Impedance Inverter based Analysis of Wireless Power Transfer Consists of Repeaters via Magnetic Resonant Coupling

Kim-Ean KOH[†], Takehiro IMURA[††], and Yoichi HORI[††]

† Department of Electrical Engineering, The University of Tokyo Japan
†† Department of Advance Energy, The University of Tokyo Japan
E-mail: †{koh,imura}@hori.k.u-tokyo.ac.jp, ††hori@k.u-tokyo.ac.jp

Abstract Wireless power transfer via magnetic resonant coupling is widely researched for various applications especially for charging electric vehicles. In order to reduce the dependency on battery capacity, charging while the vehicle is moving may be a solution. Wireless power transfer lane is constructed by embedding the transmitters and repeaters beneath the road to provide charging coverage to certain distance. The actual system will consists of arbitrary number of repeaters and receivers and therefore conventional equivalent circuit analysis will be complex. In this paper, a new and simplified analysis method using impedance inverter representation is proposed to investigate the feasibility of charging lane.

Key words Magnetic resonant coupling, Wireless power transfer, repeaters, Electric vehicle charging

1. Introduction

Using wireless charging for electric vehicle is safe and convenient compared to plugging the vehicle into power outlet. Furthermore newly introduced wireless power transfer method that is magnetic resonant coupling increases the transferable distance and robustness to positional shift compared to magnetic induction method. This mid-range transfer method also enable wireless charging while the vehicle is moving on the road.

The resonators can be embedded beneath and arranged along the road. The car is then charged while moving on the road. However "dead zones" exist in certain positions where the power do not transfer to the load [1]. Conventional equivalent circuit equations may be complex due to number of resonators involved [2] [3]. This paper analyze the condition by representing the coupling coefficient between resonators using impedance inverter and investigate the impedance seen by the power supply. The analysis method can be easily extended for a section of road that consists of one transmitter and arbitrary number of repeaters.

In the system, the transmitter and subsequent repeaters are placed side by side horizontally. Therefore, repeaters will not be overlapping each other. Assuming the transmitter and all the repeaters are the same, non-adjacent resonant coils will be separated at least one time the dimension. Therefore, the cross coupling can be ignored [4]. The analysis is limited to the case where the receiver is coupled to only either the transmitter or one of the repeaters. The case where the vehicle is between two resonator coils and receiving power simultaneously from both is not included in this investigation.

2. Derivation

An example of charging while moving system mentioned in the introduction is shown in Fig. 1. This section provides the basis needed which is impedance inverter representation and multi-receiver equations to analyse the system. Five example analysis cases are then given in the subsequent sections.

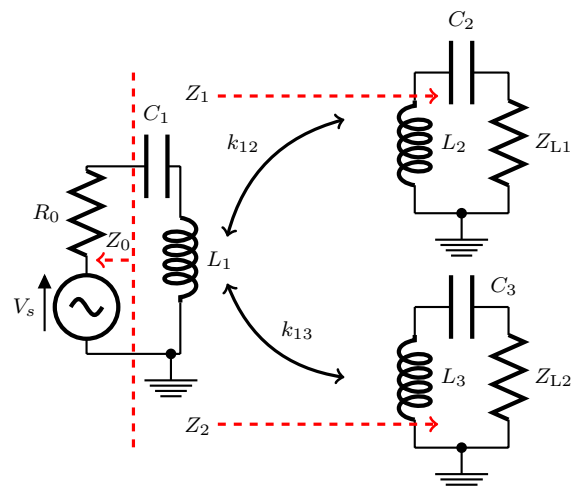Fig. 1 Illustrative of electric vehicle charging while moving.

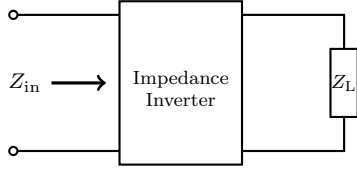
Fig. 2 Equivalent circuit of a two-receiver system.

Fig. 3　Operation of impedance inverter.

Fig. 4　Simplified two-receiver circuit.

