Coupling Coefficients Estimation of Wireless Power Transfer System via Magnetic Resonance Coupling using Information from Either Side of the System

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Abstract—Wireless power transfer via magnetic resonance coupling method has open new possibility to electric vehicle (EVs) system. One is that it allows the wireless charging system of moving vehicles, using charging lanes. However, although the efficiency of power transmission is relatively high, the efficiency still depends on the displacement of antennas. There have been several researches on methods to maintain power transmission at highest efficiency. However, in such systems, information on system parameter especially coupling coefficients between transmitting and receiving side is needed, and in charging lane system, such information is unlikely to be obtainable without communication system which will add complexity to the system. Therefore it has come to attention that parameter estimation is a crucial factor to implement the charging lane system.

This paper presents derivations of equations for estimating coupling coefficients in several configurations of wireless power transfer system, using information from only one side, either transmitting side or receiving side, of the system. The presented equations are both applicable to the case of single receiving antenna and are also generalized for the case of multiple receiving antennas. Each equation is verified with both simulations and experiments.

Index Terms—Wireless power transfer, magnetic resonance coupling, coupling coefficient, parameter estimation

I. INTRODUCTION

Recently, the development of electric vehicles (EVs) has been gaining more attentions from both consumers and research community as potential alternatives to traditional combustion engine cars. However, the main drawback of electric vehicles which still remains is their energy storage. Currently used batteries or capacitors have relatively small energy capacity; hence EVs cannot travel across long distance. Researchers have come up with various method to compensate such drawback. One of the possible methods is to incorporate wireless power transfer system into EV charging systems. Not only it will make the process of charging more convenient and safer since it helps reduce the risk of electric shock, wireless power transmission also open up new solutions of dynamically charging running vehicles.

There are several methods for wireless power transmission. But the magnetic resonance coupling method is believed to be the most appropriate for EV charging system due to this method's characteristics which allows power transmission, that is non-radiative, across longer power transfer distance with relatively high efficiency while being robust to antennas positional shift [1][2]. Several methods, such as impedance matching, have been proposed to maximize efficienty of wireless power transfer system; however, most of them require the knowledge of systems parameter [3][4][5]. And those that do not, due to the use of search algorithm, are still not fast enough for improving efficiency of moving pick-ups. Therefore it has come to attention that the parameter estimation is a crucial factor to make the system of charging lane implementable.

This paper presents derivations of equations for estimating coupling coefficients in several configurations of wireless power transfer system, using information such as voltage and current from only one side, either transmitting side or receiving side, of the system. In section II, wireless power transfer system topology used in this study will be described. Section III and IV will present derivations of estimation methods. The presented equations are both applicable to the case of single receiving antenna and are also generalized for the case of multiple receiving antennas. Each equation is verified with simulation result and experimental result shown in section V.

II. WIRELESS POWER TRANSFER SYSTEM TOPOLOGY

In this study, wireless power transfer system is modeled by equivalent circuit, in which the antenna is represented as the combination of inductor and capacitor in series with internal resistance[2]. The equivalent circuit, for configuration with one transmitting antenna and one receiving antenna, is shown in Fig. 1(a). This equivalent circuit can also be rewritten as the T-type equivalent circuit illustrated in Fig. 1(b).

Parameter L_1 , C_1 , R_1 , L_2 , C_2 and R_2 represent inductances, capacitances, and resistances of transmitting antenna and receiving antenna respectively. Power source impedance is Z_S and load impedance is Z_L . Note that, this study will only consider the case when load impedance is purely resistive, in other word, when Z_L does not have an imaginary part. Z_{in} is the input impedance when looking from power source. Mutual inductance, which is the main factor that decides how much power is transfered, is represented by L_m . And lastly, V_1 , I_1 and V_2 , I_2 are defined as voltage across and current going through power source and load respectively,



(b) T-type equivalent circuit

Fig. 1: T-type equivalent circuit for wireless power transfer

immediately before the transmitting antenna, with ϕ as phase different between V_1 and I_1 .

Estimation of coupling coefficient can be done either using only information from source side or only information from load side, depending on the application. In the next two sections, estimation equations will be presented.

III. ESTIMATION EQUATION FROM TRANSMITTING SIDE

In this section, estimation equation using only information from source side will be presented. The advantage of estimating coupling coefficient from source side is that, in the case of multiple EVs or receivers having entered the charging system, if the coupling coefficients between transmitting antenna and each receiving antennas are known, not only charging efficiency can be improved, but how much power to distribute to each EV can also be decided using the method introduced in [6].

Looking from source side, the information we can obtain are V_1 , I_1 , and ϕ . From these parameters, given $V_1 = |V_1|e^{j(\omega t+\phi)}$ and $I_1 = |I_1|e^{j\omega t}$, another information that can be retrieved is input impedance Z_{in} as shown in equation (1).

$$Z_{in} = \frac{V_1}{I_1} = \frac{|V_1|}{|I_1|} (\cos \phi + j \sin \phi)$$
(1)

where
$$Re\{Z_{in}\} = \frac{|V_1|}{|I_1|} \cos \phi$$
 and $Im\{Z_{in}\} = \frac{|V_1|}{|I_1|} \sin \phi$.

