

# Receiving Resonator Design for Efficiency Maximization in Wireless Power Transfer via Magnetic Resonance Coupling

Masahiko Tsuboka, Takehiro Imura, Hiroshi Fujimoto, Yoichi Hori  
Department of Advanced Energy, the University of Tokyo, Japan

**Abstract**— Charging system for electric vehicle using wireless power transfer has become more popular in research community. Required features of such charging system are longer power transfer distance with high efficiency and robust to positional shift. Among various wireless power transfer methods, magnetic resonance coupling method has the mentioned features, hence is most suitable for this application. Ideally, in this method, both transmitting and receiving resonators should be designed to have the same self-resonant frequency. However the self-resonant frequency tends to shift due to manufacturing error, temperature change and aging. Therefore, the power transfer frequency needs adjustment in order to transmit power with maximum efficiency. In this paper, a design method to adjust the power supply frequency according to self-resonant frequency of the receiving resonator for maximum efficiency is proposed. The proposed design method is validated by simulations and experiments.

## I. INTRODUCTION

Electric vehicle (EV) is drawing more and more attention due to environmental issues. However, many people are reluctant to switch to EV due to the need of frequent charging. In order to charge EV with ease, research on EV charging application using wireless power transfer (WPT) has been gaining attention [1]. As a method of wireless power transmission for EV charging system, three methods which are microwave transmission [2], electromagnetic induction [3] and magnetic resonance coupling, are considered. Wireless power transfer via magnetic resonance coupling method [4] which is able to transmit power across large air-gap with high efficiency, hence is suitable for EV charging [5] is used in this work.

In WPT via magnetic resonance coupling method, the transmitting and receiving resonators are designed to have the same self-resonant frequency. The power source frequency is then set to operate at the same resonant frequency. However the self-resonant frequency of each resonator tends to shift due to manufacturing error, temperature change and aging. Therefore, the power source frequency needs to be adjusted to the frequency of maximum power transfer efficiency.

In conventional method to find the point of maximum power transfer efficiency is measured, system frequency is changed toward the direction with higher efficiency until the system reaches maximum efficiency. However, such method takes a long time. In contrast, using the proposed method, the maximum efficiency is the same as the self-resonant frequency

of the receiving resonator. Therefore the system can be immediately set to the point of maximum frequency, hence a quick efficiency maximization can be realized.

There are two ways for achieving maximum efficiency. One is obtaining the information of self-resonant frequency of the receiving resonator from the receiving side using a communication system. After that, the power source is set to transfer power at such frequency, obtaining the maximum efficiency instantly. The other way is to estimate the self-resonant frequency of the receiving side, and then, use frequency control to achieve maximum power transfer efficiency, without using communication system.

In this paper, a new design method to make the frequency of maximum power transfer efficiency match with self-resonant frequency of the receiving resonator is proposed. The advantage of this method is that quick efficiency maximization can be realized using only a value of self-resonant frequency of the receiving resonator.

The remainder of this paper is organized as follow. In section II, resonator design method is introduced. Next, in section III, simulation verification is shown. Then, experiment results are demonstrated in section IV. Finally, conclusions and future work is mentioned in section V.

## II. RESONATOR DESIGN METHOD

### A. Wireless Power Transfer System

Wireless power transfer via magnetic resonance coupling method can be used to transmit power at frequencies range from kHz to GHz [6]. In the case of power transmission in kHz band, the disadvantages are larger coil size and heavier than resonator coils for MHz band. Furthermore, ohmic loss in copper coil may increase. On the other hand, transmission at kHz range has larger usable ISM frequency band and therefore is suitable for efficiency optimization through frequency tuning. In this study, wireless power transfer is conducted at kHz range.

In kHz range power transmission, WPT system can be considered as lumped parameter circuit because the circuit length is short relative to the wavelength. For analysis of WPT system, the system can be represented by equivalent circuit [7]. The equivalent circuit shown in Fig. 1 is used. The parameter  $L_1$ ,  $L_2$ ,  $C_1$ ,  $C_2$ ,  $R_1$  and  $R_2$  represent the inductance, capacitance, and resistance of the transmitting resonator and receiving resonator respectively. Subscript 1 represents the parameters of the transmitting side, and subscript 2 represents the parameters of

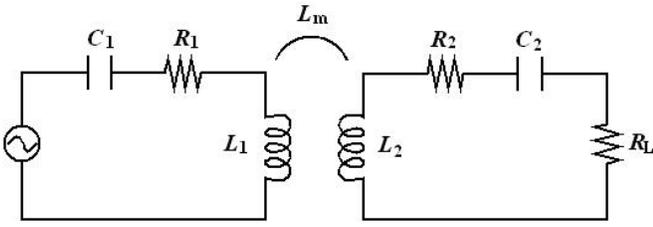


Fig. 1. Equivalent circuit of magnetic resonance coupling.

the receiving side. The parameter  $L_m$  represents the mutual inductance between the two resonators.

There are four basic topologies which are investigated in [8]. A topology is different to the others by whether the capacitor is connected in series or parallel in transmitting side and receiving side. As shown in Fig. 1, The S/S topology in which the capacitor is connected in series in both sides is often used due to simpler circuit equation.

