New Characteristics Analysis Considering Transmission Distance and Load Variation in Wireless Power Transfer via Magnetic Resonant Coupling

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Abstract - **Wireless power transfer (WPT) via magnetic resonant coupling has been attracting research attention for various applications. Conventionally, load is assumed to be constant and only transfer efficiency is studied. In actual WPT applications, the load and transfer distance change frequently. Furthermore, information such as ratio of input voltage to output voltage, ratio of input current to output current, and input impedance are needed for understanding and constructing the power transfer system. In this paper, not only the transfer efficiency but also the three parameters mentioned are studied. These parameters are then analyzed for changing load and changing transfer distance conditions using actual antennas' parameters. From the analysis results, the load resistance value for maximum efficiency exists. Secondly, improving efficiency by changing load resistance for small mutual inductance case has larger effect. The optimum load resistance also changes according to transmission distance and also the consumed power peak may not correspond to maximum efficiency. Finally, fault protection may also be necessary for the cases when the load resistance is extremely high and when the receiver antenna is not present causing high supply current.**

I. INTRODUCTION

Wireless Power Transfer (WPT) via magnetic resonant coupling which was first introduced in year 2006 has been receiving much attention from researchers and companies [1]. With this method high transfer efficiency is obtainable over relatively larger gap compared to induction method. Potential application includes charging electric vehicles [2] (Fig .1). Moreover the magnetic field in this power transfer method is non-radiative type and therefore is safe for the human body [3]. The resonant antennas used in wireless power transfer consist of coils and capacitors. Antenna design with Series-Series(SS) configuration where the coils and capacitors are connected in series is preferred in recent research [9]. Conventional studies assume the load is always constant and discussed the transfer efficiency only in terms of input power and output power [4]-[6] but not in terms of voltage and input impedance. Furthermore, the effects of changing load and changing transfer distant were not discussed [7]. However, in actual wireless power transmission applications, load and transmitting distance change frequently. Information such as the ratio of input to output voltage, the ratio of input to output current and input impedance seen from high frequency power supply are needed to understand and construct the power transfer system [8].

This paper presents not only the mathematical expression for transfer efficiency in SS-type magnetic resonant coupling but also the voltage ratio, current ratio and input impedance in terms of load and transmitting distance change. The derived equations are investigated using simulations. From these mathematical expressions, important information for wireless power transfer design such as voltage across the load and load impedance values during maximum efficiency are obtainable.

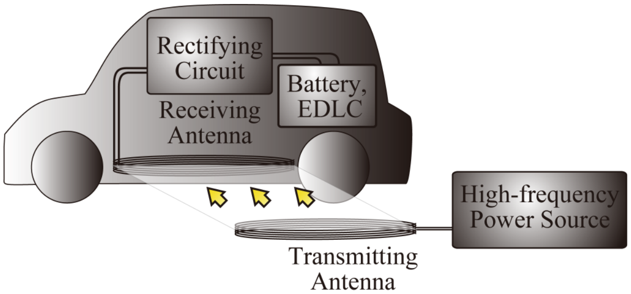


Fig. 1. Wireless power transfer for electric vehicle.

II. CHARACTERISTICS OF SELF RESONANT FREQUENCY

A. *Equivalent Circuit*

The resonant antennas used in WPT consist of coils and capacitors. Antenna design with SS configuration where the coils and capacitors are connected in series is preferred in recent research [9]. The equivalent circuit of magnetic resonance coupling method with SS configuration has already been proved in the past research [9], and is illustrated in Fig. 2. L_1 and L_2 represent the inductances of the coils. C_1 and C_2 represent the capacitances that are connected to coils in series. R_1 and R_2 are the antennas' internal losses. L_m is mutual inductance which is related to transfer distance. Transmitting antenna and receiving antenna satisfy (1). Term ω_0 in (1) is the self resonant frequency. The equivalent circuit of the wireless power transfer system is shown in Fig. 2.