An equivalent circuit of a two receiver wireless power transfer system is shown in Fig. 2. The coupling in between the top receiver and transmitter is k_{12} while the coupling in between the bottom receiver and transmitter is k_{13}. Cross coupling in between the two receivers is not considered in this analysis. The couplings between antennas are expressible in terms of impedance inverter's characteristic impedance [5]:

$$k_{12} = \frac{K_{12}}{\omega\sqrt{L_1 L_2}} \qquad k_{13} = \frac{K_{13}}{\omega\sqrt{L_1 L_3}}, \qquad (1)$$

where ω is the angular frequency of the power supply.

Impedance inverter as the name implies inverts the impedance connected to the inverter. Fig. 3 and (2) show the impedance Z_{in} looking into the impedance inverter that is connected to load, Z_L at the other end. There are many applications and many types of impedance inverter [6]. In this paper, impedance inverter is used to represent the couplings between antennas given by (1).

$$Z_{in} = \frac{K^2}{Z_L}. \qquad (2)$$

From Fig. 2, (1) and (2):

$$Z_0 = R_0$$

$$Z_1 = \frac{K_{12}{}^2}{Z_{L1} + Z_{r1}}$$

$$Z_2 = \frac{K_{13}{}^2}{Z_{L2} + Z_{r2}}$$

$$Z_{r1} = r_2 + j(\omega L_2 - \frac{1}{\omega C_2})$$

$$Z_{r2} = r_3 + j(\omega L_3 - \frac{1}{\omega C_3}). \qquad (3)$$

Where Z_1 is the impedance looking from the source to the top receiver in Fig. 2 and Z_2 is the impedance looking from the source to the bottom receiver. The term r_2 and r_3 are the internal resistance of top receiver and bottom receiver respectively. In equivalent circuit, internal resistance is in series with the inductor and capacitor [3] and is not shown in the circuit of Fig. 2.

Z_1 and Z_2 appear in series connection when viewing from the source [7] and therefore circuit of Fig. 2 can be simplified into Fig. 4.

3.　Case Analysis

A section of charging while moving system which consist of a transmitter and three repeaters embedded beneath the road is used as an example. The cases when the electric vehicle above the road is coupled only to either the transmitter or one of the repeaters are considered in this section.

Assuming the vehicle is at the beginning of the charging system and is coupled only to the transmitter as shown in Fig. 5.
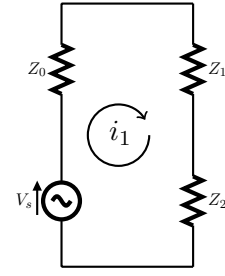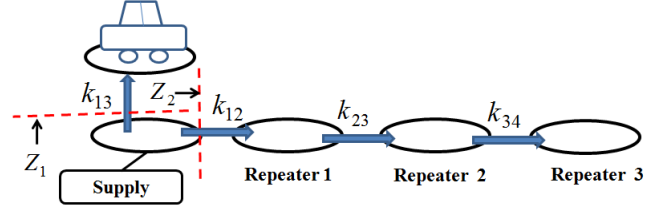

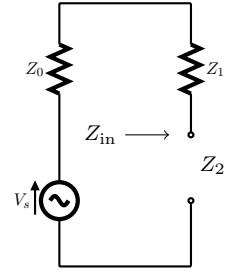Fig. 5　Case I analysis.

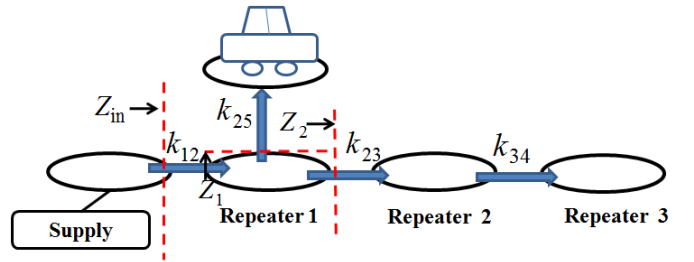Fig. 6　Equivalent circuit of Case I.
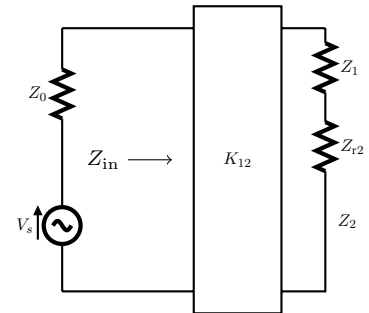

Fig. 7　Case II analysis.