Fig. 1 shows the equivalent circuit of magnetic resonant coupling. The efficiency is measured by power amplification  $A_p$ , which is the ratio of power between output and input as expressed in (1).

$$A_p(\omega) = \frac{\text{Re}[V_2 \cdot \bar{I}_2]}{\text{Re}[V_1 \cdot \bar{I}_1]} = \frac{R_L}{\text{Re}[(AR_L + B)(CR_L + D)]} \quad (1)$$

$$A = \frac{1}{\omega L_m} \left\{ \left( \omega L_1 - \frac{1}{\omega C_1} \right) - jR_1 \right\} \quad (2)$$

$$B = \frac{1}{\omega L_m} \left[ \left\{ R_2 \left( \omega L_1 - \frac{1}{\omega C_1} \right) + R_1 \left( \omega L_2 - \frac{1}{\omega C_2} \right) \right\} - j \left\{ R_1 R_2 - \left( \omega L_1 - \frac{1}{\omega C_1} \right) \left( \omega L_2 - \frac{1}{\omega C_2} \right) + (\omega L_m)^2 \right\} \right] \quad (3)$$

$$C = -j \frac{1}{\omega L_m} \quad (4)$$

$$D = \frac{1}{\omega L_m} \left\{ \left( \omega L_2 - \frac{1}{\omega C_2} \right) - jR_2 \right\} \quad (5)$$

### B. Design Method

The frequency of maximum efficiency,  $f_{\max}$ , is calculated by differentiating  $A_p$  with respect to  $\omega$  and solving for the frequency when the equation is zero. The relation between the frequency of maximum efficiency,  $f_{\max}$ , and the self-resonant frequency of receiving resonator,  $f_2$ , is shown in (6). Coefficient of  $f_2$  is defined as expressed in (7). Considering (6), the frequency of maximum efficiency does not depend on the transmitting resonator parameters such as the inductance, capacitance, and resistance.

$H$  depends only on the receiving resonator parameters, and if  $H$  is equal to 1, maximum efficiency frequency is the same as the resonant frequency of receiving resonator. Therefore, the maximum efficiency is obtained by setting the source frequency to the resonant frequency of receiving resonator.  $R_L$  in (7) is the load and cannot be changed. On the other hand,  $L_2$ ,  $C_2$ ,  $R_2$  can be designed freely. Therefore, it is possible to match the point of maximum efficiency to the self-resonant frequency of receiving resonator.

The design method is described as follow. Focusing on  $C_2/L_2$  in (7), Fig. 2 is plotted with  $R_L=50 \Omega$ ,  $f_2=120 \text{ kHz}$ .  $R_2$  in Fig. 2 are the value of the coil simulation using JMAG. In the simulation, the value of the coil's resistance and inductance can

$$f_{\max} = \frac{1}{\sqrt{1 - \frac{C_2(R_L + R_2)^2}{2L_2}}} \cdot f_2 = H \cdot f_2 \quad (6)$$

$$H = \frac{1}{\sqrt{1 - \frac{C_2(R_L + R_2)^2}{2L_2}}} \quad (7)$$

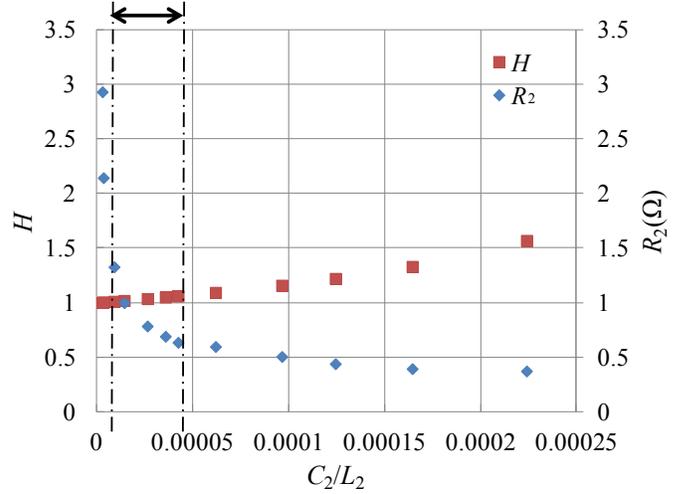


Fig. 2.  $H$  and  $R_2$  versus  $C_2/L_2$ .

be obtained when the coil has 10 to 60 turns. The coils are made of 2 mm copper wire wound into a spiral with 46 cm outer diameter and 3 mm pitch.

If  $L_2$  is relatively larger than  $C_2$ ,  $C_2/L_2$  is smaller,  $H$  becomes closer to 1 as shown in Fig. 2. It is simpler to make  $C_2$  small. However, in case of making  $L_2$  larger, the coil needs to be wound with more turns, resulting in higher internal resistance and increase in size and weight of the coil. Because internal resistance contributes to efficiency decrease, it is necessary to balance the value of internal resistance value and  $C_2/L_2$ . Therefore, the appropriate pair of  $C_2$  and  $L_2$  is within the range specified by dot-dashed lines in Fig. 2.

