Loss Reduction in Antenna for Wireless Power Transfer by Magnetic Resonant Coupling  
- Relation between Inter-wire Capacitance and the Antenna Loss -

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ABSTRACT: The wireless charging system of electric vehicles will be of high power. Due to environmental problems and the heat emission design of the machines, it is not easy to improve the efficiency of the power transmitting system. While improving the efficiency of the overall system is important, it is especially significant to improve the efficiency of the transmitting and receiving antennas. Improving the antennas is necessary as it accounts for a large part of the overall efficiency and it has strong relation to the transfer characteristics. In this paper, the importance of reducing the antenna loss for high power implementation is discussed. We showed that the antenna losses not only affecting the transfer efficiency but also the maximum power consumed by the load (the available power) by deriving an equation. Through mathematical equations and experiments, we also proved that the antenna loss is caused by both the resistance of the antenna wires and the inter-wire capacitance of the antenna.

KEY WORDS: Wireless power Transfer, Magnetic Resonance Coupling, Inter-wire Capacitance

1. Introduction

From the standpoint of energy issues and environmental issues, the move from vehicles that use internal combustion engines to electric vehicles is inevitable. However, since current electric vehicles house batteries with low energy density and therefore require frequent charging, methods of charging must be made as simple as possible. Also, charging methods would be carried out by the user who would not have had specialist training, so safe charging procedures that have no risk of electrification are sought. With that, one method that is effective in solving those problems is applying wireless power transfer technology to electric vehicles. With this method, there are no cables to be connected, charging can be achieved simply by parking the vehicle, and furthermore, there is no human contact and therefore it is an easy and highly safe charging method (Fig. 1).

When charging electric vehicles wirelessly, some air gap has to be set aside, and high efficiency must be maintained. From numerous wireless power transfer methods, magnetic resonance coupling (which makes use of magnetic resonance phenomena) can obtain high-efficiency transfer over long ranges as well as remain safe for the human body, making it the most suitable power transfer method for electric vehicles.

Regarding the power delivered to electric vehicles, it is approximately 50 kW for fast charge [1], and in consideration of environmental issues and heat-dissipative design, it is important that the transmitting and receiving antennae have as low loss as possible. In this paper, focus is centered on loss minimization of the transmitting and receiving antennae. Equations that show how antenna loss affects both efficiency and maximum power are introduced. Next, it is demonstrated through mathematically and experimentally that not only copper loss but also line capacitance are causes of antenna loss.

2. Antenna’s Loss and Efficient Power

2.1 Antenna for Wireless Power Transfer

Wireless power transfer is carried out through transmitting antenna and receiving antennas. Transmitting and receiving antennas used in wireless power transfer via magnetic resonance coupling method is shown in Fig. 2, and the equivalent circuit for the antenna is illustrated in Fig. 3[2]. In this circuit, $L_1$, $C_1$ and $L_2$, $C_2$ are parameters for transmitting antenna and receiving antenna respectively which are determined by the shape of the antenna, $R_1$, $R_2$ are loss in antenna, and $L_{m}$ is mutual inductance whose value is determined by air gap between antennas.
Fig. 2 Antenna for Magnetic Resonance Coupling Method

(a) Experimental Setup  (b) Equivalent Circuit of Antenna

Fig. 3 Antenna’s Structure and Equivalent Circuit

2.2 Relationship between Loss and Efficient Power

Antenna’s loss can simply be seen to have effect on the efficiency of power transfer. However, the maximum power that can be consumed by load, in other words, available power also affects the efficiency. This section will elaborate on this matter. First, consider a simple circuit which contains internal resistance as shown in Fig. 4.

![Simple Circuit with Internal Resistance](image)

In this circuit, when resistance $R_L$ is changed, the maximum available power that can be consumed by the load $P_{L_{\text{max}}}$ and the value of the resistance at that instant $R_{L_{\text{max}}}$ can be derived by equation (1) and (2) written below

$$P_{L_{\text{max}}} = \frac{V_1^2}{4r} \quad (1)$$

$$R_{L_{\text{max}}} = r \quad (2)$$

Next, with the above setup, consider the antenna used in magnetic resonance coupling method. The equivalent circuit in Fig. 3 is transformed into T-type circuit with power source and load added to it as shown in Fig. 5.

![Equivalent Circuit of Magnetic Resonance Coupling Method](image)

The shape of transmitting antenna and receiving antenna are identical, in other words, $L_1=L_2=L$, $C_1=C_2=C$, $R_1=R_2=R$. Here, when $R_L$ is changed, the maximum power that can be consumed at the resistor load, available power $P_{L_{\text{max}}}$ is found. Frequency that can be used for power transfer, or self-resonant frequency is expressed in equation (3). Here, the input and output voltage ratio of the transmitting antenna $A_v$ can be expressed with equation (4) [3]. Hence, the expression for power in load can be derived into equation (5). From this equation, the point where maximum $P_L$ occurred, named $P_{L_{\text{max}}}$ can be calculated by equation (6). The value obtained is the efficient power for the load.