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}} \tag{1}$$

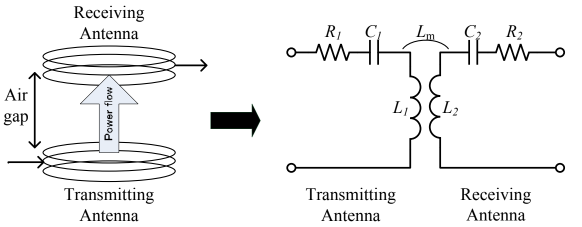


Fig. 2. Equivalent circuit of magnetic resonant coupling with SS configuration.

B. Definition of the Four Characteristics

In this section, the four characteristics used to study the phenomena of magnetic resonant coupling are defined. Fig. 3 shows T-type equivalent circuit of the wireless power transfer, and is terminated by a load and a power source on each side. Properties to be defined are A_V, A_I, A_P, Z_{in1}. Where A_V is ratio of output to input voltage and A_I is ratio of output to input current as shown by (2) and (3). A_P is the ratio of output to input power as shown in (4). A_P is the same as transmitting efficiency. Power ratio is the product of A_V and complex conjugate of A_I. Z_{in1} is the input impedance seen from the power source as shown in (5). From (2)-(5), the relation of power supply's voltage and power supply's current can be known.

$$A_V = \frac{V_2}{V_1} \tag{2}$$

$$A_I = \frac{I_2}{I_1} \tag{3}$$

$$A_P = \frac{V_2 \cdot \overline{I_2}}{V_1 \cdot \overline{I_1}} = \left(\frac{V_2}{V_1}\right) \cdot \overline{\left(\frac{I_2}{I_1}\right)} = A_V \cdot \overline{A_I} \tag{4}$$

$$Z_{in1} = \frac{V_1}{I_1} \tag{5}$$

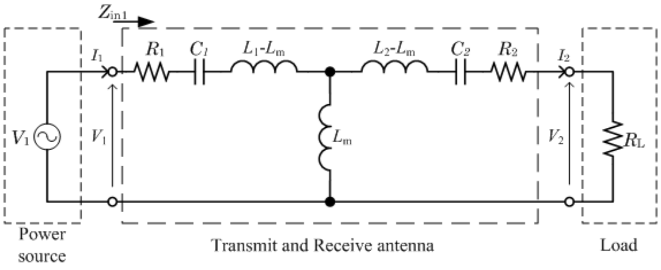


Fig. 3. Equivalent circuit of wireless power transfer system terminated by power supply and load at each side.

C. Equations of the Characteristics in Self Resonance

Next, each characteristics of SS-type magnetic resonant coupling is described. Equation (6) is derived from the equivalent circuit in Fig. 3. The elements in the matrix are described by (7), (8), and (9).

$$\begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{6}$$

$$Z_{11} = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \tag{7}$$

$$Z_{12} = Z_{21} = j\omega L_m \tag{8}$$

$$Z_{22} = R_2 + R_L + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) \tag{9}$$

The supply frequency is usually the same as antenna's self-resonant frequency given by (1). Substituting the self-resonant frequency, (2)-(5) become (10)-(13).

$$A_V\Big|_{\omega=\omega_0} = j\frac{\omega_0 L_m R_L}{R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2} \tag{10}$$

$$A_I\Big|_{\omega=\omega_0} = j\frac{\omega_0 L_m}{R_L + R_2} \tag{11}$$

$$A_P\Big|_{\omega=\omega_0} = \frac{(\omega_0 L_m)^2 R_L}{(R_L + R_2)\left(R_1 R_L + R_1 R_2 + (\omega_0 L_m)^2\right)} \tag{12}$$

$$Z_{in1}\Big|_{\omega=\omega_0} = \frac{V_1}{I_1} = R_1 + \frac{(\omega_0 L_m)^2}{R_L + R_2} \tag{13}$$

Equation (10) and (11) contain only imaginary component. This shows that voltage and current are phase shifted by 90 degree from input to output regardless of the load impedance and transmitting distance. On the other hand, (12) contains only real component. This shows that power waveform does not shift phase from input to output. Equation (13) also contains only real component showing that power factor is 100%

III. ANALYSIS OF CHANGING LOAD VALUE AND TRANSMITTING DISTANCE

A. Analysis Method

Using the derived equations, the change of each characteristic corresponding to load, R_L and transmitting distance (which affects mutual inductance L_m) is calculated. The antennas' parameters are set to be the same as the actual antennas. L_1 and L_2 are 800 uH, C_1 and C_2 are 2000 pF and R_1 and R_2 are 1.2 Ω. Table 1 shows the L_m values of a few transmitting distances.