Fig. 8　Equivalent circuit of Case II.

Impedance Z_1 is the impedance looking from the transmitter towards the load through coupling k_{13}. Coupling k_{13} can be represented by an impedance inverter with characteristic impedance K_{13}. Therefore from (2),

$$Z_1 = \frac{K_{13}^2}{Z_L + Z_{r1}}, \tag{4}$$

where Z_L is the load impedance. Again using impedance inverter representations for the transmission path that contains all the repeaters:

$$Z_2 = \frac{K_{12}^2}{\frac{K_{23}^2}{\frac{K_{34}^2}{Z_{r4}}+Z_{r3}} + Z_{r2}}. \tag{5}$$

The term Z_{r1} is the impedance sum of the inductor, capacitor and internal resistance of the receiver. Similarly, Z_{r2}, Z_{r3} and Z_{r4} are for the first repeater, second repeater and third repeater respectively. In all five cases, all the resonators are having the similar resonant frequency and the source is supplying a.c. power near this resonant frequency. Also internal resistance should be small for magnetic resonant coupling [8]. Therefore both the real part and imaginary part of these impedance are small and impedance Z_2 will be large. If impedance Z_2 is assumed to be large enough compared to the power supply's output impedance, this impedance can be assumed open circuited. According to Sec. 2, impedance Z_1 and Z_2 appear as series when seen from the power supply. Thus the equivalent circuit of case I is as shown in Fig. 6 and power supply's input impedance given by (6). The load appears as if open circuited to the supply and almost all the power are reflected back instead traveling to the load.

$$Z_{in} = Z_1 + Z_2 \tag{6}$$

In case II, the vehicle travels to above the first repeater and coupled to only that repeater as shown by Fig. 7, the impedance becomes:

$$Z_1 = \frac{K_{25}^2}{Z_L + Z_{r1}}$$

$$Z_2 = \frac{K_{23}^2}{\frac{K_{34}^2}{Z_{r4}} + Z_{r3}}$$

$$Z_{in} = \frac{K_{12}^2}{Z_1 + Z_2 + Z_{r2}}. \tag{7}$$

Impedance Z_2 is close to zero and is assumed short-circuited. Therefore the equivalent circuit is as shown in Fig. 8. Impedance seen by the supply is Z_{in} and power is able to transferred to the load.

Fig. 9 shows case III which is when the vehicle is above the second repeater. Impedance now becomes:

$$Z_1 = \frac{K_{35}^2}{Z_L + Z_{r1}}$$

$$Z_2 = \frac{K_{34}^2}{Z_{r4}}$$

$$Z_{in} = \frac{K_{12}^2}{\frac{K_{23}^2}{Z_1+Z_2+Z_{r3}} + Z_{r2}}. \tag{8}$$

Impedance Z_2 is again close to ∞ and therefore impedance
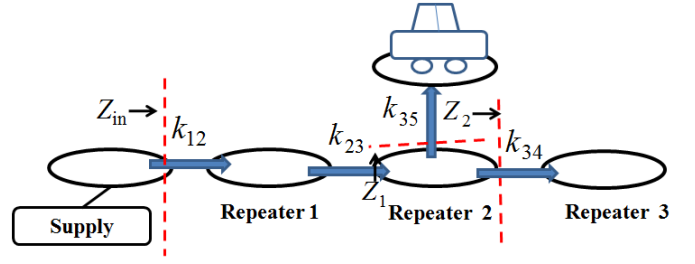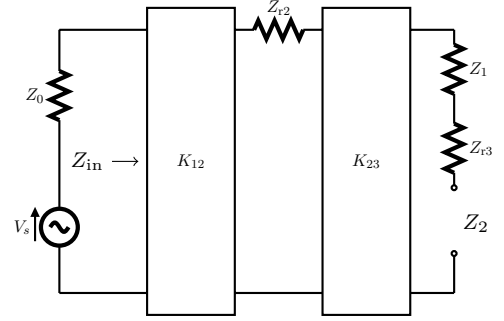

Fig. 9　Case III analysis.

