Novel Band-Pass Filter Model for Multi-Receiver Wireless Power Transfer via Magnetic Resonance Coupling and Power Division

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***Abstract*—Recently medium range wireless power transfer had been extensively researched for applications such as consumer electronics products, portable devices, robotics and electric vehicles. Coupled-mode theory and equivalent circuit model representations are the more recognized models used to describe and design the system mathematically. Band-pass filter model is relatively new, using this model the physical wireless power transfer system is representable in relatively simpler equations compared to coupled-mode theory and equivalent circuit model. Methodology for multi-receiver is derived using band-pass filter model and impedance matching is achieved. Newly proposed methodology allows controllable power division among receivers. Controllable power division is a very important feature for an effective wireless power transfer system in real applications. When powering multiple devices, the devices nearer to the transmitter tend to absorb more power compared to the farther devices, past literature had never addressed this issue of wireless power transfer system. With this new methodology, not only impedance matching is achieved, but also the ratio of power delivered to each receiver end is controllable.**

I. INTRODUCTION

Power transmission is defined as a three-step process in which: 1) dc electrical power is converted into RF power, 2) the RF power is then transmitted through space to some distant point, and 3) the power is collected and converted back into DC power at the receiving point [1]. In early history, the most notable work on power transmission by radio waves was done by Nikola Tesla around the end of 19th century. He was interested in the concept of resonance and sought to apply the principle to the transmission of electrical power from one point to another without wires. After Nikola Tesla, there was not much interest on wireless power transfer due to the lack of high frequency power supply and lack of device to convert the high frequency power into DC power.

Unlike the time of Tesla, existing electrical-wire grid is able to transmit power almost everywhere, every building and every room. Therefore wireless power transfer with medium range capability of few tens of centimetres to few metres is sufficient for many applications. In year 2006, Witricity team in Massachusetts Institute of Technology lighted a 60-watt light bulb that was placed two metres away from the power source without any wired connection in between [2]. The wireless transfer of energy in this experiment is based on the principle of magnetic resonant coupling which is an efficient wireless non-radiative mid-range energy transfer method. This experiment had ever since ignited extensive research interest in mid-range wireless power transfer for various applications such as portable electronics devices, robotics, electric vehicles [3], medical sensors and implantable devices [4] and position sensing [5].

Along with the introduction of mid-range wireless power transfer technology, coupled mode theory which is based on oscillation and propagating wave theory is proposed to explain the exchange of energy between antennas [2] [6]. Later equivalent circuit is introduced and is utilised extensively to model wireless power transfer system and to interface with the electrical portion of the system [7]–[13]. A desirable wireless power transfer system for various applications must at least have minimal transmission loss, selectivity and ability to support many loads simultaneously. Using equivalent circuit model, the transmission equation for multiple-load system quickly becomes complex or rigorous to be analysed [9] [11].

Therefore band-pass filter model is proposed in this paper. The physical wireless power transfer is representable in relatively simpler equations compared to coupled-mode theory and equivalent circuit model. Methodology for multi-receiver is derived using band-pass filter model and impedance matching is achieved. Newly proposed method allows controllable power division among receivers thus fulfilling the mentioned conditions of selectivity, minimal transmission loss and ability to support multiple loads. Section II of this paper explains the representation of a wireless power transfer system with a band-pass filter circuit, Section III details the impedance matching method using band-pass filter circuit model. Section IV explains the extension of impedance matching method in Section III for multi-receiver system and power division methodology. Next, simulation results of power division methodology are given in the result section.

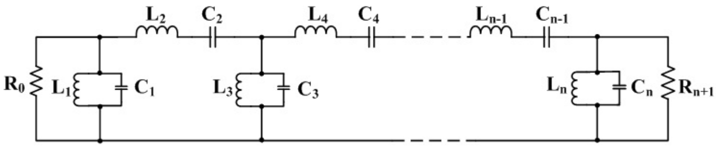

Fig. 1. Generic Band-pass Filter.

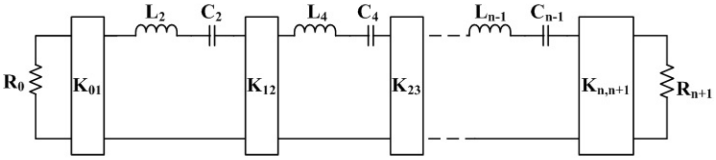

Fig. 2. Alternative Structure of Band-pass Filter Using Only Series Resonators and Impedance Inverter.