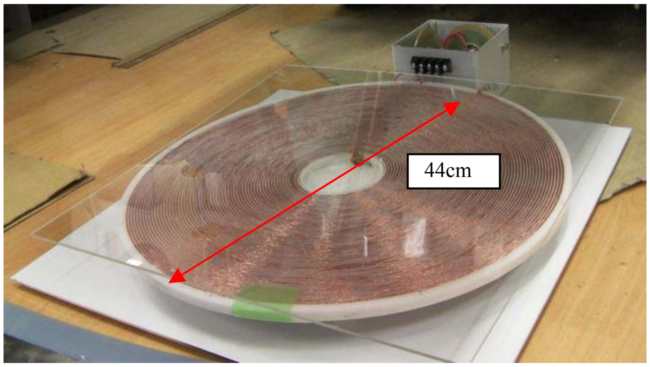


Fig. 4. The actual antenna with the parameters used in calculation.

Table 1 Lm vs transfer distance

L_m [uH]	Transfer distance [cm]
15	50
26	40
49	30
102	20
240	10

B. Analysis Result of Changing R_L

Fig. 5, Fig. 6, Fig. 7 and Fig. 8 show the plot of A_P, A_V, A_I and Z_{in1} correspondings to changing R_L.

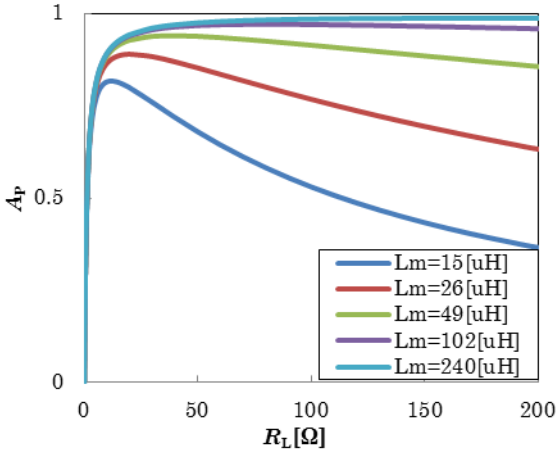
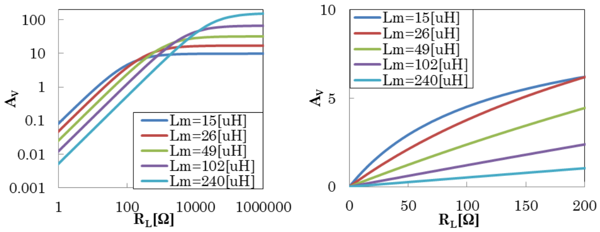
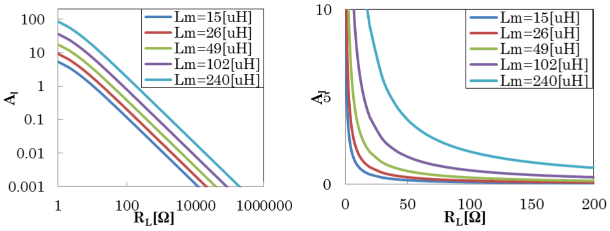


Fig. 5. A_P when R_L changes.



(a) logarithmic plot (b)linear plot

Fig. 6. A_V when R_L changes.



(a) logarithmic plot (b)linear plot

Fig. 7. A_I when R_L changes.

