Infinity Norm Approach for Precise Force Control of Manipulators Driven by Bi-articular Actuators

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Abstract—In recent years there has been increasing interest in manipulators presenting animal musculoskeletal characteristics such as bi-articular actuators. Manipulators driven by bi-articular actuators usually have more actuators than joints, presenting therefore actuator redundancy. In this paper a new approach based on $\infty$–norm to resolve actuators redundancy is proposed.

The proposed method is compared with the Phase Different Control (PDC) approach, which is based on human muscle activation level patterns. It is shown that the infinity norm approach produces no error in output force, while the PDC approach produces non-zero error. Moreover, in this paper, a PDC approach with non-linear model that eliminates error in output force is also proposed. However the PDC approach with non-linear model is more complex than the proposed infinity norm approach.

I. INTRODUCTION

Robotic manipulators presenting animal musculoskeletal characteristics such as bi-articular actuators have been proposed for more than two decades [1]. In recent years there has been increasing attention on such animal inspired robotic arms, both in hardware and control design aspects [2] [3] [4]. Manipulators driven by bi-articular actuators usually have more actuators than joints, presenting therefore actuator redundancy.

A widely used approach to resolve actuator redundancy in robot applications [5] [6] [7] is the Phase Different Control (PDC) approach, which is based on human muscle activation level pattern [8]. According this approach, the relationship between the output force at the end effector and the actuator joint torques can be determined using a linear model based on muscle activation level pattern, which have been derived from observation of electromyography activity of human muscles.

In our previous work [9], an approach based on $\infty$–norm for actuator redundancy resolution was proposed as a model to optimize the design of manipulator driven by bi-articular actuators. It have been shown that, under the same maximum actuator joint torques conditions, $\infty$–norm allows to obtain a greater maximum output force at the end effector in respect to Moore-Penrose pseudoinverse matrix based approach.

In this paper, the proposed $\infty$–norm approach is compared to the PDC approach. It is shown that the PDC approach presents an error between the desired output force at the end effector and the obtained one. This error is not present in the case of the proposed $\infty$–norm approach.

The error in force output for the PDC approach is due to the linearization of the actuator activation level patterns. This aspect is also analyzed in this paper, where a PDC approach with non-linear model to eliminate error in output force is proposed. This non-linear model considers the geometry and configuration of the manipulator using sinusoidal functions. Therefore it is more complex than the proposed $\infty$–norm approach, which is instead based only on linear functions for resolution of actuator redundancy.

In Section II main features and kinematics model of robotic manipulators equipped with bi-articular actuators are described. In Section III, two approaches for actuator redundancy resolution — PDC and $\infty$–norm — are introduced. In Section IV the proposed $\infty$–norm approach is compared with the PDC approach at first (Section IV-A and Section IV-B), and then a PDC approach with non-linear model to eliminate error in output force is proposed (Section IV-C). Finally, in Section V, the advantages of the $\infty$–norm approach are summarized.

II. CHARACTERISTICS AND MODELING OF ROBOTIC ARM WITH BI-ARTICULAR ACTUATORS

In conventional manipulators each joint is driven by one actuator. On the contrary, animal limbs present a complex musculoskeletal structure based on two types of muscles:

1) Monoarticular muscles, which produce a torque on one joint.
2) Bi-articular muscles, which produce the same torque on two consecutive joints at the same time. Gastrocnemius is an example of bi-articular muscle in the human leg.

A simplified model of the complex animal musculoskeletal system is shown in Fig. 1. This model is based on 6 contractile actuators — extensors ($e_1$, $e_2$ and $e_3$) and flexors ($f_1$, $f_2$ and $f_3$) — coupled in three antagonistic pairs.

- $e_1$–$f_1$ and $e_2$–$f_2$: pairs of monoarticular actuators which produce torques about joints 1 and 2, respectively.
- $e_3$–$f_3$: pair of bi-articular actuators which produce torque about joints 1 and 2 contemporaneously.
The kinematics model of the arm driven by bi-articular actuators of Fig. 1 is shown in Fig. 2 where:

- $T_1$ and $T_2$ are total torques at joint 1 and 2, respectively.
- $\tau_1$ and $\tau_2$ are torques produced monoarticular actuators at joints 1 and 2 respectively, calculated as:
  \[
  \tau_1 = (f_1 - e_1)r
  \]
  \[
  \tau_2 = (f_2 - e_2)r
  \]
  where $r$ is the distance between the joint axis and the point where the force is applied.
- $\tau_3$ is the bi-articular torque produced at both joints:
  \[
  \tau_3 = (f_3 - e_3)r
  \]
- $F$ is a general force at the end effector.

