Multi-receiver and Repeater Wireless Power Transfer via Magnetic Resonance Coupling – Impedance Matching and Power Division Utilizing Impedance Inverter

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***Abstract*—Future applications of wireless power transfer will include powering various devices in a room, charging electric vehicles in a parking area, charging moving robots and so on. Therefore practical wireless power transfer must be able to support complicated configurations for example combination of multi-receiver and repeaters. Many past works have discussed on methods for improving efficiency and more recently extended the methods to multi-receiver system. However controllable power division among receivers is also an important feature as receivers nearer to the transmitter tend to absorb more power compared to further antennas. In this paper, a new impedance matching and power division method utilizing impedance inverters only at receiver sides is proposed. The mathematical equations in the proposed method are then generalized for arbitrary number of receivers and arbitrary number of repeaters.**

***Index Terms*—Magnetic Resonance Coupling, Multi-Receiver and Repeaters, Power Division, Wireless Power Transfer**

I. INTRODUCTION

An ideal wireless power transfer must be able to transfer power efficiently regardless of the receiving end at least in the effective region. However magnetic resonance coupling method is efficient only in a fixed distance and orientation. When the receiver is moved away from its optimal operating point, the efficiency falls off rapidly [1]. Furthermore, in a wireless power transfer consist of multiple receivers, the receiver nearer to the transmitter tend to absorb more power [2]. Many past papers have proposed different ways to resolve the efficiency issue but not on power distribution.

Paper [3]-[5] explored the possibilities of multi-receiver system using either equivalent circuit or coupled mode theory. Efficiency analysis at different conditions are provided but methods for improving efficiency and power distribution are not proposed. Often wireless power transfer is analyzed using equivalent circuit [3] [6]-[11], however the equations for system with more antennas quickly becomes complex or rigorous to be analyzed [3][9]. Therefore, band-pass filter representations is proposed by [12][13]. The design equations are very simple even with many repeaters added in the power transfer system. However the method is impractical due to inapplicable to multi-receiver and all the antennas' positions need to be controllable.

Other attempts include adding and adjusting a third coil to improve the transfer efficiency [14][15]. The method however is limited to specific case. Frequency tracking method where the frequency of the source is varied for different conditions has also been proposed [1][16][17]. Efficiency improvement using this method is only obtainable when the antennas are strongly coupled. In practical applications, the wireless power transfer should stay inside an allowable industrial, scientific and medical band which is narrow. Therefore, tuning frequency is not a feasible method for wireless power transfer. In [7][18], impedance matching circuit is inserted in the transmitter side based on equivalent circuit model. Transfer efficiency is optimized regardless of the receiving end. However controllable power distribution is not possible using this method.

In a multi-receiver system, not only the transfer efficiency is important but also the power distribution among the receivers. Power distribution depends on both the load impedances and the relative positions of the receivers to the transmitter. Assuming identical load connected to each receiver, the receiver nearer to the transmitter tend to absorb most of the power while the further receiver may not obtain enough to function properly. In this paper, method for impedance matching and controllable power division is proposed. The design equations are derived and then generalized for arbitrary number of receivers and arbitrary number of repeaters. Simulations using LTspice were performed to validate the new method.

II. MULTI-RECEIVER WIRELESS POWER TRANSFER

Fig. 1 shows an equivalent circuit of a two-receiver wireless power transfer. The coupling in between the top receiver and transmitter is k_{12} while the coupling in between the bottom receiver and transmitter is k_{13}. Cross coupling in between the two receivers is assumed to be zero in this method. The couplings between antennas are expressible in terms of impedance inverter [19]:

$$k_{12} = \frac{K_{12}}{\omega_0\sqrt{L_1 L_2}} \qquad k_{13} = \frac{K_{13}}{\omega_0\sqrt{L_1 L_3}}, \qquad (1)$$