II. BAND-PASS FILTER MODEL REPRESENTATION

Fig. 1 shows a generic band-pass filter, which consists of series resonators and alternating with shunt resonators. This arrangement is avoidable by the use of "impedance inverter". An example is given in Fig. 2 where all the shunt resonators are substituted with impedance inverters and all the remaining resonators are of series type. Impedance inverter as the name implies invert the load impedance or load admittance connected to it, and therefore is usable to transform shunt-connected elements to series-connected elements or vice versa [14]. Fig. 3 and (1) shows the impedance, Z_{in} looking into the impedance inverter that is connected to a load, Z_{L}.

$$Z_{\text{in}} = \frac{K^2}{Z_{\text{L}}} \tag{1}$$

where:

K = Characteristic Impedance of Inverter

This alternative structure of band-pass filter will be used to model the wireless power transfer system. Fig. 4 explains the analogy of a physical one transmitter to one receiver wireless power transfer system to this type of band-pass filter circuit. Power source with internal termination resistor is represented by a termination resistor, R_0, couplings are represented by impedance inverters, K_{01}, K_{12} and K_{23}, antennas are represented by series resonators, L_1, C_1 pair and L_2, C_2 pair and finally the load is represented by termination resistor, R_{L}. The impedance inverter in between resonators is not a physical circuit element but is virtualised to represent the coupling in between antennas. Note that new couplings between power supply or load and resonator antennas are also defined in this methodology. In [15], the relation of termination resistor to their adjacent resonators is termed as "External Quality Factor", *Q*. The reciprocal of this quantity expresses the coupling strength between the termination resistor and resonator and is termed as "External Coupling Coefficient", *k* [16].

The equations relating coupling coefficients, *k* to impedance inverter's characteristic impedance, *K* are given in (2). Any type of filters namely high-pass filters, low-pass filters, band-stop filters and band-pass filters are derivable from corresponding low-pass prototype filters. For instance, a Butterworth response, three-stage band-pass filter is derivable from a Butterworth response, three-stage low-pass prototype filter. Once the response of the filter is fixed, the low-pass prototype filter element values in (2) is obtainable from filter design tables in any filter design book [15].

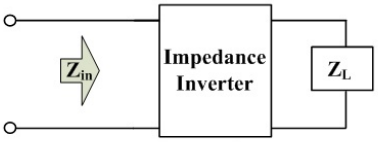

Fig. 3. Operation of Impedance Inverter.

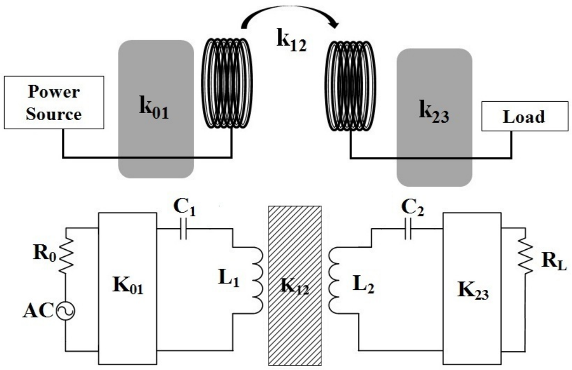

Fig. 4. Analogy of a Wireless Power Transfer System to Band-pass Filter Circuit.

$$\begin{aligned} k_{01} &= \frac{K_{01}^2/R_0}{\omega_0 L_1} = \frac{w}{g_0 g_1} \\ k_{12} &= \frac{K_{12}}{\omega_0 \sqrt{L_1 L_2}} = \frac{w}{\sqrt{g_1 g_2}} \\ k_{23} &= \frac{K_{23}^2/R_{\text{L}}}{\omega_0 L_2} = \frac{w}{g_2 g_3} \end{aligned} \tag{2}$$

where:

