Mathematical and Experimental Verification of Efficient Force Transmission by Biarticular Muscle Actuator

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Abstract: This paper proves that the biarticular muscle structure which is common in most of animals improves the force generation efficiency in a two-link manipulator. To this end, the statics of two configurations - two joint-actuators configuration and one joint-actuator and one biarticular actuator configuration - are compared using two types of Jacobian. Moreover, the statics are simplified to clarify this difference and an analytical solution about the efficiency of the biarticular muscle structure is proposed. Finally, experimental results verify this advantage of the biarticular muscle structure.

Keywords: Robots manipulators, Modeling, Mechatronic systems

1. INTRODUCTION

The biarticular muscle is an intrinsic mechanism that is shared by most of animals from the lancelet (Tsuji [2010]) to our human. Even though the biarticular muscle mechanism is such a fundamental mechanism, it has been neglected in the design of robots. However, it has started to be highlighted again in robotics recently (Tsuji [2010], Iida [2008], Klein [2008], Hosoda [2010], Niiyama [2010]).

This biarticular muscle structure was introduced to the robotics several decades ago (Hogan [1985]), but it was not fully applied to robotics. Recent research, however, proves that in spite of the redundancy, the biarticular muscle has many advantages and can improve control of humanoid robots (Iida [2008], Klein [2008], Hosoda [2010], Niiyama [2010]).

In the physiology, the biarticular muscle is said to play a significant role making power transfer in human motions (Jacobs [1996], Schenau [1987]). Three pairs of muscles activated in an antagonistic way with a phase difference can generate a well-shaped force hexagon at the end-effector providing a simplified force generation (Kumamoto [1994]).

In the robotics, the biarticular muscle is adopted to develop robots that mimic animals (Tsuji [2010],Niiyama [2007]) showing that the biarticular muscle can simplify motion control.

However, these advantages of the biarticular muscle have been shown only in empirical ways, until we developed a novel statics based on the biarticular muscle system and clarified the simplification by the biarticular muscle system in a numerical way (Oh [2009]).

Based on this simple statics, we could suggest a novel feedback control which can control the direction of the reaction force at the end-effector without any force feedback (Oh [2010]). This could be achieved due to the simplicity in the statics of the biarticular muscle structure.

Recently, we also have proved that the biarticular muscle can distribute the necessary torques for the desired forces at the end-effector in an optimal way (Valerio [2010]), and
it coincides with the physiological data of human subjects (Kumamoto [1994]). In spite of these advantages, there still is a question about whether a robot really needs the biarticular muscle actuator. This paper addresses this problem proving the biarticular muscle has more efficient torque transmission characteristics. To clarify this point, the statics of a two-link manipulator with two different configurations (two joint-torques configuration and one monoarticular muscle torque on the first joint and the biarticular muscle torque configuration) are analyzed and compared in this paper.

First, in Section 2 we compare the required torques to generate a constant force at the end-effector in a numerical way using Jacobian. Then in Section 3, a simple representation way of the statics for a general two-joint manipulator is derived as another comparison tool. Finally, in Section 4 efficiency of the biarticular muscle actuator is verified using experiments.

2. COMPARISON OF THE STATICS BETWEEN JOINT ACTUATOR AND BIARTICULAR MUSCLE ACTUATOR

2.1 Configuration of a two-link manipulator driven by joint actuators and biarticulated actuator

![Fig. 2. Configuration of Two-link Manipulator](image)

Figure 2 is the configuration of a two-link manipulator, where two actuators are located in two joints, while Figure 3 illustrates another type of a two-link manipulator with a biarticular muscle actuator represented by a linear motor. The muscle torques that are generated by $f^e_f$ in Figure 1 is described as $\tau^m_1$ in Figure 3.

The biarticular muscle structure is embedded in various ways in the robotics. Yoshida [2009] and Tsuji [2010] use the pulleys to transmit a torque generated by a motor to two joints, Iida [2008] and Klein [2008] use the springs, and Hosoda [2010] and Niyama [2010] use the McKibben muscles as passive actuators for the biarticular muscle. Planetary gears are also used to realise the biarticular muscles (Umemura [2010], Kimura [2010]). Fujimoto [2010] develops a small linear actuator that can work as a biarticular muscle, which is illustrated in Figure 3. A linear force $F_m$ can produce a torque $\tau^m_3$ in two joints working as a biarticular muscle actuator.