This Z_{in} parameter can be calculated regardless of the number of the receivers. Therefore estimation equation from source side will be derived mainly in terms of Z_{in} .

A. With single receiver

From the equivalent circuit, input impedance Z_{in} can be calculated, and the derived relations are shown in equation (2) and (3) when $Re\{Z_{in}\}$ and $Im\{Z_{in}\}$ represent real part and imaginary part of Z_{in} and ω is defined as power sources operating frequency respectively.

$$Re\{Z_{in}\} = R_1 + \frac{(\omega L_m)^2 (Z_L + R_2)}{(Z_L + R_2)^2 + (Z_{A2})^2}$$
(2)

$$Im\{Z_{in}\} = Z_{A1} - \frac{(\omega L_m)^2 (Z_{A2})}{(Z_L + R_2)^2 + (Z_{A2})^2}$$
(3)

where $Z_{A1} = \omega L_1 - \frac{1}{\omega C_1}$ and $Z_{A2} = \omega L_2 - \frac{1}{\omega C_2}$.

However, considering measurement taking, the reliable value of imaginary part of Z_{in} is quite difficult to obtain since it varies quite drastically relative to slightly shifted frequency point. Therefore it is more practical to use measurement of real part in estimation process. Hence, deriving from (2), L_m can be estimated using equation (4), assuming load value and antenna parameters are known.

$$L_m = \frac{1}{\omega} \sqrt{\frac{[Re\{Z_{in}\} - R_1] \left[(Z_L + R_2)^2 + Z_{A2}^2\right]}{Z_L + R_2}} \quad (4)$$

B. With multiple receivers

For the case of system with multiple receivers, input impedance can be derived similarly. Equivalent circuit for system with multiple receivers is illustrated in Fig. 2, where subscription 2 and 3 represent component of load 1 and 2 respectively and so on. In this study, cross-coupling between each load will not be considered due to the fact that this system is aiming for EV charging system, in which receiving antennas in each EVs will have neglegible effect on each other. Z_{in} can be expressed as shown in (5).

$$Z_{in} = R_1 + \left(j\omega L_1 + \frac{1}{j\omega C_1}\right) + \sum_{i=2}^{m+1} \frac{(\omega L_{1i})^2}{Z_i + R_i + j\omega L_i + \frac{1}{j\omega C_i}}$$
(5)
where m = the number of receiving antennas

In order to solve equation (5) for all mutual inductance values, the necessary number of linearly independent equations equals to the number of loads. Controllable parameter in the transmitting side is source frequency. Therefore, once enough information is obtained by performing frequency sweep, mutual inductance values can be calculated by solving equation systems (6) expressed below.



Fig. 2: Equivalent circuit for system with multiple receivers

$$\begin{bmatrix} Z_{in(1)} \\ Z_{in(1)} \\ \vdots \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ \vdots \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & \cdots \\ Q_{21} & \ddots & \cdots \\ \vdots & \cdots & Q_{mm} \end{bmatrix} \begin{bmatrix} L_{12}^2 \\ L_{13}^2 \\ \vdots \end{bmatrix}$$
(6)
where $P_j = R_1 + j\omega_j L_1 + \frac{1}{j\omega_j C_i}$
$$Q_{ij} = \frac{\omega_j^2}{Z_i + R_i + j\omega_j L_i + \frac{1}{j\omega_j C_i}}$$
and $m =$ total number of receivers.

IV. ESTIMATION EQUATION FROM RECEIVING SIDE

In this section, estimation equation using only information from load side will be presented. The advantage of estimating coupling coefficient from the load side is when there are more than one receivers in the system. With only one available source, it is more practical to put control from the load side so all loads can match with the one source. Although estimating from load side can only achieve the information of mutual coupling between source and itself but not with other load, the estimation can be done with much less information comparing to source side when the number of loads increases.

A. With single receiver

In the case of single receiver, the relation between mutual coupling L_m and voltage across the load be expressed as shown in equation (7) when V_1 is source voltage and V_2 is voltage across load Z_2 .

$$L_m = \frac{1}{2\omega} \left[\frac{V_1}{V_2} Z_2 \pm \sqrt{\left(\frac{V_1}{V_2} Z_2\right)^2 + 4R_1(R_2 + Z_2)} \right]$$
(7)

However, due to the characteristics of V_2 , which is that it has a peak as illustrated in 3, and which sign in equation (7) should be used to perform estimation depedns on which side of the peak the system is currently at. Therefore it is not possible to determine L_m with only one set of input sample, which leads to the limitation of estimation from load side using only a single set of information. However, estimation of coupling



Fig. 3: V_2 characteristics

coefficient from load side can still be performed with the same method as the case of multiple receivers, although additional information is required.