ω_0 = Resonance Angular Frequency

w = Fractional Bandwidth of the Filter Response

g_0, g_1, g_2 = Low Pass Prototype Filter Element Values

III. IMPEDANCE MATCHING METHODOLOGY USING BAND-PASS FILTER MODEL

The design of wireless power transfer system differs from the design of filters for other applications in the sense that the response is already fixed beforehand by the gap in between transmitting antenna and receiving antenna. The shape of the filter response such as roll-off rate, bandwidth and ripple are not significant in wireless power transfer system, the only condition that matters is obtaining close to zero attenuation in frequency region around the resonance frequency. Therefore the filter's response type used is Butterworth response which

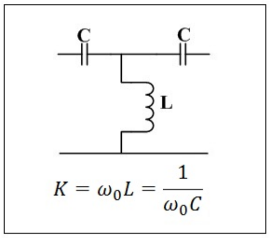

Fig. 5. Impedance Inverter Circuit used.

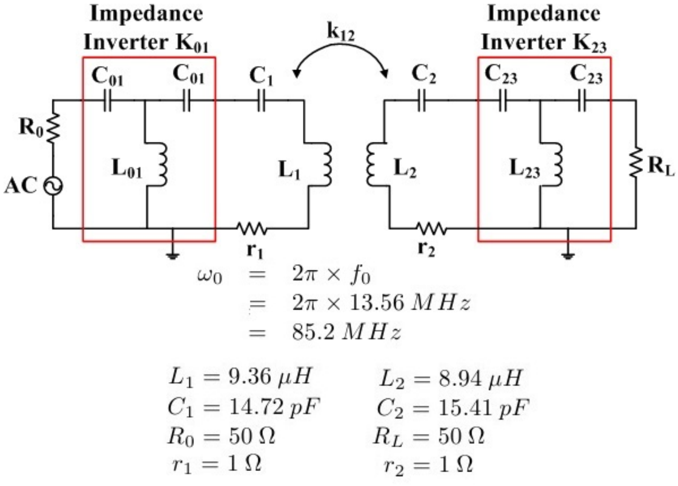

Fig. 6. Simulation Circuit used in LTSpice.

is the simplest form, (3) gives the low pass prototype filter element values in (2) for this type of response.

$$g_0 = 1 \qquad g_1 = \sqrt{2} \qquad g_2 = \sqrt{2} \qquad g_3 = 1 \tag{3}$$

Substituting (3) into (2), the coupling coefficients are obtained as below:

$$k_{01} = k_{12} = k_{23}. \tag{4}$$

The coupling between antennas, k_{12}, is fixed by the gap, the external couplings k_{01} and k_{23} are modified so that (4) is fulfilled. The external couplings are modifiable by changing the characteristic impedance of the impedance inverters, [17] lists some ways to realise an impedance inverter. The impedance inverter circuit used in this paper is shown in Fig. 5 where the equation relating the characteristic impedance to the lumped element values is also given. Calculations and simulations using LTspice are performed for two different coupling coefficients, k_{12} to demonstrate this impedance matching method. Circuit parameters are given in Fig. 6. Both r_1 and r_2 are the intrinsic resistances of the antennas, these parameters are included in the simulations so that the simulation results will be as close as possible to actual system. The antennas simulated are with 5 turns, 15.5 cm radius, 5 mm pitch, open type spiral antennas and resonate at 13.56 MHz.

A. *Case I:* k_{12}*=0.04*

Using (4), impedance matching is achieved when:

$$k_{01} = k_{23} = k_{12} = 0.04.$$

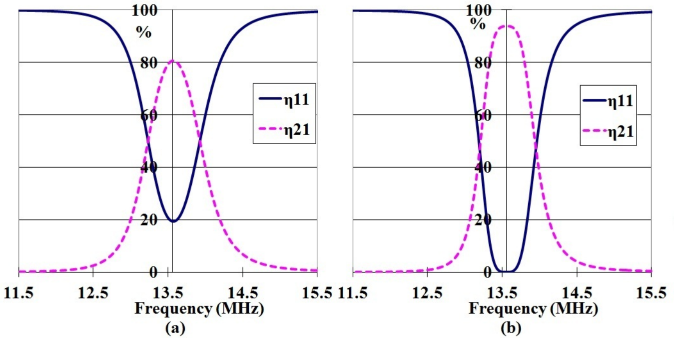

Fig. 7. Simulation Result of Case I: a) Before Matching. b) After Matching.