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (3)$$

$$A_v = \frac{V_2}{V_1} = \frac{\omega_0 L_m R_1}{(\omega_0 L_m)^2 + R \cdot R_L + R_2} \quad (4)$$

$$|P_L| = \left| \frac{(V_1 A_v)^2}{R_L} \right| = \frac{(V_1 \cdot \omega_0 \cdot L_m)^2 R_L}{(\omega_0 L_m)^2 + R \cdot R_L + R_2^2} \quad (5)$$

$$P_{L_{\text{max}}} = \frac{V_1^2}{4R} \cdot \frac{1}{1 + \left( \frac{R}{\omega_0 L_m} \right)^2} \quad (6)$$

Considering the above equation, there are two methods to increase $P_{L_{\text{max}}}$ as following:

- Increase the voltage of power source, $V_1$
- Reduce $R$

Regarding the first method, $P_{L_{\text{max}}}$ can be increased, however, the voltage of other parts of the system will also increase, damage such as in antenna’s insulator can happen more easily, equipment will become more difficult to design, resulting in cost rise. Moreover, this method will not improve the efficiency of power transfer. Therefore, the second method which is to reduce $R$, or to decrease the loss in antenna, will increase the available power, which results in increasing efficiency, making it a vital point in implementing this system in high power.

Next, mathematical analysis is conducted based on this equation. Then, concrete values were inserted, and a graphs is plotted. Parameter are set to be close to values of the real antenna as following: $L=1000\mu\text{H}, C=1000\mu\text{F}, R=5, V_1=100\text{V}$. $L_m$ is varied between 5, 10, 25, 50, 100$\mu\text{F}$. The changes to $P_{L_{\text{max}}}$ when $R$ is varied is plotted in Fig. 6, and the changes to $P_L$ when $R_L$ is varied is plotted in Fig. 7. From the figure, observation below can be made:

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• For $P_{L_{\text{max}}}$, when $R$ decreases, the efficient power $P_{L_{\text{max}}}$ increases.
• The bigger $L_m$ is, in other words the smaller the gap is, the bigger the $P_{L_{\text{max}}}$ becomes.
• The smaller $L_m$ is, the smaller the $R_L$ of the maximum extractable power becomes.

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### 3. Relation between Inter-wire Capacitance and Loss

#### 3.1 Previous Research on Antenna Loss

The antenna loss can be separated into radiation loss and resistance loss. The loss through radiation is insignificant, and the loss is mainly through the resistance of the conductive wire, namely the ohmic loss. Therefore, reducing the ohmic loss is vital to reduce the antenna loss. One of the most common ways to reduce the antenna loss is by increasing the surface area of the antenna to reduce the ohmic loss through skin effect. The skin effect is a phenomenon that occurs when high frequency current flows through a wire. The current gathers at the surface of the wire, which causes the resistance to increase. Thus, by increasing the surface area of the wire, the increase in resistance due to this effect can be reduced.

However, in previous research[4], the actual loss and calculated value does not match, even when the skin effect is considered. Also, that paper does not explain in detail why an increase in loss occurs when the pitch of the antenna element is shrunk. Therefore, the inter-wire capacitance will be introduced as a parameter, and its relation to the antenna loss is studied.

#### 3.2 Relational Expression of Inter-wire Capacitance and Input Impedance

In previous works, the equivalent circuit of a single antenna is shown as Fig. 8. Here, the input impedance $Z_{in}$ can be expressed with Equation (7). In this equation, during resonance, the $\omega$ when $I_m[Z_{in}]=0$, the resistive part of $Z_{in}$ is fixed at $R_i$.

$$Z_{in} = R_i + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$$  \hspace{1cm} (7)

![Fig. 6 Antenna’s Resistance vs Efficient Power](image1.png)

![Fig. 7 Load Resistance vs Load Power](image2.png)

![Fig. 8 Equivalent circuit of single Antenna (previous)](image3.png)

However, in this case, the impedance characteristics of the experimental results do not match with the analysis results. Moreover, the results also do not match at the resonance point, even when the skin effect is considered. This shows that there exists some parasitic components that were not considered in previous works. Thus, this time, the capacitance between the windings of the antenna wires, the “inter-wire capacitance” will be studied. The Inter-wire capacitance is the parasitic capacitance that is generated in between two wires in the antenna coils (Fig. 8). Fig. 9 shows the equivalent circuit of a single antenna when the inter-wire capacitance is considered, where $Z_{in}$ can be expressed as equation (8). Here $C_p$ is the inter-wire capacitance. Through this equation, it is possible for $Z_{in}$ to be higher than $R_i$ during resonance, depending on the parameters.

$$Z_{in} = \frac{R_i + j\omega L_1}{1 + j\omega C_p(R_i + j\omega L_1)} - j\frac{1}{\omega C_1}$$  \hspace{1cm} (8)

$$= \frac{R_i}{(C_p R_i \omega)^2 + (1 - C_p L_1 \omega^2)^2}$$

$$+ j\left(\frac{\omega L_1(1 - C_p L_1 \omega^2) - C_p R_i \omega^2}{(C_p R_i \omega)^2 + (1 - C_p L_1 \omega^2)^2} - \frac{1}{\omega C_1}\right)$$
Next, the frequency characteristics will be analyzed by inputting some concrete values into Equation (8). The analysis results are shown in Fig. 10, and the parameters and results are summarized in Table 1.