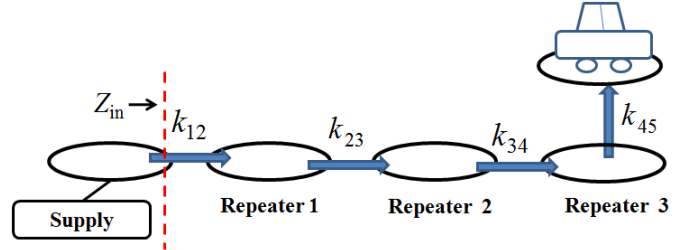Fig. 10　Equivalent circuit of Case III.
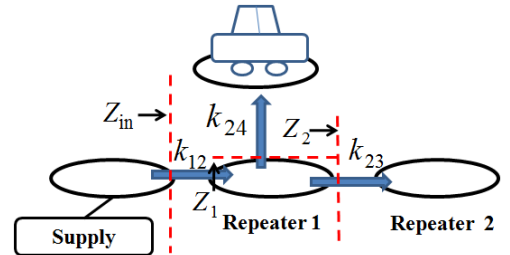

Fig. 11　Case IV analysis.

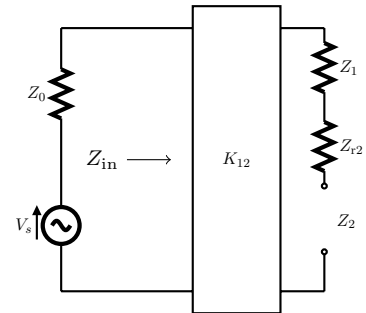

Fig. 12　Case V analysis.

Fig. 13　Equivalent circuit of Case V.

Z_{in} in (8) is also close to ∞ compared to the power supply's output impedance. Therefore the equivalent circuit shown in Fig. 10.

In case IV, the vehicle arrives at above the third repeater as shown in Fig. 11. The impedance seen by the supply is only impedance Z_{in} shown by below equation and the power can be transferred to the load.

$$Z_{in} = \frac{{K_{12}}^2}{\frac{{K_{23}}^2}{\frac{{K_{34}}^2}{\frac{{K_{45}}^2}{Z_{\text{L}}+Z_{\text{r1}}}+z_{r4}}+Z_{\text{r3}}}+Z_{\text{r2}}}. \tag{9}$$

When the charging system contains even number of repeaters, the "dead zones" will appear at certain position similar to case I and case III. However the reason is not due to open circuited input impedance but instead short circuited input impedance. Case V as shown in Fig. 12 is an example of this condition. Impedance in this case can be calculated as:

$$Z_1 = \frac{{K_{24}}^2}{Z_{\text{L}} + Z_{\text{r1}}}$$

$$Z_2 = \frac{{K_{23}}^2}{Z_{\text{r3}}}$$

$$Z_{\text{in}} = \frac{{K_{12}}^2}{Z_1 + Z_2 + Z_{\text{r2}}}. \tag{10}$$

Impedance Z_2 is close to ∞ and therefore impedance Z_{in} in (10) is close to 0. The equivalent circuit is a shown as Fig. 13. The power supply sees a almost short circuited input impedance and therefore almost all power are reflected back to the supply and is not transferred to the load.

From above 5 analysis cases, when the charging system contains odd number of repeaters, the "dead zones" will happen when the receiver is coupled either only to the transmitter, second repeater or all other even number repeaters. On the other hand, when the charging system contains even number of repeaters the "dead zones" will happen when the receiver is coupled only to first repeater or all other odd number of repeaters. This due to the open circuited impedance is inverted odd number of times and the power supply will see a short circuited input impedance.