Even though there are various realization ways, they share a common point: the torque generated by the biarticular muscle actuator affects two joints at the same time. In this paper, this characteristic is described using the muscle torque representation; $\tau^m_1, \tau^m_2$ are the torques generated by monoarticular muscles force of two joints $f^e_1,f^e_2$, and $\tau^m_3$ is the torque generated by a biarticular muscle force $f^e_3$ in Figure 1. For the comparison, the torques of joint actuators in Figure 2 are represented as $T^j_1$ and $T^j_2$ in this paper.

Under this description, the relation between these two kinds of torques is defined as the following.

$$\begin{pmatrix} T^j_1 \\ T^j_2 \end{pmatrix} = \begin{pmatrix} \tau^m_1 + \tau^m_3 \\ \tau^m_2 + \tau^m_3 \end{pmatrix} \tag{1}$$

2.2 Statics expressed by two types of Jacobian

The statics that represents the balance between force generated at the end-effector and the applied joint torques is usually described using Equation (2) where force $F_e$ in Figure 2 is described as $F_e = (f_x, f_y)^T$.

$$\begin{pmatrix} T^j_1 \\ T^j_2 \end{pmatrix} = J^c \begin{pmatrix} f_x \\ f_y \end{pmatrix} \tag{2}$$

$$J = \begin{pmatrix} -l_1 \sin \theta_1 - l_2 \sin(\theta_1 + \theta_2) - l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_2 \cos(\theta_1 + \theta_2) \end{pmatrix} \tag{3}$$

Now we define the statics for the muscle actuator configuration in Figure 3. However there is a redundancy - three torques to generate two dimensional force. In order to deal with this problem, we intentionally ignore the second monoarticular muscle torque $\tau^m_2$.

There is another reason to ignore $\tau^m_2$. By removing $\tau^m_2$, the comparison we address here becomes clear: when we are given two actuators, which configuration is more efficient for a two-link manipulator, two joint-actuators configuration where two joint torques $T^j_1, T^j_2$ generate $F_e$, or one joint motor and one biarticular motor configuration where one monoarticular muscle torque $\tau^m_1$ and one biarticular muscle torque $\tau^m_3$ generate the force? This configuration difference can be illustrated as Figure 4 and 5.

Under this $\tau^m_1, \tau^m_3$ configuration (Figure 5), the torques necessary to generate the force $F_e$ can be described as follows Oh [2009].

$$\begin{pmatrix} \tau^m_1 \\ \tau^m_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} T^j_1 \\ T^j_2 \end{pmatrix}$$
Fig. 6. Desired Force and Necessary Torques \( (\theta_2 = \frac{1}{3} \pi) \)

The lengths of two links \( l_1, l_2 \) are set same to 1m, \( \theta_2 \) is set to \( \frac{1}{3} \pi \) and \( \theta_1 \) to 0 to make the comparison easier.

The maximum necessary torques to generate 1N force at the endeffector are different in two cases. In the case of the biarticular muscle actuator which is represented by the green solid line, the maximum for the two actuators is the same 10Nm. On the other hand, in the case of the two joint-actuators that is represented by the red dashed line, the maximum for \( T^m_1 \) is more than 1.73Nm. This means we need a bigger actuator to generate the same force, when we use the two joint-actuator configuration.

Fig. 7. Desired Force and Necessary Torques \( (\theta_2 = \frac{1}{6} \pi) \)

Figure 7 is the necessary torques with \( \theta_2 = \frac{1}{6} \pi \). The maximum of \( T^m_1 \) is around 1.93Nm, which shows the less \( \theta_2 \) is, the larger \( T^m_1 \) torque becomes necessary. In a two-link manipulator that is supposed to work with various posture, the actuator for \( T^m_1 \) needs to have substantial torque output in order to deal with a force task under a small \( \theta_2 \) condition. On the other hand, with the biarticular muscle configuration, we can fully utilize the torques of two actuators regardless of \( \theta_2 \).