Using (2), the required characteristic impedances of the two inverters are calculated to be:

$$\begin{aligned} K_{01} &= \sqrt{k_{01}\omega_0 L_1 R_0} = 39.94\,\Omega \\ K_{23} &= \sqrt{k_{23}\omega_0 L_2 R_{\mathrm{L}}} = 39.03\,\Omega. \end{aligned}$$

Finally, the required capacitances and inductances of the two impedance inverters are calculated to be:

$$\begin{aligned} C_{01} &= \frac{1}{\omega_0 K_{01}} = 294\,pF \\ L_{01} &= \frac{K_{01}}{\omega_0} = 0.47\,\mu H \\ C_{23} &= \frac{1}{\omega_0 K_{23}} = 301\,pF \\ L_{23} &= \frac{K_{23}}{\omega_0} = 0.46\,\mu H. \end{aligned}$$

Fig. 7 shows the simulation results of before impedance matching and after impedance matching. Reflection ratio, η_{11} is reduced to almost zero, if not none and transmission ratio, η_{21} is improved significantly around the resonance frequency, 13.56 MHz after impedance matching. The intrinsic resistances of both antennas are accounted for the transmission loss that is still exist after impedance matching.

B. *Case II:* k_{12}*=0.15*

Using the same calculation method of Case I:

$$\begin{aligned} &k_{01} = k_{23} = k_{12} = 0.15 \\ &K_{01} = 77.34\,\Omega \quad K_{23} = 75.58\,\Omega \\ &C_{01} = 152\,pF \quad L_{01} = 0.91\,\mu H \\ &C_{23} = 155\,pF \quad L_{23} = 0.89\,\mu H. \end{aligned}$$

Fig. 8 shows the simulation result of before impedance matching and after impedance matching. Similar to case I, reflection ratio is reduced to almost zero, if not none and transmission ratio is improved significantly around the resonance frequency after impedance matching.

IV. POWER DIVISION METHODOLOGY

The concept of defining and modifying coupling coefficients in between termination resistances and adjacent resonators is extendable to multi-receiver wireless power transfer system.

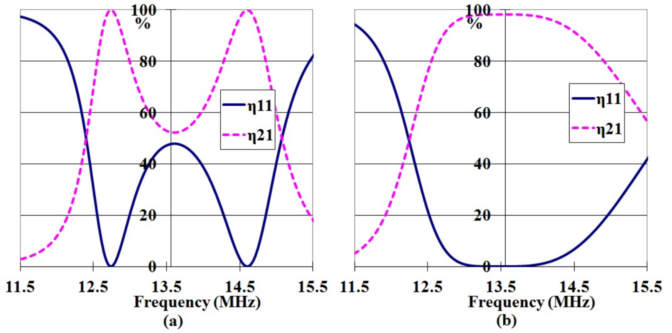

Fig. 8. Simulation Result of Case II: a) Before Matching. b) After Matching.

Besides impedance matching to suppress reflection power, the very significant feature of this methodology is allowing controllable power division among receivers. In a multi-receiver system, the receiver placed nearer to the transmitter antenna tends to extract most of the supply power leaving very little power to the other load. This issue can be seen in simulation result in the next section. The dependency of power distribution among loads to relative position to transmitter antenna is not desirable in a practical wireless power transfer system.

Consider a two-receiver wireless power transfer system represented by the circuit diagram of Fig. 9. In order to distinct the two receivers, all circuit parameters of the top antenna are subscripted with ‘a’ and subscripted with ‘b’ for the bottom antenna. The external coupling coefficient in between power supply’s termination and transmitter resonator antenna, k_{01} is also differentiated into two parts, $k_{01\mathrm{a}}$ and $k_{01\mathrm{b}}$. The reason of doing this is the two receivers needed to be matched individually to the transmitter antenna. Cross coupling k_{ab} in the circuit diagram is assumed to be zero in this paper. The steps to perform impedance matching and power distribution are listed as below.