4. Simulation Result

All five analysis cases are simulated using LTspice. Percentage reflected power, transferred power and power supply's input impedance are plotted. Circuit parameters chosen are listed below.

$$\begin{aligned}
\omega_0 &= 2\pi \times 13.56\,\text{MHz} \\
L_1 &= L_2 = L_3 = L_4 = L_5 = 10\,\mu\text{H} \\
C_1 &= C_2 = C_3 = C_4 = C_5 = 13.78\,\text{pF} \\
R_0 &= Z_{\text{L1}} = 50\,\Omega \\
r_1 &= r_2 = r_3 = r_4 = r_5 = 1\,\Omega,
\end{aligned}$$

where L_1 is the inductance the transmitter, L_2, L_3, and L_4 are the inductance of the first, second and third repeater respectively, and L_5 is the inductance of the receiver. Same subscripts apply to the capacitance and internal resistance. The supply is outputting a.c power with frequency near chosen ω_0. Finally R_0
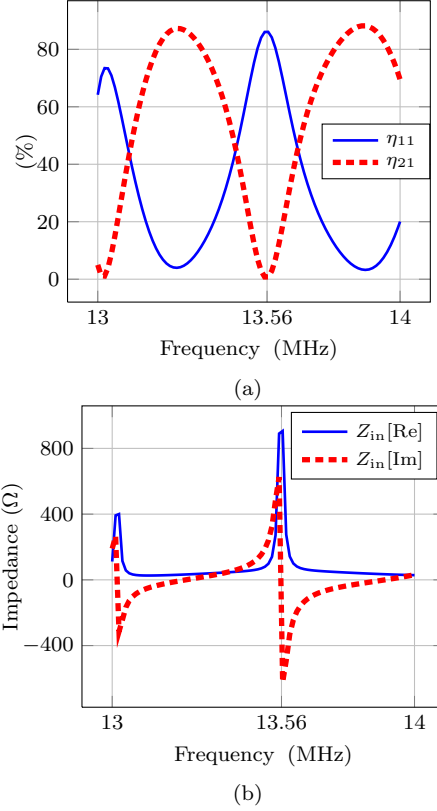

Fig. 14 Simulation result of case I: a) transfer and reflection plot. b) input impedance.

and Z_{L1} are the supply output impedance and load impedance respectively.

4.1 Case I

The coupling coefficients in Fig. 5 are chosen to be:

$$k_{12} = k_{23} = k_{34} = k_{13} = 0.06.$$

From (1), the characteristic impedance of the inverter representing each coupling coefficient is:

$$\begin{aligned}
K_{12} &= K_{23} = K_{34} = K_{12} \\
&= 2\pi \times 13.552 \times 10^6 \times 0.06 \times \sqrt{10\,\mu \times 10\,\mu} \\
&= 51\,\Omega.
\end{aligned}$$

Although simulation result is presented, the system still can not be in perfect resonance due to rounding errors and limited resolution. The input impedance is calculated for point 13.552 MHz which is close to the chosen resonant frequency.

$$\begin{aligned}
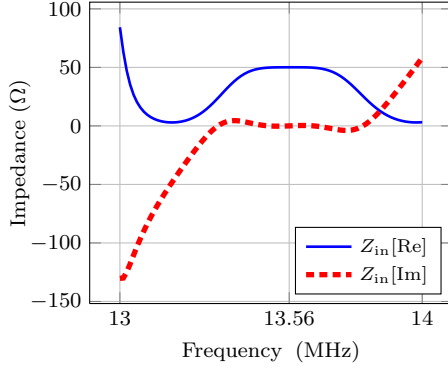Z_1 &= \frac{51^2}{50 + 1 - j0.74} = (51 + j0.74)\,\Omega \\
Z_2 &= \frac{51^2}{\frac{51^2}{\frac{{K_{34}}^2}{1-j0.74}+1-j0.74} + 1 - j0.74} = (840 + j622)\,\Omega \\
Z_{\text{in}} &= (891 + j623)\,\Omega.
\end{aligned}$$