3. ANALYTICAL COMPARISON BY SIMPLIFIED STATICS EXPRESSION

3.1 Derivation of simplified statics

Since \( J_{abs} \) that describes the statics of the biarticular muscle configuration is fit for rotation transformation, the relationship between \( F_e \) and \( \tau_1^m, \tau_3^m \) can be more simplified when \( F_e \) are given as \( F_e = (F \cos \theta_f, F \sin \theta_f) \) (Oh [2009]).

\[
\begin{bmatrix} F_{m}^{e} \\ \tau_{m}^{1} \\ \tau_{m}^{3} \end{bmatrix} = (J_{abs})^T \begin{bmatrix} F \cos \theta_f \\ F \sin \theta_f \\ F l_2 \sin(\theta_f - \theta_{12}) \end{bmatrix} = \begin{bmatrix} F l_1 \sin(\theta_f - \theta_{1}) \\ F l_2 \sin(\theta_f - \theta_{12}) \end{bmatrix} \tag{6}
\]

In order to simplify Equation (6) more, the reference frame can be set along with the first link so that \( \theta_1 \) can be set to 0 without loss of generality, which makes Equation (6).

\[
\begin{align*}
\tau_1^m &= F l_1 \sin \theta_f \\
\tau_3^m &= F l_2 \sin(\theta_f - \theta_2) \tag{7}
\end{align*}
\]

Using this simplified \( \tau_1^m, \tau_3^m \), the statics in Equation (2) can also be transformed into the trigonometric functions. By substituting the required muscle torque in Equation (7) to Equation (1), the statics in Equation (2) is expanded as the following.
\begin{align}
T_1 &= F l_1 \sin \theta_f + F l_2 \sin(\theta_f - \theta_2) \\
T_2 &= F l_2 \sin(\theta_f - \theta_2),
\end{align}
(8)

This statics can also be more simplified as follows using the combination of sine functions with the parameters of \( l_m \) and \( \theta_m \) defined as Figure 8. Detailed derivation process is explained in Oh [2011].

\begin{align}
T_1 &= F l_m \sin(\theta_f - \theta_m) \\
T_2 &= F l_2 \sin(\theta_f - \theta_2),
\end{align}
(9)

Fig. 8. Parameters for Statics of Two-link Manipulator

3.2 Efficient force generation by biarticular muscle actuator

The comparison done in Section 2.3 using Jacobian can be revisited using the suggested simplified statics. Figure 9 is the required torques \( T_1 \) and \( T_2 \) in order to generate a force of 1N at the end effector with the direction of \( \theta_f \) under the configuration \( l_1 = l_2 = 1m, \theta_2 = \frac{\pi}{2} \). Necessary torques are illustrated with regard to the angle \( \theta_f \).

![Fig. 9. Necessary Torques to Generate a Force at the End effector](image)

This figure provides some general points that can be revealed only by this simplified statics. \( T_1 \) = \( T_2 \) when the force direction \( \theta_f \) is set to 0 (which is the direction of the first link) regardless of the parameters \( l_1, l_2, \theta_2 \), since \( l_m \sin \theta_m \) is same as \( l_2 \sin \theta_2 \) as is shown in Figure 8. There are four specific angles \( \theta_a, \theta_b, \theta'_a, \theta'_b; \theta_a = \theta_m \) is the angle where \( T_1 \) cannot contribute to the force at the end effector since the moment arm of the torque \( T_1 \) is zero (Figure 10 explains this characteristic), \( \theta_b = \theta_2 \) is the same angle for the case of \( T_2 \), while \( \theta'_a = \frac{\pi}{2} + \theta_m \) is the angle where \( T_1 \)

 contribute most to the force and \( \theta'_b = \frac{\pi}{2} + \theta_2 \) is the same angle for \( T_2 \).

Figure 10 shows these angles in terms of the robot configuration; \( \theta_a \) is same with \( \theta_m \), the direction in which the first joint and the end effectors are aligned and \( \theta_b \) is identified with \( \theta_2 \), the direction of the second link. \( \theta'_a \) and \( \theta'_b \) are the angles \( \frac{\pi}{2} \) away from \( \theta_a \) and \( \theta_b \). This relationship is in good agreement with the explanation of Figure 9.

Magnitude \( F l_m \) and \( F l_2 \) in Equation (9) explain the drawback of the joint actuator configuration. \( F l_m \) changes with regard to \( \theta_2 \) as in Figure 10. In the range of \( 0 \leq \theta_2 \leq \frac{\pi}{2} \), \( l_m \) is greater than \( l_2 \) (under the assumption \( l_1 = l_2 \)) which means larger torque is required for \( T_1 \) than \( T_2 \). When \( \theta_2 \geq \frac{\pi}{2} \), smaller \( T_1 \) is good enough to generate a specific force at the end effector.