Step 1: Match one of the receiver antennas to the transmitter. Using band-pass filter circuit model, there are actually two ways to perform impedance matching. One of the method is already discussed in section III that is modifying external coupling coefficients so that (4) is fulfilled. This method is simple, however the equation exert constraint for multi-receiver system as will be seen in later steps. Another way is still using band-pass circuit model but performing equivalent circuit matching. This matching method will be explained in next subsection “Equivalent Circuit Matching”. Assuming using either way, $k_{01\mathrm{a}}$ and $k_{23\mathrm{a}}$ had been determined with a given $k_{12\mathrm{a}}$ fixed by the gap between receiver ‘a’ and the transmitter antenna.

Step 2: Calculate $k_{01\mathrm{b}}$:

$$\begin{aligned}&\text{Desired ratio of power division, } x\\ &= \frac{\text{power received by antenna ‘b’}}{\text{power received by antenna ‘a’}}\end{aligned}$$

$$k_{01\mathrm{b}} = x \times k_{01\mathrm{a}}. \tag{5}$$

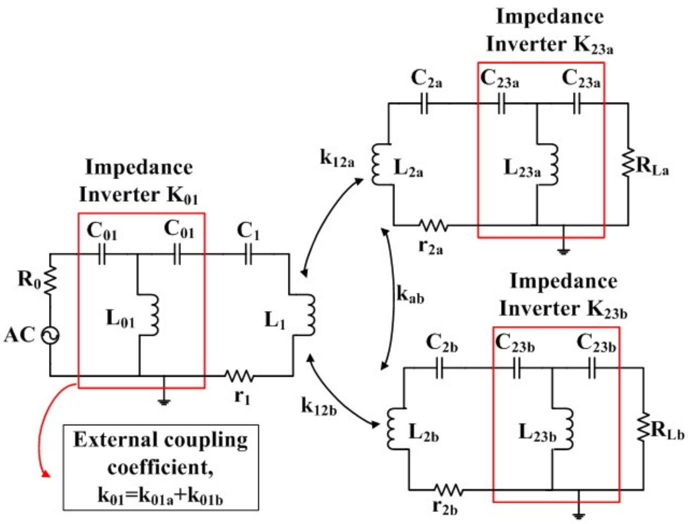

Fig. 9. Circuit of Two Receivers System.

Step 3: Match receiver ‘b’ to the transmitter. Since $k_{12\mathrm{b}}$ is already fixed by the gap in between receiver ‘b’ and transmitter and $k_{01\mathrm{b}}$ is fixed in step 2, the only parameter that is utilisable is $k_{23\mathrm{b}}$. Therefore the only way to perform impedance matching for this transmission path is using equivalent circuit matching which will be explained in the next subsection.

Step 4: Calculate k_{01}:

$$k_{01} = k_{01\mathrm{a}} + k_{01\mathrm{b}}. \tag{6}$$

Step 5: Using obtained k_{01}, $k_{23\mathrm{a}}$ and $k_{23\mathrm{b}}$, calculate all the required lumped element values in impedance matching\inverter circuits using (2) and equation in Fig. 5.

A. Equivalent Circuit Matching

The band-pass filter circuit model of Fig. 4 is redrawn in Fig. 10 and impedances viewing from points that will be used in calculations are labelled. Assuming step 1 and step 2 in the power division methodology are done and external coupling coefficient in between load and receiver, k_{23} is the only parameter that is adjustable for impedance matching. In Fig. 10, the impedance to the left of point ‘Y’, Z2 is defined as the source impedance and the impedance to right of point ‘Y’, Z3 is defined as the load impedance. Any point on the circuit can be chosen to be the “breaking point”, point ‘Y’ is chosen for convenience in this case. In order to obtain maximum power transfer from one stage of the circuit to another stage with no power reflections, the load impedance must equal to the source impedance [18]. Therefore in this impedance matching method, Z3 is equalised to Z2 through (7) to (9).

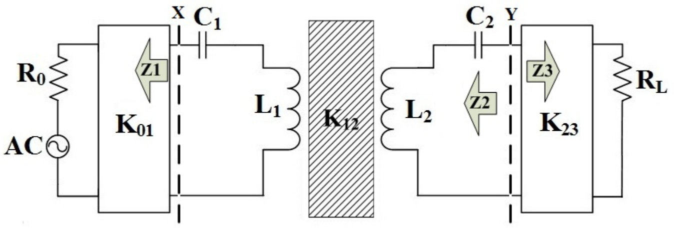

Fig. 10. Band-pass Circuit Model Redrawn.