The kinematics of this system can be therefore expressed by:
\[
\begin{align*}
T_1 &= \tau_1 + \tau_3 \\
T_2 &= \tau_2 + \tau_3
\end{align*}
\]

Manipulator equipped with bi-articular actuators have numerous advantages: dramatical increase in range of end effector impedance which can be achieved without feedback [1], realization of path tracking and disturbance rejection using just feedforward control [10], improvement of balance control for jumping robots that do not use force sensors [11]. Moreover, multi-joints actuators such as tri-articular actuators, can increase the efficiency in output force production [6].

Another advantage of arm equipped with bi-articular actuators is the ability to produce a maximum output force at the end effector in a more homogeneously distributed way [12].

In Fig. 3 the maximum output force at the end effector for a two-link traditional manipulator and a arm equipped with bi-articular actuators is shown for comparison. In the case of traditional manipulator, 2 actuators with maximum joint torque $T_1 = T_2 = 10$ Nm are considered. On the other hand, for the bi-articular actuators driven arm three actuators with maximum joint torque $\tau_1 = \tau_2 = \tau_3 = 6.66$ Nm are taken into account. Therefore the total maximum torque in the two cases is the same. The conventional quadrilateral shape becomes an hexagon for arms driven by bi-articular actuators, which can therefore produce a maximum force at the end effector more homogeneously distributed in respect to output force direction. This aspect is peculiar for application which interact with humans such as rehabilitation robots, as well as for jumping and waking robots [11] [13].

### III. ACTUATOR REDUNDANCY RESOLUTION’S METHODS

A two-link manipulator with the kinematics shown in Fig. 2 is driven at least by three actuators, resulting in actuator redundancy. Given $\tau_1$, $\tau_2$ and $\tau_3$, it is possible to determine $T_1$ and $T_2$, and so $F$ by using the transpose Jacobian:
\[
\begin{pmatrix}
T_1 \\
T_2
\end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix}
\]

where $f_x$ and $f_y$ are the orthogonal projection of $F$ on the axis $x$ and $y$, respectively. On the other hand, given $F$, and therefore $T_1$ and $T_2$, it is generally not possible to determine uniquely $\tau_1$, $\tau_2$ and $\tau_3$ (Eq. 4).
In the following two approaches to resolve actuator redundancy — linear PDC and ∞-norm — are described.

**A. Phase different control approach**

In Fig. 4 the output force at the end effector of a manipulator driven by bi-articular actuators is shown. There are 6 main force vectors, each due to one of the 6 contractile actuators — e1, e2, e3, f1, f2, f3 — which therefore form an hexagon [8].

Fig. 5 shows six muscle activation level patterns in respect to the output force direction θf which is expressed in respect to points A-F of Fig. 4. These patterns are obtained linearizing the data obtained from electromyography observation of human muscle activation under static conditions [8].

Given a desired force at the end effector with magnitude $|F^{\text{des}}|$ and angle $\theta_f^{\text{des}}$:

1) Calculate the values in degrees of points A-F of Fig. 5 on the basis of the actual configuration of the manipulator.

2) Calculate the activation level of each muscle — e1, e2, e3, f1, f2, f3 — using the activation level patterns by setting $\theta_f^{\text{des}} = \theta_f$.

3) The desired actuator joint torques are:

$$
\begin{align*}
\tau_1 &= (f_1 - e_1) r \frac{|F^{\text{des}}|}{|F^{\text{max}}|} \\
\tau_2 &= (f_2 - e_2) r \frac{|F^{\text{des}}|}{|F^{\text{max}}|} \\
\tau_3 &= (e_3 - f_3) r \frac{|F^{\text{des}}|}{|F^{\text{max}}|}
\end{align*}
$$

where $|F^{\text{max}}|$ is the magnitude of the maximum force that can be produced in the direction $\theta_f^{\text{des}}$ in the actual configuration of the manipulator.

In order to realize force control, if $|F^{\text{des}}| < |F^{\text{max}}|$, a precise value of $|F^{\text{max}}|$ is required, and therefore the manipulator Jacobian is necessary. On the other hand, if just the application of maximum output force is required, the manipulator Jacobian is not necessary, as in this condition $\frac{|F^{\text{des}}|}{|F^{\text{max}}|} = 1$.