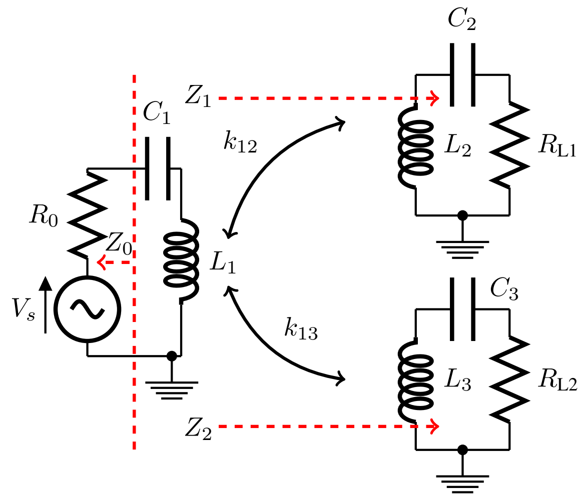

Figure 1. Equivalent circuit of a two-receiver system.

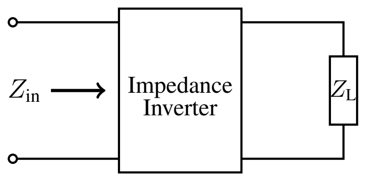

Figure 2. Operation of impedance inverter.

where K_{12} and K_{13} are inverters' characteristic impedance.

Impedance inverter as the name implies inverts the impedance connected to the inverter. Fig. 2 and (2) show the impedance Z_{in} looking into the impedance inverter that is connected to load, Z_L. There are many applications and many types of impedance inverter [20]. In this paper, impedance inverter is used to represent the couplings between antennas given by (1) and also impedance matching which will be explained in later part of this section.

$$Z_{\text{in}} = \frac{K^2}{Z_{\text{L}}}. \tag{2}$$

External coupling coefficient is the ratio of the resonator's termination resistance to the resonator's "reactance slope parameter" [12][19]:

$$k_{01} = \frac{R_0}{\omega_0 L_1} \qquad k_{23,1} = \frac{R_{\text{L1}}}{\omega_0 L_2} \qquad k_{23,2} = \frac{R_{\text{L2}}}{\omega_0 L_3}. \tag{3}$$

Assuming all the antennas posses similar resonant frequencies and that the power source is also operating at very near these resonant frequencies, the impedances of all the antennas are therefore ignorable. From Fig. 1, (1), (2) and (3):

$$Z_0 = R_0 = k_{01}\omega_0 L_1$$

$$Z_1 = \frac{{K_{12}}^2}{R_{\text{L1}}} = \frac{{k_{12}}^2}{k_{23,1}}\omega_0 L_1$$

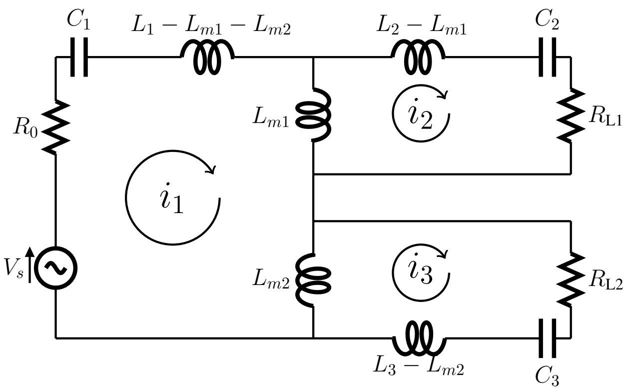

Figure 3. Alternative circuit of two-receiver wireless power transfer.

$$Z_2 = \frac{{K_{13}}^2}{R_{\text{L2}}} = \frac{{k_{13}}^2}{k_{23,2}}\omega_0 L_1. \tag{4}$$

Where Z_1 is the impedance looking from the source to the top receiver in Fig. 1 and Z_2 is the impedance looking from the source to the bottom receiver.

