-10 \), i.e., \( K=5R_s-10R_o \).
2. Put the disturbance observer's estimation speed by eq.(15).
3. Design the speed controller by eqs.(17) and (18).

**SIMULATION RESULTS OF THE SLOW RESONANCE RATIO CONTROL**

Here we will show the time response simulation of the "slow resonance ratio control" At \( n=5 \), the speed command \( \omega_c=1 \) is given to observe its command response. At \( n=25 \), step disturbance of \( T_q=0.5 \) is given to see the disturbance response.

In this simulation, the model constants include 10–20\% errors, and backlash (+/-0.01) and torque limiter (+/-1.2) are introduced. We can know that the performances of the proposed method are same or even superior to other methods, e.g., the resonance ratio control, the optimal PID control, and even the state feedback control.[10][11][15]

The speed controller here is designed for 1-inertia system without considering vibration suppression. Such an independent design has a great advantage in actual industrial application systems.

**EXPERIMENTAL RESULTS OF DISTURBANCE OBSERVER BASED CONTROLLERS**

**Experimental setup**

Figure 8 illustrates the "Torsional Vibration System Experimental Setup" specially made by Mitsubishi Heavy Industry. It consists of two brushless DC (BLDC) motors, changeable backlash and friction mechanism, the load equipment and so on. The torques of BLDC motors are controlled fast and precisely enough by two high performance motor drivers.

Sensor information from shaft encoders and tacho-generators are read into the microcomputer via counter boards and A/D converters. After some control calculations, the torque commands are outputted to the drivers via D/A converters. Control algorithm is written by C language. We developed some more convenient programs, e.g., a frequency response measurement algorithm using M-series signal.
Figure 8: Experimental setup of torsional vibration system.

Table 1 gives the experimented control methods and their parameters. The inertia moments are converted to the load-side (i.e., the torsional shaft side) quantities. We implemented

1. original disturbance observer designed for 1-inertia system,
2. fast resonance ratio control using fast disturbance observer,
3. slow resonance ratio control proposed in this paper.

The speed controllers of (1) and (3) are designed for 1-inertia system, and in (2) we used Manabe Polynomial method\cite{8}. In all experiments, IP speed controllers are used by putting $b=0$ in Figure 4.

Table 1- Tested control methods and the parameters

<table>
<thead>
<tr>
<th>controller parameters</th>
<th>disturbance observer for 1 axis</th>
<th>fast resonance ratio control</th>
<th>slow resonance ratio control</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>system parameters</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inertia moment of motor</td>
<td>$J_{M0}=4.016 \times 10^{-3}$ [kgm²]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inertia moment of load</td>
<td>$J_L=2.921 \times 10^{-3}$ [kgm²]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>stiffness constant</td>
<td>$K_L=39.21$ [Nm/rad]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>resonant frequency</td>
<td>$\omega_0=152.3$[rad/s], $\omega_a=115.9$[rad/s]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>anti-resonant frequency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inertia ratio</td>
<td>$R_0=J_L/J_{M0}=0.7273$, $H_0=1.314$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>resonance ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>control period</td>
<td>$T_s=1$[ms]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>parameters in speed control</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_p$ (proportional gain)</td>
<td>0.804</td>
<td>0.435</td>
<td>0.535</td>
</tr>
<tr>
<td>$K_i$ (integral gain)</td>
<td>26.6</td>
<td>14.26</td>
<td>17.71</td>
</tr>
<tr>
<td><strong>parameters in vibration control</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K$ ($=R/R_0$)</td>
<td>-</td>
<td>3.025</td>
<td>2.368</td>
</tr>
<tr>
<td>$\omega_q$ ($=1/T_q$) *</td>
<td>2.0$\omega_0$</td>
<td>3.0$\omega_0$</td>
<td>1.7$\omega_0$</td>
</tr>
</tbody>
</table>

* $T_q$: time constant of the disturbance observer

(a) command response

(b) disturbance response

(c) frequency response from $\omega_{ref}$ and $T_L$ to $\omega_L$

Figure 9: Original disturbance observer.

(a) command response

(b) disturbance response

(c) frequency response from $\omega_{ref}$ and $T_L$ to $\omega_L$

Figure 10: Fast resonance ratio control.
Figure 11(a), the speed reference from $T_L$ to $\omega_m$ can be suppressed effectively. However, the transfer function from $T_L$ to $\omega_m$ has a harmful frequency peak around 200[rad/s]. Due to this peak, relatively big high frequency vibration remains in the motor torque.

The slow resonance ratio control shown in Figure 11 gives sufficiently stable vibration-less responses in frequency characteristics and motor torque waveforms. This is because there are no fast parts in the proposed controller.

**CONCLUSION**

In this paper, we proposed the “slow resonance ratio control” as an effective torsional system control method. We gave the explicit formula of the optimal estimation speed of the disturbance observer. We confirmed its superior performances by simulation and real experiment. In the original “fast resonance ratio control”, we needed to design the speed controller considering the vibration suppression, where we dealt with the total system's transfer function or the state space representation. On the contrary, in the proposed slow resonance ratio control, we can use the speed controller independently designed for 1-inertia system. This is a great advantage in most industrial applications.

**REFERENCES**