**B. ∞-norm approach**

Given the desired joint torques $T_1$ and $T_2$, three actuators joint torques $\tau_1$, $\tau_2$ and $\tau_3$ can be calculated using ∞-norm by resolving the following problem (it is assumed that $\tau_i^{\text{max}} = \tau_1^{\text{max}} = \tau_3^{\text{max}}$, where $\tau_i^{\text{max}}$ is the maximum joint torque the actuator $i$ can produce):

$$
\begin{align*}
\text{minimize} & \quad \max\{|\tau_1|, |\tau_2|, |\tau_3|\} \\
\text{subject to} & \quad T_1 = \tau_1 + \tau_2 \\
& \quad T_2 = \tau_2 + \tau_3
\end{align*}
$$

A closed form solution of the problem can be determined on the basis of the values of $T_1$ and $T_2$ as follows:

- if $T_1 T_2 \leq 0$

  $$
  \begin{align*}
  \tau_1 &= \frac{T_1 - T_2}{2} \\
  \tau_2 &= \frac{T_2 + T_1}{2} \\
  \tau_3 &= \frac{T_1 + T_2}{2}
  \end{align*}
  $$

- if $T_1 T_2 > 0$ and $|T_1| \leq |T_2|

  $$
  \begin{align*}
  \tau_1 &= T_1 - T_2 \\
  \tau_2 &= \frac{T_2}{2} \\
  \tau_3 &= \frac{T_1}{2}
  \end{align*}
  $$

- if $T_1 T_2 > 0$ and $|T_1| > |T_2|

  $$
  \begin{align*}
  \tau_1 &= \frac{T_1}{2} \\
  \tau_2 &= T_2 - \frac{T_1}{2} \\
  \tau_3 &= \frac{T_1}{2}
  \end{align*}
  $$

The proof of these 3 equations is reported in [9].

Given a generic force at the end effector $F$, the actuators inputs $\tau_1$, $\tau_2$ and $\tau_3$ can be calculated in the following way.

1) Calculate the joint torques $\mathbf{T}$:

$$
\mathbf{T} = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = J^T \begin{pmatrix} F_x \\ F_y \end{pmatrix}
$$

2) According to calculated $T_1$ and $T_2$, the desired actuators inputs can be directly determined using linear equations:
linear functions to resolve actuator redundancy.

There is error between desired (Fig. 7(a)) and output force (Fig. 7(b)). The desired output force direction at the end effector $\theta$ for both the PDC and $\infty$-norm approaches is small for $\theta_2 = 120^\circ$ (Fig. 8), but is significant for $\theta_2 = 30^\circ$ (Fig. 9) where $\text{err}_{\text{max}} \approx 27^\circ$ and $\text{err}_{\text{max}} = 1.7$.

Therefore the proposed $\infty$-norm approach is based on the Jacobian to determine the required joint torques, and uses only linear functions to resolve actuator redundancy.

IV. RESULTS

At first the PDC and $\infty$-norm approaches are compared with respect to two aspects:

1) Precision in output force at the end effector (Section IV-A): it is shown that the PDC approach presents error in output force due to the linearization of the actuator activation level patterns.

2) Input torque patterns to obtain the maximum output force at the end effector (Section IV-B)

Afterwards (Section IV-C), a PDC approach with non-linear model that eliminates error in output force is proposed. By using the PDC approach with non-linear model there is no error in force output, however it is more complex than the $\infty$-norm approach due to the presence of non-linear equations for the resolution of actuator redundancy.

A. PDC VS $\infty$-norm: precision of output force

Fig. 6 shows the maximum output force at the end effector of a two-link manipulator with $l_1 = l_2 = 10$ m using both the PDC and $\infty$-norm approaches. The maximum joint torques are $\tau_1^{\text{max}} = \tau_2^{\text{max}} = \tau_3^{\text{max}} = 10 \text{ Nm}$ in both cases, $\theta_1 = 0^\circ$ and $\theta_2 \in \{30, 60, 90, 120, 150^\circ\}$. The shape of the hexagon representing the maximum output force is equal for the PDC and $\infty$-norm approaches. However, for the PDC approach there is error between desired ($\text{F}^{\text{des}}$) and output force ($\text{F}$).

Fig. 7 shows the maximum output force at the end effector for both the PDC and $\infty$-norm approach when $\theta_2 = 120^\circ$ (Fig. 7(a)) and $\theta_2 = 30^\circ$ (Fig. 7(b)). The desired output force direction at the end effector $\theta_{\text{f}^{\text{des}}}$ varies from 0 to 360$^\circ$ every 5$^\circ$.

For the $\infty$-norm approach there is no error in output force. For the PDC approach the error in output force direction $(\text{err}_\theta = \theta_{\text{f}^{\text{des}}} - \theta_\theta)$ and magnitude ratio $(\text{err}_\text{f} = \frac{|\text{F}|}{|\text{F}^{\text{des}}|})$ is small for $\theta_2 = 120^\circ$ (Fig. 8), but is significant for $\theta_2 = 30^\circ$ (Fig. 9) where $\text{err}_{\text{max}} \approx 27^\circ$ and $\text{err}_{\text{max}} = 1.7$.