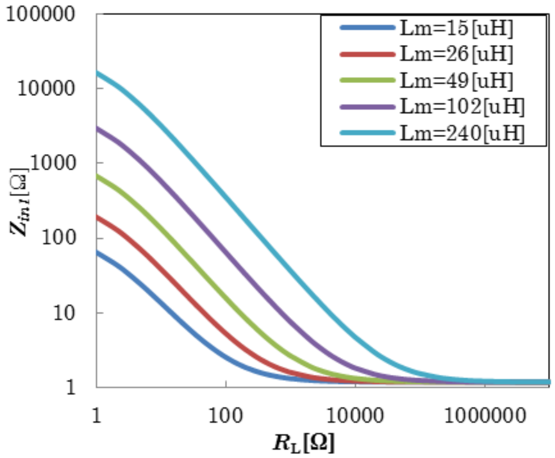


Fig. 8. Z_{in1} when R_L changes.

Fig. 5 shows A_P peaks at certain R_L values. Transmitting efficiency is affected by load resistance. Therefore optimizing load value for high efficiency is important. Moreover waveform of A_P is precipitous meaning that optimizing load resistance has larger effect when the transfer distance is far. Equation (14) describes R_{L_APmax}, and (15) describes A_{Pmax}. R_{L_APmax} is the load resistance during maximum efficiency. A_{Pmax} is maximam eficiency when R_L is optimized.

$$R_{L\_AP\max} = \sqrt{R_2\left(\frac{(L_m\omega_0)^2}{R_1}+R_2\right)} \tag{14}$$

$$A_{P\max} = \frac{(\omega_0 L_m)^2 R_{L(AP\max)}}{\left((\omega_0 L_m)^2+2R_1R_2\right)+2R_2\left((\omega_0 L_m)^2+R_1R_2\right)} \tag{15}$$

Fig. 6 shows that A_V is increasing with increasing R_L and becomes saturated when R_L reaches a certain value. When the primary side is powered, the voltage at the secondary side increases with the load resistance. This voltage may increase to a dangerous level without proper control. A_{V_sat} which is the saturation voltage ratio when R_L is infinite is expressed in (16).

$$A_{V\_sat} = \frac{\omega_0 L_m}{R_1} \tag{16}$$

Fig. 7 shows that A_I decreases with increasing R_L. From (4), A_P is the product of A_V and A_I. Furthermore, from Fig. 5 and Fig. 7, A_P decreases due to decreasing A_I when R_L is high whereas A_V is saturated at this point. On the other hand, efficiency declines when R_L is extremely low due to decreasing A_V.

Fig. 8 shows that Z_{in1} decreases when R_L increases. The input impedance is low when R_L increases and no load condition occur. Power supply current, I_1 increases. Result of Fig. 6 and Fig. 8 shows that no load (R_L is infinite) may caused damage to both the load and the power supply.

C. *Analysis Results of Changing L_m*

Fig. 9, Fig. 10, Fig. 11, and Fig. 12 show the plot of A_P, A_V, A_I, Z_{in1} respectively when L_m changes.

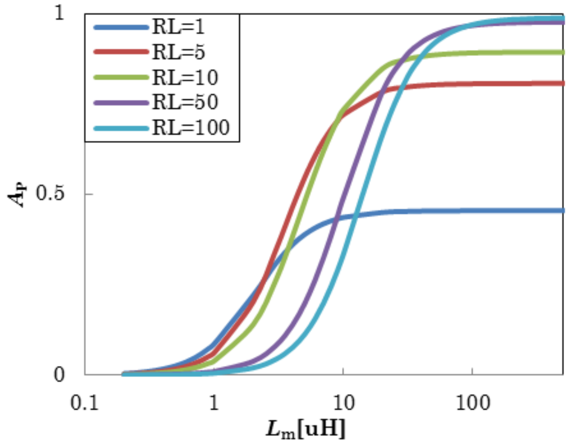


Fig. 9. A_P when L_m changes.