By (1), $Z1$ and $Z2$ are:

$$\begin{aligned} Z1 &= \frac{K_{01}^2}{R_0} \\ Z2 &= \frac{K_{12}^2}{Z1} \\ &= \frac{K_{12}^2}{K_{01}^2/R_0}. \end{aligned} \tag{7}$$

Where the impedances of series resonators are assumed to be very close to zero and are ignorable. To achieve impedance matching:

$$\begin{gathered} Z3 = Z2 \\ \frac{K_{23}^2}{R_\mathrm{L}} = \frac{K_{12}^2}{K_{01}^2/R_0}. \end{gathered} \tag{8}$$

Substituting (2) into (8):

$$K_{23} = k_{12} \times \sqrt{\frac{\omega_0 L_2 R_\mathrm{L}}{k_{01}}}. \tag{9}$$

Where k_{01} and k_{12} are obtained from step 1 and step 2 of power division methodology.

V. RESULT

Calculation and simulation results of two multi-receiver cases are given in this section to demonstrate the newly proposed power division methodology.

A. *Case I: Equal Power Division*

Simulation circuit in LTspice is as shown in Fig. 9. Given below circuit parameters, the objective is to achieve impedance matching and equal power distribution between the two receiver antennas.

$$\omega_0 = 2\pi \times 13.56\,MHz = 85.2\,MHz$$

$$\begin{array}{ccc} L_1 = 9.36\,\mu H & L_\mathrm{2a} = 8.94\,\mu H & L_\mathrm{2b} = 9.06\,\mu H \\ C_1 = 14.72\,pF & C_\mathrm{2a} = 15.41\,pF & C_\mathrm{2b} = 15.21\,pF \\ R_0 = 50\,\Omega & R_\mathrm{1a} = 50\,\Omega & R_\mathrm{1b} = 50\,\Omega \\ r_1 = 1\,\Omega & r_{2a} = 1\,\Omega & r_{2b} = 1\,\Omega \\ k_\mathrm{12a} = 0.04 & k_\mathrm{12b} = 0.15 & k_\mathrm{ab} = 0 \end{array}$$

Step 1: As $k_\mathrm{12a} = 0.04$, by using (4):

$$k_\mathrm{01a} = k_\mathrm{23a} = 0.04.$$

Step 2: Using (5):

$$\begin{aligned} &x = 1 \\ &k_\mathrm{01b} = x \times k_\mathrm{01a} = 0.04. \end{aligned}$$

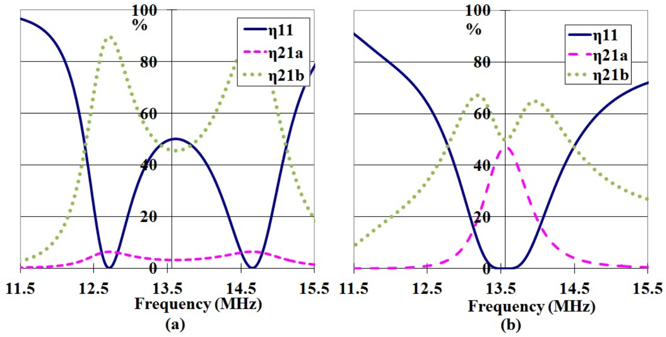

Fig. 11. Simulation Result of Power Division Case I: a) Before Matching. b) After Matching.

Step 3: Using (9):

$$\begin{aligned} K_\mathrm{23b} &= k_\mathrm{12b} \times \sqrt{\frac{\omega_0 L_\mathrm{2b} R_\mathrm{Lb}}{k_\mathrm{01b}}} \\ &= 147\,\Omega. \end{aligned}$$

Step 4: Using (6), $k_{01} = 0.08$.