Fig. 10 shows the maximum error in output force direction (Fig. 10(a)) and magnitude ratio (Fig. 10(b)) with respect to $\theta_2$ for several link length ratios. For the $\infty$-norm approach there is no error between desired and output force. On the other hand, for the PDC approach the error in output force depends on $\theta_2$, and on arm link length ratio and can increase exponentially when as the manipulator is closer to singular configurations. This error is due to the fact that the PDC approach is based on linear actuator activation patterns.

B. PDC VS $\infty$-norm: actuator activation level patterns

Fig. 11 shows the actuator activation level patterns for the PDC and $\infty$-norm approaches. A manipulator with $l_1 = l_2$, $\theta_1 = 0^\circ$ and $\theta_2 = 30^\circ$ is taken into account without loss of generality.

The actuator activation level patterns for the PDC approach are calculated as $e_1 - f_1$, $e_2 - f_2$ and $e_3 - f_3$, which is equal to a situation in which each actuator torque joint is
A manipulator with \( l \) for the PDC approach is to use the following non-linear model.

**C. PDC approach with non-linear model for elimination of error in output force**

Given a generic force at the end effector \(|F^{des}|\) and \( \theta_f^{des} \), a possible way to determine the actuators joint torques \( \tau_1 \), \( \tau_2 \) and \( \tau_3 \) that do not produce any output force error at end effector for the PDC approach is to use the following non-linear model. A manipulator with \( l_1 = l_2 \), \( \theta_1 = 0^\circ \) and \( \tau_1^{max} = \tau_2^{max} = \tau_3^{max} \) is considered.

1) Instead of using \( \theta_f = \theta_f^{des} \) in the graph in Fig. 5, calculate \( \theta_f \) on the basis \( \theta_f^{des} \) using one the following equations:

- **if** \( 0 \leq \theta_f^{des} \leq \frac{\pi}{2} \):
  \[
  \theta_f = \frac{\theta_f^{des} \sin(\theta_f^{des})}{2 \sin(\theta_f^{des} - \theta_f^{des})} 
  \]
  (13)

- **if** \( \frac{\pi}{2} < \theta_f^{des} \leq \theta_2 \):
  \[
  \theta_f = \frac{\theta_f^{des} \sin \left( \theta_f^{des} - \frac{\theta_f^{des}}{2} \right) \sin(\pi - \theta_2)}{\sin \left( \frac{\theta_f^{des}}{2} \right) \sin(\pi - \theta_f^{des})} + \frac{\theta_2}{2} 
  \]
  (14)

- **if** \( \theta_2 \leq \theta_f^{des} \leq \pi \):
  \[
  \theta_f = \frac{(\pi - \theta_2) \sin(\theta_f^{des} - \theta_2) \sin \left( \frac{\theta_f^{des}}{2} \right)}{\sin(\pi - \theta_2) \sin \left( \frac{\theta_f^{des}}{2} - \theta_2 \right)} + \theta_2 
  \]
  (15)

- **if** \( \pi \leq \theta_f^{des} \leq \pi + \frac{\theta_2}{2} \):
  \[
  \theta_f = \frac{\theta_f^{des} \sin(\theta_f^{des})}{2 \sin(\theta_f^{des} - \theta_f^{des})} + \pi 
  \]
  (16)

**Fig. 8.** \( \theta_2 = 120^\circ \)

**Fig. 9.** \( \theta_2 = 30^\circ \)

**Fig. 10.** \( \theta_f^{max} \) and \( \theta_f^{min} \) in respect to \( \theta_2 \)
In this paper, a new approach based on $\infty - \text{norm}$ to resolve actuator redundancy in robotic arms driven by bi-articular actuators is proposed and compared with the PDC approach. The shape of the hexagon representing the maximum output force at the end effector is the same for the PDC and $\infty - \text{norm}$ approaches. However for the PDC approach there is an error between the desired and output force. This error depends on three factors — the desired force direction, the angle between the links ($\theta_2$) and on the link length ratio — and can increase when the manipulator moves towards singular configurations. On the other hand, there is no error in output force when using the $\infty - \text{norm}$ approach.

In addition, in this paper, a PDC approach with non-linear model to eliminate error in output force is proposed. By using the PDC approach with non-linear model there is no error in output force. However the non-linear model is based on sinusoidal functions, and therefore is more complex than the proposed $\infty - \text{norm}$ approach, which is instead based only on linear functions for actuator redundancy resolution.

**REFERENCES**