Circuit in Fig. 1 is redrawn in Fig. 3 with mutual inductance terms to derive impedance matching and power division equations. The current loop equations for Fig. 3 are given as:

$$\begin{aligned} V_s &= i_1 R_0 - i_2 j\omega_0 L_{\text{m1}} - i_3 j\omega_0 L_{\text{m2}} \\ 0 &= i_2 R_{\text{L1}} - i_1 j\omega_0 L_{\text{m1}} \\ 0 &= i_3 R_{\text{L2}} - i_1 j\omega_0 L_{\text{m2}}. \end{aligned} \tag{5}$$

Solving current i_2 and i_3 in terms of i_1 and replace the load impedances and mutual inductances with (1) and (3):

$$\frac{V_s}{i_1} = k_{01}\omega_0 L_1 + \frac{{k_{12}}^2}{k_{23,1}}\omega_0 L_1 + \frac{{k_{13}}^2}{k_{23,2}}\omega_0 L_1. \tag{6}$$

Equation (6) implies that circuit in Fig. 1 and Fig. 3 can be simplified into circuit shown in Fig. 4. The first term in (6) is the power supply termination impedance, the second term is impedance Z_1 and third term is impedance Z_2 in Fig. 1. Using maximum power transfer theorem [21], impedance matching is achieved when:

$$\begin{aligned} Z_0 &= Z_1 + Z_2 \\ k_{01} &= \frac{{k_{12}}^2}{k_{23,1}} + \frac{{k_{13}}^2}{k_{23,2}}. \end{aligned} \tag{7}$$

and because of same current flowing through Z_1 and Z_2, power division ratio is:

$$\begin{aligned} Z_1 &: Z_2 \\ \frac{{k_{12}}^2}{k_{23,1}} &: \frac{{k_{13}}^2}{k_{23,2}}. \end{aligned} \tag{8}$$

For controllable power division and impedance matching without having to change the antennas' positions, the external coupling coefficients of all the receivers should

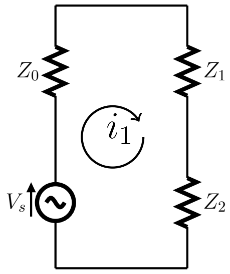

Figure 4. Simplified two-receiver circuit.

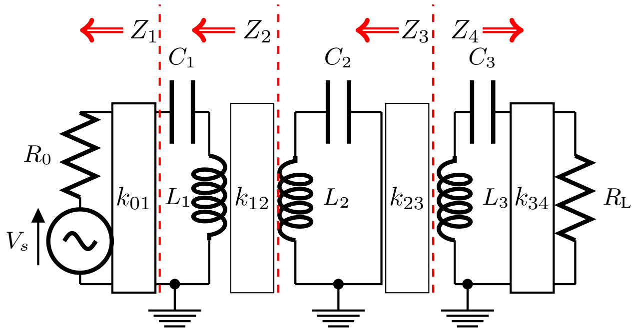

Figure 5. Equivalent circuit for wireless power transfer with repeater.

be modifiable. Impedance inverter circuits are inserted in between the receiver antennas and the load so that the new external coupling coefficients' values are [19]:

$$k_{23,1} = \frac{{K_{23,1}}^2/R_{L1}}{\omega_0 L_2} \qquad k_{23,2} = \frac{{K_{23,2}}^2/R_{L2}}{\omega_0 L_3}. \tag{9}$$

Where $K_{23,1}$ and $K_{23,2}$ are the characteristic impedances of the inverters to be implemented in the top receiver and bottom receiver respectively. The impedance inverter circuit implemented throughout this paper is the same as in [2].

In addition to deriving impedance matching and power division method, (6) also provides important information regarding multi-receiver wireless power transfer. Firstly, receivers appear as series connection viewing from the source. Secondly, assuming same antennas are utilized in the system, power distribution is proportional to square of coupling coefficients and inversely propotional to the load impedances.

III. New Impedance Matching Method for System with Repeaters

Although wireless power transfer via magnetic resonance coupling is able to trasmit power more efficiently compared to induction method, the transmittable distance is still limited to a few meters. This range is extendable by using repeater antennas. However as more antennas are added into the system, exisiting equivalent circuit equations quickly become complicated. Paper [22] resort to search algorithms and [9] uses computer aided design (CAD). Band-pass filter design method is simple but is impractical due to stringent conditions exerted by band-pass filter equations. In this paper, not only power division methods, but also new impedance matching method for wireless power transfer with repeaters is proposed. The new method combines the advantages of both existing equivalent circuit and band-pass filter method.