Step 5: Calculate all the lumped element values of the three impedance inverters:

$$\begin{aligned} K_{01} &= \sqrt{k_{01}\omega_0 L_1 R_0} = 56.48\,\Omega \\ K_\mathrm{23a} &= \sqrt{k_\mathrm{23a}\omega_0 L_\mathrm{2a} R_\mathrm{La}} = 39.03\,\Omega \\ K_\mathrm{23b} &= 147\,\Omega\ \text{(from step 3)} \end{aligned}$$

$$\begin{aligned} C_{01} &= \frac{1}{\omega_0 K_{01}} = 208\,pF \\ L_{01} &= \frac{K_{01}}{\omega_0} = 0.66\,\mu H \\ C_\mathrm{23a} &= \frac{1}{\omega_0 K_\mathrm{23a}} = 301\,pF \\ L_\mathrm{23a} &= \frac{K_\mathrm{23a}}{\omega_0} = 0.46\,\mu H \\ C_\mathrm{23b} &= \frac{1}{\omega_0 K_\mathrm{23b}} = 80\,pF \\ L_\mathrm{23b} &= \frac{K_\mathrm{23b}}{\omega_0} = 1.73\,\mu H. \end{aligned}$$

Fig. 11(a) shows the simulation result when there are no impedance matching\inverter circuit inserted. Due to impedance mismatched, the reflection ratio, η_{11} is around 50%. Antenna 'b' which possesses stronger coupling with the transmitter antenna compared to antenna 'a' absorbs most of the remaining power. The simulation result after power division methodology is shown in Fig. 11(b), reflection ratio is suppressed to almost none, and both antennas received almost equalised power as desired.

B. *Case II: 80% - 20% Power Division*

In order to demonstrate the effectiveness of this power division methodology, a case where 80% of the power is forced to the farther antenna from the transmitter and remaining 20%

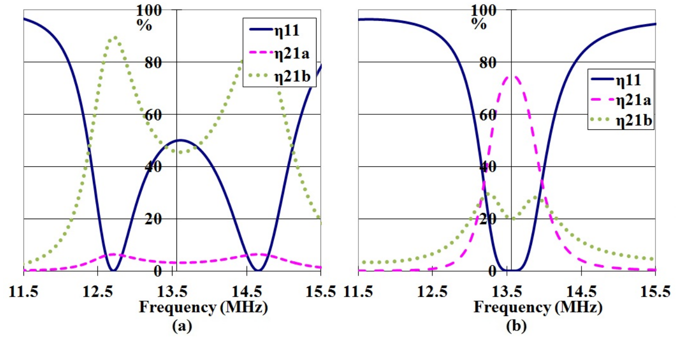

Fig. 12. Simulation Result of Power Division Case II: a) Before Matching. b) After Matching.

to the nearer antenna is simulated. All the circuit parameters are the same as in case I except for the power division ratio. Using the same calculation method of Case I power division:

$$\begin{array}{ccc} k_{01\mathrm{a}} = 0.04 & k_{23\mathrm{a}} = 0.04 & x = 4 \\ k_{01\mathrm{b}} = 0.01 & K_{23\mathrm{b}} = 295\,\Omega & k_{01} = 0.05 \\ K_{01} = 44.65\,\Omega & K_{23\mathrm{a}} = 39.03\,\Omega & C_{01} = 263\,pF \\ L_{01} = 0.52\,\mu H & C_{23\mathrm{a}} = 301\,pF & L_{23\mathrm{a}} = 0.46\,\mu H \\ C_{23\mathrm{b}} = 40\,pF & L_{23\mathrm{b}} = 3.46\,\mu H. & \end{array}$$

Similar to result of Case I power division, impedance matching is achieved. After power division methodology exerted, the farther receiver antenna from transmitter antenna obtains almost 80 % of the power and the nearer antenna obtains 20 % as desired. One interesting observation from simulation results of both cases is that although power division is exerted, the farther antenna tends to suffer more loss.

VI. CONCLUSION

Recently medium range wireless power transfer via magnetic resonance coupling is researched extensively for various applications. Band-pass filter circuit model is proposed to represent the wireless power transfer system. Impedance matching is performed by modifying the external coupling coefficients using impedance inverters. This impedance matching method is extended to multi-receiver system and new power division methodology is proposed. Due to restriction of pure band-pass matching, equivalent circuit matching is also proposed in multi-receiver and power division methodology. Simulation results validate the proposed methodology.

Future works of this research will include performing experiment for above cases, considering cross-coupling in multi-receiver system and power division methodology and design of wireless power transfer system combining multi-stage and multi-receiver system.

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