From Fig. 14(a), reflection ratio, η_{11} is high at around frequency of interest, 13.56 MHz. The transfer ratio, η_{21} to the load is nearly zero. This is due to high impedance seen by the supply as shown by the impedance plot in Fig. 14(b) and by above calculation in this case.
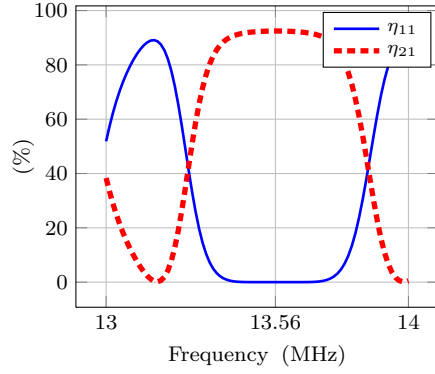
Fig. 15 Simulation result of case II: a) transfer and reflection plot. b) input impedance.

Fig. 16 Simulation result of case III: a) transfer and reflection plot. b) input impedance.

4.2 Case II

The coupling coefficients in Fig. 7 are chosen to be:

$$k_{12} = k_{23} = k_{34} = k_{25} = 0.06.$$

Similar to Case I, the characteristic impedance of the inverter representing each coupling coefficient is:

$$K_{12} = K_{23} = K_{34} = K_{25} = 51\ \Omega.$$

The input impedance calculated for point 13.552 MHz:

$$Z_{in} = (49 + j2)\ \Omega.$$

In this case impedance seen by the supply is close to 50 Ω around chosen resonant frequency as shown by the impedance plot in Fig. 15(b) and by above calculation. Therefore reflection ratio, η_{11} in Fig. 15(a) is almost none. Most of the available power is being transferred to the load as shown by η_{21} plot. Some power is loss due to the internal resistance of the resonators.

4.3 Case III

The coupling coefficients in Fig. 9 are chosen to be:
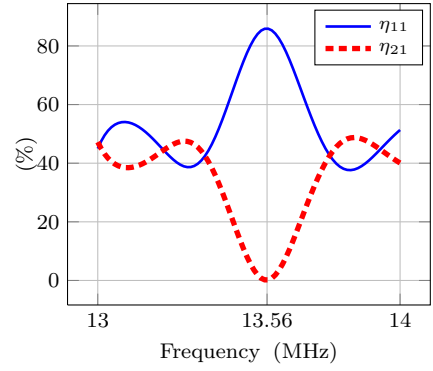
$$k_{12} = k_{23} = k_{34} = k_{35} = 0.06.$$

The characteristic impedance of the inverters representing each coupling coefficient is:

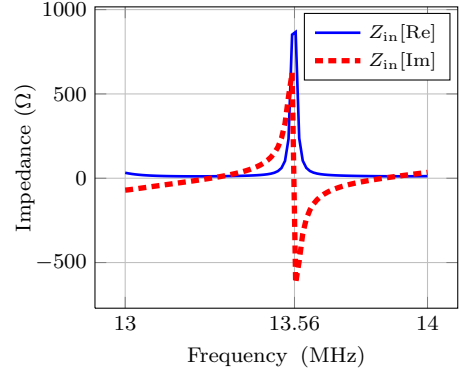$$K_{12} = K_{23} = K_{34} = K_{35} = 51\ \Omega.$$

The input impedance calculated for point 13.552 MHz:

$$Z_{in} = (853 + j622)\ \Omega.$$

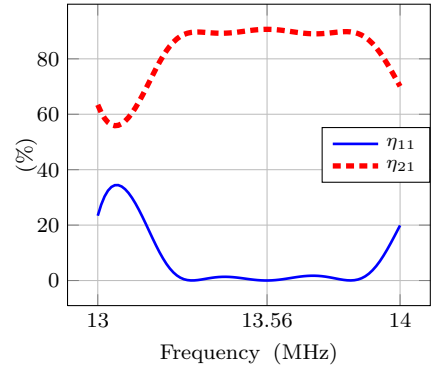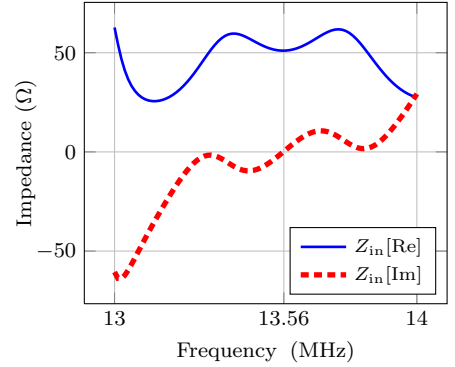Again the impedance seen by the power supply in this case is