Consider a wireless power transfer with a repeater antenna in between the transmitter and reciver and cross coupling does not exist. The equivalent circuit is shown in Fig. 5. Assuming impedance matching is to be performed by modifying impedance Z_4 in Fig. 5, therefore

$$Z_4 = Z_3. \tag{10}$$

In order to satisfy (10), the required k_{34} is to be calculated. Using (2) and Fig. 5:

$$\begin{aligned} Z_1 &= R_0 \\ Z_2 &= \frac{{K_{12}}^2}{Z_1} \\ Z_3 &= \frac{{K_{23}}^2}{Z_2}. \end{aligned} \tag{11}$$

Again assuming the system is in resonance and therefore impedance of the antennas are ignorable. Using the same form of equations as in (1) and (3) and equalizing Z_4 to Z_3, the required k_{34} is derived:

$$k_{34} = k_{01} \times \frac{{k_{23}}^2}{{k_{12}}^2}. \tag{12}$$

An impedance inverter can be inserted in the box labelled k_{34} in Fig. 5 to realise (12).

IV. Generalization Equation for System With Both Multi-receiver and Repeaters

The equations derived in Sec. II and Sec. III are expandable for wireless power transfer with arbitrary number of repeaters and arbitrary number of receivers. Consider a wireless power transfer with m number of receivers and $(n-2)$ number of repeaters for each receiver in Fig. 6. Impedance inverters are inserted in between every receiver antenna and the corresponding load for impedance matching and power division. The number of repeaters in each transmission path need not be the same. Cross coupling between all the repeaters and receivers are asummed to be zero. External coupling coefficient k_{01} in (13) can be broken down into m-number of parts:

$$k_{01} = \frac{R_0}{\omega_0 L_1} = \sum_{i=1}^{i=m} k_{01,i}. \tag{13}$$

According to (8), the percentage of power received by each receiver will be:

$$i \mid_{i=1 \ to \ m} = \frac{k_{01,i}}{k_{01}} \times 100\%. \tag{14}$$

Finally, the external coupling coefficient of each receiver has to be:

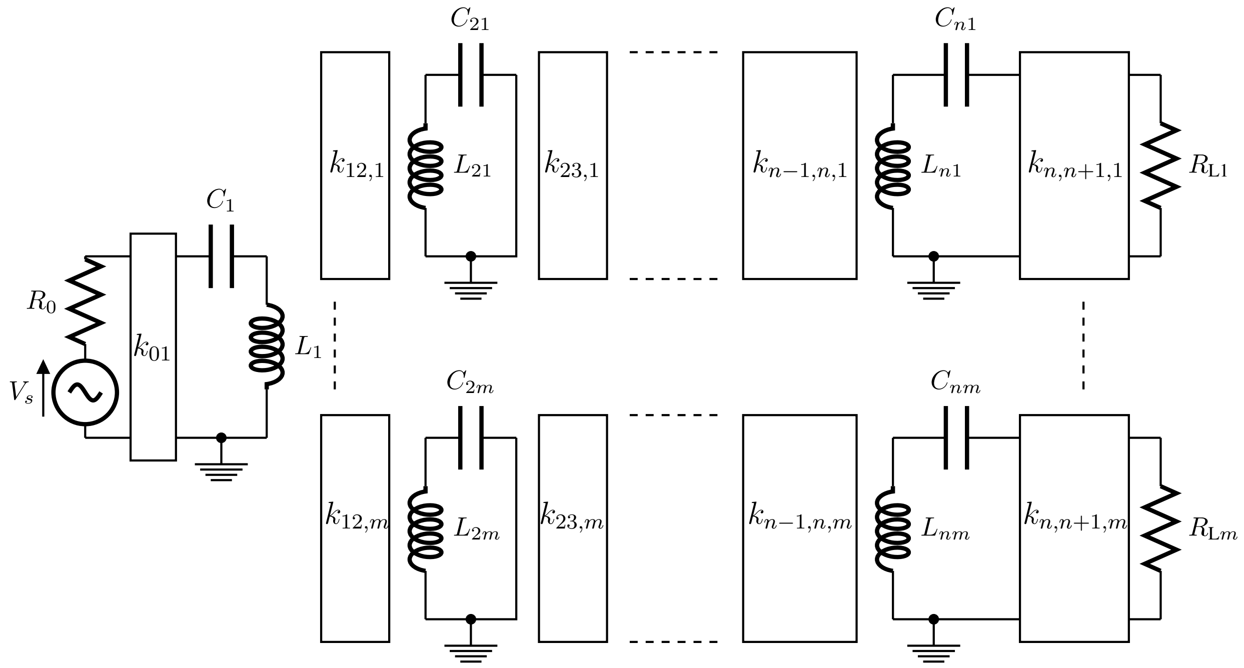

Figure 6. Wireless power transfer with arbitrary number of receivers and repeaters.

$$k_{n,n+1,i}\mid_{i=1\ to\ m} = k_{01,i}{}^{(-1)^{n+1}} \times \prod_{j=1}^{j=n-1} k_{j,j+1,i}{}^{2(-1)^{j+1+n}}. \tag{15}$$

Where $k_{n,n+1,i}\mid_{i=1\ to\ m}$ is realisable using impedance inverters:

$$k_{n,n+1,i}\mid_{i=1\ to\ m} = \frac{(Kn,n+1,i|_{i=1\ to\ m})^2/R_{Li}}{\omega_0 L_{ni}}. \tag{16}$$

V. Simulation Result

Calculations and simulation results of multi-receiver and repeater cases are given in this section to demonstrate the proposed power division method. The equivalent circuit of a two-receiver system and with one repeater inserted between the first receiver and the transmitter is simulated using LTspice. Cross coupling in between the receivers and repeaters are assumed to be zero in this paper. The element values of the system are listed below:

$$\begin{aligned}
\omega_0 &= 2\Pi \times 13.56\,\text{MHz}\\
L_1 &= L_{21} = L_{31} = L_{22} = 10\,\mu\text{H}\\
C_1 &= C_{21} = C_{31} = C_{22} = 13.8\,\text{pF}\\
R_0 &= R_{L1} = R_{L2} = 50\,\Omega\\
k_{12,1} &= k_{23,1} = 0.05\\
k_{12,2} &= 0.1.
\end{aligned}$$

The internal resistances of the antennas are set to zero in this simulation. The power ratio $P_1 : P_2$ for this first simulation is set to 1, where P_1 is the power received by load R_{L1} and P_2 is the power received by load R_{L2} . Using (13) to (16):

$$\begin{aligned}
k_{01} &= 0.0587\\
k_{01,1} &= k_{01,2} = 0.0293\\
k_{34,1} &= 0.0293\\
k_{23,2} &= 0.3413\\
K_{34,1} &= 35\,\Omega\\
K_{23,2} &= 121\,\Omega.
\end{aligned}$$

Fig. 7(a) shows the simulation result before applying the method. Due to impedance mismatched, the reflection ratio, η_{11} is around 35% at the resonance frequency 13.56 MHz. The transmission efficiency, η_{21} to the first receiver, which is separated with the transmitter by a repeater antenna is around 16%. The transmission efficiency, η_{31} to the third receiver is around 48%. The simulation result after inserting inverter circuits with characteristic impedance calculated above is shown in Fig. 7(b). Reflection ratio is suppressed to almost none, and both receivers obtain almost equalised power as desired.