According to the graph, there are two points where $\text{Im}(Z_{in})=0$. The lower resonance frequency is called the series resonance frequency $f_0$, and the other one is called the anti-resonance frequency $f_a$. The power transferred is conducted at $f_0$, and the resistance at this frequency $R_{f0}$ is equivalent to the loss generated. It is obvious that the real part of $f_a$ has increased. $f_a$ becomes lower when $C_p$ is increased. This causes an increase in loss as $R_{f0}$ becomes bigger when $f_a$ and $f_0$ moves closer to each other. In other words, an increase in $C_p$ will lead to an increase in antenna loss.

Table 1 Characteristics when inter-wire capacitance is varied.

<table>
<thead>
<tr>
<th>$C_p$[pF]</th>
<th>250</th>
<th>500</th>
<th>750</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$[uH]</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>$C_1$[pF]</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>$R_1$[Ω]</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$f_a$[KHz]</td>
<td>318.3</td>
<td>225.3</td>
<td>183.8</td>
<td>159.1</td>
</tr>
<tr>
<td>$f_0$[KHz]</td>
<td>142.4</td>
<td>130.0</td>
<td>120.3</td>
<td>112.6</td>
</tr>
<tr>
<td>$R_{f0}$[Ω]</td>
<td>15.6</td>
<td>22.5</td>
<td>30.6</td>
<td>40.0</td>
</tr>
</tbody>
</table>

4. Verification through experiment

4.1 Experimental Outline

Next, two antennas with different shapes were created, and the inter-wire capacitance and loss of each antenna were compared. The wireless power transfer antennas are spiral, as in Fig. 2, and has a small diameter so that it can be installed into a car. One way to miniaturize the antennas is to make the coil multilayered as in Fig. 11(a). The characteristics of this antenna is compared with that of a regular single layered antenna, and the changes in anti-resonance point and loss due to the inter-wire capacitance is studied. Both antennas has a similar equivalent circuit.

(a) single(1) layered antenna  (b) double(2) layered antenna

Fig. 11 multi layered antenna. Shape diagram
4.2 Experimental Procedures
The antenna inductance \( L_1 \) of both the double layered antenna and single layered antenna were designed to be the equal. Next, \( L_1 \) and the antenna direct current resistance \( R_{DC} \) were measured using an LCR meter. Then, \( f_0 \), \( f_A \) and \( R_{f0} \) were measured using a Vector Network Analyzer (VNA).

4.3 Experimental Results: Comparison of Both antennas
The experimental results are shown in Table 2.

<table>
<thead>
<tr>
<th>Wire type</th>
<th>1 layer</th>
<th>2 layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside dimension [cm]</td>
<td>47.4</td>
<td>30.4</td>
</tr>
<tr>
<td>( L_1 ) [\mu\text{H}]</td>
<td>1013.2</td>
<td>997.3</td>
</tr>
<tr>
<td>( R_{DC} ) [\Omega]</td>
<td>1.974</td>
<td>1.624</td>
</tr>
<tr>
<td>( f_0 ) [kHz]</td>
<td>156.4</td>
<td>156.9</td>
</tr>
<tr>
<td>( f_A ) [kHz]</td>
<td>1364</td>
<td>611</td>
</tr>
<tr>
<td>( R_{f0} ) [\Omega]</td>
<td>5.02</td>
<td>7.42</td>
</tr>
</tbody>
</table>

The following can be seen by comparing the single layer and double layer antenna.
- As they are designed to have similar inductance, the double layered antenna has a smaller size
- Both antennas are designed to have similar \( L_1 \).
- Both antenna are designed to have similar \( f_0 \).
- The double layered antenna has a much lower \( f_A \). It can be predicted that the \( C_p \) increased due to multilayerization.
- The \( R_{f0} \) of the double layered antenna is bigger, despite having a smaller \( R_{DC} \). This shows that the loss is higher due to a higher \( C_p \) matching the calculation. From this result, it can be said that the loss will worsen due to an increase in \( C_p \) when miniaturization through multilayerization is conducted.

5. Summary
5.1 Summary
In this paper, the loss at the transmission antenna of the wireless power transfer system via magnetic resonance is studied. Firstly, by expressing the antenna loss with an equation, we proved that the antenna loss do not only affect the power transfer efficiency, but also greatly affects the maximum extractable power from the power source. The relation between the voltage of the power source with airgap, antenna loss, and available power were analyzed based on concrete values that is close to the actual conditions. Next, we showed that it is important not only to reduce the ohmic loss, but also the inter-wire capacitance in order to reduce the antenna loss through an equation. Finally, by comparing the characteristics of two antennas with different shapes, we managed to experimentally verify that the inter-wire capacitance affects the loss.

5.2 Future Works
Through experiments, we now know that when miniaturizing an antenna by making it multilayered, the inter-wire capacitance generated will lead to an increase in loss. However, miniaturization of the antenna is necessary to implement this system. Therefore, we need to develop an antenna design that can be miniaturized without increasing the inter-wire capacitance.

References