This point is the significant difference between the statics strategies of the general two joint-actuators manipulator and the biarticular actuator manipulator. Compared with Equation (9), the magnitudes of the necessary torques are constant in Equation (7). This implies that the output of the actuator installed in the first joint can be utilized efficiently regardless of \( \theta_2 \), with the biarticular muscle actuator; the biarticular muscle torque can improve the efficiency of the actuators in terms of the force generation.

For example, in the worst case where \( \theta_2 \) is set to 0, the maximum torque required for \( T_1 \) is \( F l_1 + l_2 \), while the maximum torque required for \( T_2 \) is still \( F l_1 \) to generate the same force at the end effector.

4. VERIFICATION OF EFFICIENCY OF BIARTICULAR MUSCLE ACTUATOR BY EXPERIMENTS

4.1 Experimental setup

Six motors are used to realize the six muscles in Figure 1. Figure 11 is the side view of our experimental setup which shows the 6 motors. The outputs of the motors are connected to the two links through wires and pulleys. The lengths of two links are 0.112m all, and the radius of the pulleys is 22mm.

Two types of experiments are conducted. In the first experiment, the torque patterns to generate a force circle at the end effector are given to each motor and the input torques and the output force obtained. In the second experiment, the force is measured with one pair of motor torques fixed while the other pair of motor output varies.
4.2 Necessary torques to generate a certain pattern of force

In order to evaluate static force, the endeffector is constrained and connected to a force sensor. The configuration and coordinate is explained in Figure 13. The measured force is described according to this frame and in this coordinate is not 0 anymore.

In order to get a force circle with the magnitude of 9N, the torque patterns of $T^1_j$ and $T^2_j$ according to Equation (9) and the patterns of $\tau^m_1$, $\tau^m_3$ according to Equation (7) are given in each configuration with $F = 9$. $\theta_2$ is set to $\frac{\pi}{4}$. 36 points of $\theta_j$ from 0 to $2\pi$ are chosen, and the converged forces are measured with each $\theta_j$. Figure 14 and 15 are the results where the red circles are the desired force and the blue stars are measured force.

Even though there are some errors due to the transmission losses in the wires, the forces can be generated as desired. According to the coordinate definition in Figure 13, the vertical $y$ axis corresponds the angle $a'$ in Figure 9 where the biggest $T^1_j$ torque is necessary. Figure 14 and 15 verify that the biarticular muscle configuration can generate the same force with less torque output from the motor.

4.3 Comparison of force generations by joint actuator and biarticular muscle actuator

Different torque patterns are given here. First the torque for the first joint ($T^1_1$ and $T^1_3$) is fixed and the other torque ($T^2_1$ and $T^2_3$) is given from 0 to the same value of the first joint torque. The measured force is shown in Figure 16. Secondly, $T^3_1$ and $\tau^m_3$ are fixed and the first joint torque changes from 0 to the same value of the other torque. Figure 17 is the result.

The red circles are of the two joint-actuators configuration and the blue stars are of the biarticular muscle motor configuration. Lines are simulation results calculated from the suggested statics. Figure 16 indicates that $T^2_2$ cannot contribute any force in the $y$ direction in which direction the largest torque is necessary for $T^1_j$ while $\tau^m_3$ can. In Figure 17, even though the torques are same in $T^2_2$ and $\tau^m_3$, the force in the $y$ direction starts from the different positions, which means $\tau^m_3$ can contribute the force generation to where the first link torque needs to be large, resulting in the efficient force generation in all direction.

5. CONCLUSIONS

Here, the statics of a two-link manipulator under two different configurations - two joint-actuators configuration
and one joint motor and one biarticular muscle actuator configuration - are analyzed in a mathematical way and by several experiments.

The results show that the configuration with two joint-actuators has a drawback that it needs to have a large actuator for the first joint to generate a certain force, while the biarticular muscle configuration does not. The necessary maximum torque for the first joint is kept constant in the biarticular muscle configuration regardless of $\theta_2$. On the other hand, it changes leading to larger torque requirement in a certain range of $\theta_2$ in the two joint-actuator configuration.

As is said in Section 1, the advantage of the biarticular muscle is not only this efficient force generation. The biarticular muscle robotics must be a significant topic that should be studied more.

REFERENCES