In the second simulation, power ratio $P_1 : P_2$ is set to be $\frac{7}{3}$. Using the same calculation steps as the previous simulation case, the characteristic impedance of the inverters to be inserted in the first receiver and second receiver are respectively:

$$\begin{aligned}
K_{23,1} &= 42\,\Omega\\
K_{23,2} &= 156\,\Omega.
\end{aligned}$$

Fig. 8 shows the simulation result after applying the method. Similar to equal power distribution case, reflection

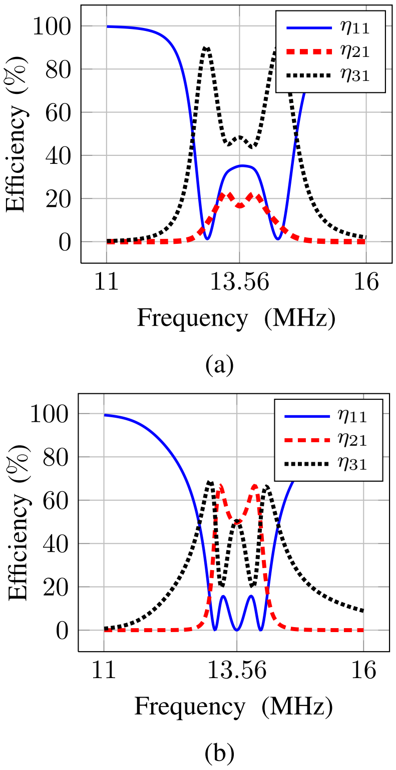

Figure 7. Simulation result of equal power division: a) before impedance matching. b) after impedance matching.

ratio, η_{11} is suppressed to almost none. Transmission efficiency, η_{21} to the first receiver is around 70% and transmission efficiency, η_{31} to the third receiver is around 30% as desired.

Above two simulation cases are performed without considering internal resistances of the antennas to validate the proposed equations. The equal power distribution case with internal resistances of all the antennas set to 1 Ω is simulated. Fig. 9 shows that the frequency response with 1 Ω internal resistance is close to the response of Fig. 7(b). Therefore it is reasonable to omit the values in the method's equations as magnetic resonance coupling antennas should have low internal resistances [23]. Reflection ratio, η_{11} is still close to none, however both transmission efficiencies are lower in this case with η_{21} equal to 46.7% and η_{31} equal to 48.5%. Derivation of loss equations for various configuration of wireless power transfer is reserved for future work.

VI. Conclusion

In a multi-receiver wireless power transfer via magnetic resonance coupling, not only high transfer efficiency but also controllable power division is important. New impedance matching and power division method is proposed and generalized for arbitrary number receivers and arbitrary number of repeaters. Due to systematic circuit representation using impedance inverter, the design equations are simple. Impedance matching and power division conditions are in terms of coupling coefficients. In order to exert controllable power division without having to change the antennas' position, the external coupling coefficients of the receivers can be modified by inserting impedance inverter circuit in between receiver and the corresponding termination resistor.

Future work of this research includes derivation of loss equations, cross coupling consideration and construction of automatic impedance matching and power division system.

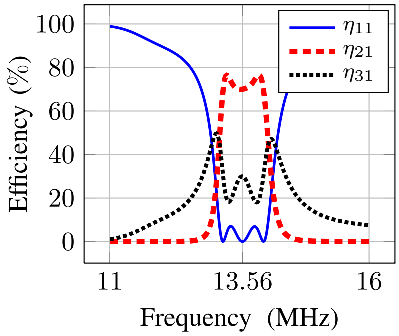

Figure 8. Simulation result of 70%-30% power division after impedance matching.

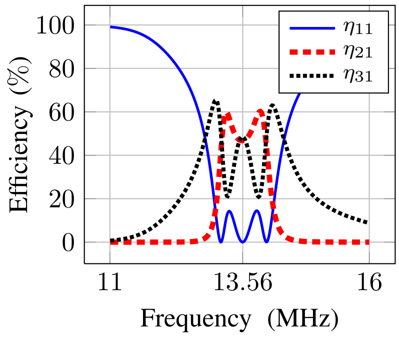

Figure 9. Simulation result of equal power distribution considering internal resistance after impedance matching.

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