Fig. 17 Simulation result of case IV: a) transfer and reflection plot. b) input impedance.
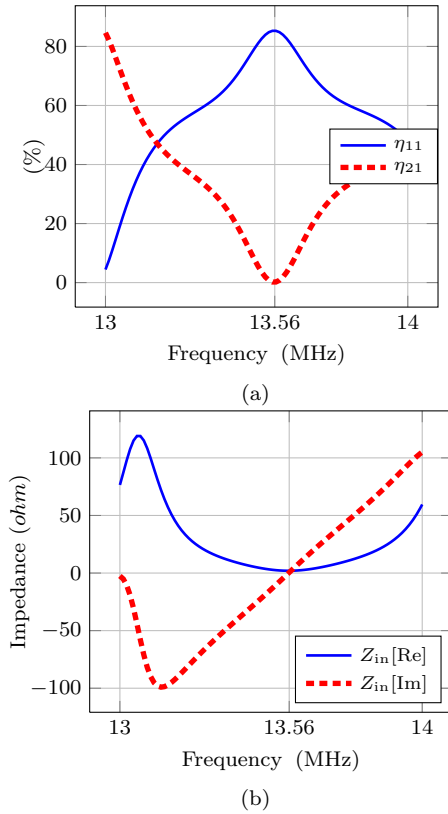
Fig. 18 Simulation result of case V: a) transfer and reflection plot. b) input impedance.

high compared to the output impedance as shown in Fig. 16(b). Therefore almost all the power are reflected as indicated by reflection ratio, η_{11} plot in Fig. 16(a) and no power is being transferred to the load.

4.4 Case IV

The coupling coefficients in Fig. 11 are chosen to be:

$$k_{12} = k_{23} = k_{34} = k_{45} = 0.06.$$

In this case, the supply only sees one transmission path towards the load. The load impedance is inverted once with each repeater. Power is being transferred as indicated by the transmission ratio, η_{21} plot in Fig. 17(a). As the input impedance is close to 50 Ω as shown in Fig. 17(b), the reflection ratio, η_{11} is close to none. Some power is loss due to the internal resistance of the resonators.

4.5 Case V

The coupling coefficients in Fig. 12 are chosen to be:

$$k_{12} = k_{23} = k_{34} = k_{24} = 0.06.$$

The input impedance calculated for point 13.552 MHz:

$$Z_{\text{in}} = (1 + j0.7)\ \Omega.$$

In this case, almost all the power are reflected as indicated by reflection ratio, η_{11} plot in Fig. 18(a). Almost no power is transferred to the load as indicated by η_{21} plot. The reason is that the impedance seen by the power supply is nearly short circuited as shown by the plot in Fig. 18 (b) and by above calculation.

5. Conclusion

In a wireless power transfer via magnetic resonant coupling containing repeaters, there are positions where when the receiver is placed, almost no power can be transferred to the load. Wireless charging system for moving vehicle contains transmitter and repeaters arranged along the road. However, the mentioned dead zone condition appears when the vehicle is above interval of resonators. This paper provides mathematical explanation on the condition by using impedance inverter representations. The small impedance of the repeaters is inverted and is seen by the power supply as close to open circuited. This impedance is in series with the load and therefore almost no power is being transferred to the load. When the charging system contains odd number of repeaters, the condition happens when the receiver is coupled either only to the transmitter, second repeater or all other even number repeaters. When the charging system contains even number of repeaters the condition happen when the receiver is coupled only to first repeater or all other odd number of repeaters. The Loss calculation due to internal resistance is not included in this study. From this analysis, solution for this condition will be derived in the future.

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