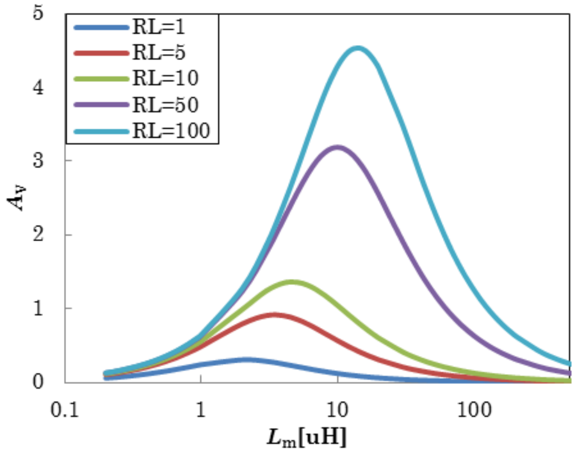


Fig. 10. A_V when L_m changes.

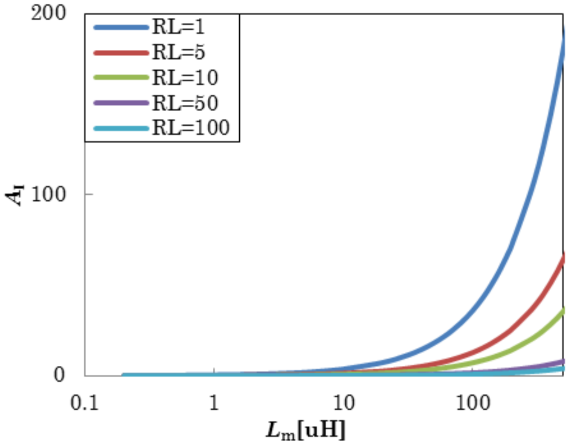


Fig. 11. A_I when L_m changes.

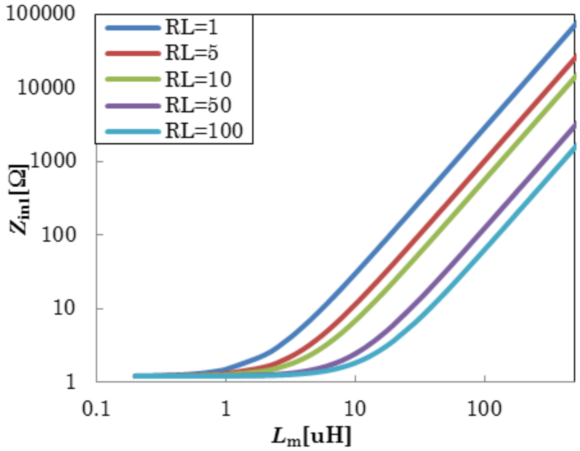


Fig. 12. Z_{in1} when R_L changes

Fig. 9 shows that A_P is large if L_m is large regardless of the value of R_L. This means that for small transfer distance, the efficiency is always high. Fig. 10 shows that A_V peaks at a certain value of L_m. In other words, secondary side's voltage is maximized at a fixed transfer distance. In this condition, the power consumption is maximized but not the transfer efficiency.

$$L_{m\_AV\max} = \frac{\sqrt{R_1(R_L + R_2)}}{\omega_0} \tag{17}$$

$$A_{V\max} = \frac{R_L}{2\sqrt{R_1(R_L + R_2)}} \tag{18}$$

Fig. 11 shows that A_I is large when L_m is large. When the transmission distance is near, efficiency increases due to increasing current. Fig. 12 shows that input impedance, Z_{in1} decreases with lower L_m. For further transmission distance, the input impedance is low as if the receiver antenna does not exist. Therefore, the power supply current (I_1) may increase to a dangerous level if there is no fault protection circuit.

IV. CONCLUSION

The characteristics of wireless power transfer which are the voltage ratio, current ratio, and input impedance and transfer efficiency are studied mathematically. The analysis is performed not only for constant load case, but also for changing load and changing transfer distant using the parameters of actual antennas.

From the analysis results, the optimal load resistance value for maximum efficiency exists. Secondly improving efficiency by changing load resistance for small mutual inductance case has larger effect. The optimum load resistance also changes according to transmission distance and also the consumed power peak may not correspond to maximum efficiency. Finally fault protection may also be necessary for the cases when the load resistance is extremely high and when the receiver antenna is not present causing high supply current.

Future work will include performing experiments to verify the mathematical analysis in this paper.

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