B. With multiple receivers

In the case of multiple receiving antennas, the estimation is currently done for constant voltage source, assuming source voltage V_1 is known. However, information from only one instance is not enough to estimate mutual inductance parameter. But if two sets of information, in this case, two voltages across load for different load values, are provided, the estimation can be done by equation (8).

$$L_{12} = \frac{V_1}{\omega} \frac{Z_{2a} V_{2b} (R_2 + Z_{2b}) - Z_{2b} V_{2a} (R_2 + Z_{2a})}{V_{2a} V_{2b} (Z_{2b} - Z_{2a})}$$
(8)

where V_{2a} = voltage across load when load is Z_{2a} and V_{2b} = voltage across load when load is Z_{2b} .

V. SIMULATION AND EXPERIMENTAL RESULTS

In order to verify the proposed equation, simple simulation is performed. The antenna which will be used in future experiment is a short-type with self-resonance frequency of approximately 900kHz, shown in Fig. 4. The detailed antenna parameters are in TABLE I.

The simulation is performed for the following setup, which is illustrated in Fig. 5. The transmitting antenna is fixed in place while the receiving antenna is placed at several distance away from the transmitting antenna. For the case of multiple receivers, the second receiving antenna is placed on the opposite side of the transmitting antenna to avoid cross coupling between the two receivers. Note that, in the actual application of charging lane, two receivers equipped in different vehicles will be too far apart, therefore cross coupling between them should be negligible.



Fig. 4: 900kHz antenna

TABLE I: Antenna Parameters

	Res. Freq. [kHz]	$R[\Omega]$	$L[\mu H]$	C[pF]
Transmitting	900.065	1.043	95.562	327.197
Receiving#1	900.821	1.059	97.459	320.289
Receiving#2	902.008	1.058	117.906	268.413



Receiving Antenna #2

Fig. 5: Wireless power transfer system setup

A. Estimation Result from Transmitting Side with Single Receiver (Reference Value)

For this study, since the coupling coefficient or k estimation equation from transmitting side is derived from direct backcalculation, the k value obtained by calculation from this case will be used as a reference value. The calculation is performed for the case when resonant frequency is 900 kHz and load impedance is 50Ω . The expected k value is shown in Fig. 6.

B. Estimation Result from Transmitting Side with Multiple Receivers

For this case, with enough information obtained by frequency sweep, k values between source and each receiver can be estimated. When there are two receivers, information from







Fig. 7: Simulation result for k_{12} and k_{13} estimation from transmitting side

two frequency points are necessary. The frequencies used in simulation and experiments are 898 kHz, 900 kHz and 902 kHz, whereas load impedance is fixed at 50 Ω . Expected value of k_{12} and k_{13} are calculated in simulation whose result is shown in Fig. 7. The plot shows k_{12} and k_{13} versus the distance between transmitting antenna and the first receiving antenna which is moved to several distance. The result of k_{12} matches with reference value while k_{13} is constant as expected since the second receiving antenna is fixed in place. The experimental result is shown in Fig. 8. The experimental result, except for the range of extremely short transmitting gap, is mostly consistent with expected value from simulation.

C. Estimation Result from Receiving Side

Due to the fact that estimation equations are the same whether there are only one or multiple receivers, only estimation result in the case of multiple receivers are presented.





Fig. 8: Experiment result for k_{12} and k_{13} estimation from transmitting side using frequency sweep

More information on receiving side can be gained by changing load, or impedance sweep. The impedance values used in simulation and experiment are 25Ω , 50Ω and 100Ω , whereas transmitting frequency is fixed at 900 kHz. As mentioned in earlier section that by estimation from receiving side, only the k between itself and source can be obtained. Simulation results in Fig. 9 shows that estimated k value is consistent with the reference value regardless of the existence of an extra receiving antenna. The experimental shown in Fig. 10 is also mostly consistent with simulation result. The estimation errors occur in experiments at larger transmission gap is believed to be due to radiation loss.

VI. CONCLUSION

This paper presents derivations of equations for estimating coupling coefficients in several configurations of wireless



Fig. 9: Simulation result for k_{12} estimation from receiving side



Fig. 10: Experiment result for k_{12} estimation from receiving side using impedance sweep

power transfer system, mainly for single transmitting antenna with single and multiple receiving antennas. Since the estimation is aiming for usage in dynamic EV charging system, the ideal solution is to be able to back-calculate the value of coupling coefficient directly from measurement. Such calculation can be done when estimation is done from the source side in one transmitting and one receiving antenna configuration. However, in other case, more information is needed. The derived relations show that, estimation of coupling coefficient is still possible theoretically by sweeping controllable parameter to obtain more linearly independent information. The proposed equations are verified with simulation result and experimental results.

Future work will include investigation of estimation sensitivities and the actual system implementation for charging lane application.

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