### III. SIMULATION

In order to verify the design method, the difference between the frequency of maximum efficiency and the self-resonant frequency of receiving resonator is investigated using parameters of several resonators that have the same self-resonant frequency but are constructed from different values of inductance and capacitance. Table I shows the parameters used in the simulation.

The parameter  $f_0$  represents the self-resonant frequency. The inductance in Case 1 is designed to be smaller than that in Case 2 though the resonant frequency is the same. The difference between Case 1 and Case 2 is the value of  $H$ . In Case 1, the value of  $H$  is not equal to 1, but  $H$  in Case 2 is about 1 as shown in Table I. It is predicted that, in case of  $H$  is equal to 1, maximum efficiency frequency is the same as the resonant frequency of receiving resonator. The resonator's parameters in Case 2 are the values used for both simulation and experiment.

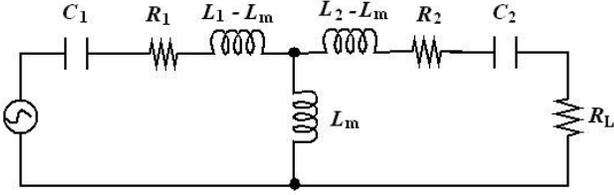


Fig. 3. T-type equivalent circuit of magnetic resonance coupling.

TABLE I  
RESONATOR PARAMETERS

		$f_0$ (kHz)	$L$ ( $\mu$ H)	$C$ (pF)	$R$ ( $\Omega$ )	$H$
Case 1	a1	114.0	60	32488	1.5	1.88
	a2	127.8	60	25843	1.5	1.53
Case 2	b1	114.0	808	2414	2.7	1.00
	b2	127.8	775	2000	2.1	1.00

TABLE II  
RELATIONSHIP BETWEEN  $k$  PARAMETER AND AIR-GAP

gap (cm)	$k$
10	0.30
20	0.13
30	0.06

Assuming the case of 10% difference between the self-resonant frequency of transmitting resonator and receiving resonator, resonant frequency of resonators are designed to be 114.0 kHz, 127.8 kHz. In order to design resonant frequency, capacitances of the resonators are adjusted appropriately. Table II shows the relation between  $k$  parameter and air-gap, where  $L_m$  is derived by substituting  $k$  into (8). Fig. 3 shows the T-type equivalent circuit used in analysis which is transformed from equivalent circuit shown in Fig. 1. Qucs is used as a simulator. In this analysis  $R_L$  and  $R_2$  are set to 50  $\Omega$  and 1.5  $\Omega$  respectively.

$$L_m = k\sqrt{L_1 L_2} \quad (8)$$

The simulation result of Case 1 is shown in Fig. 4. In the simulation, a1 is transmitting resonator and a2 is receiving resonator. From Fig. 4, the frequency of maximum efficiency is approximately 199 kHz, which is significantly different from the self-resonant frequency of the receiving resonator which is 127.8 kHz. The simulation result of Case 2 is shown in Fig. 5. In the simulation, b1 is transmitting resonator and b2 is receiving resonator. From Fig. 5, the frequency of maximum efficiency is almost the same as the resonant frequency of the receiving resonator which is 127.8 kHz. And, the frequency of maximum efficiency does not change with respect to air-gap. The maximum efficiency in Case 1 is under 0.8 in any air-gap shown in Fig. 4, but the maximum efficiency in Case 2 is over 0.8 shown in Fig. 5. The change in efficiency is steeper for larger air-gap. Therefore, the control of transmitting frequency is necessary in this case.

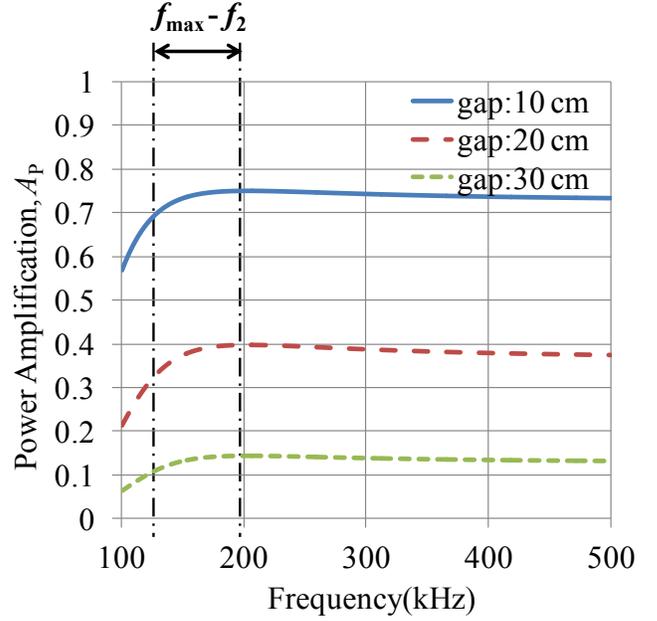


Fig. 4. Simulation result in Case 1.

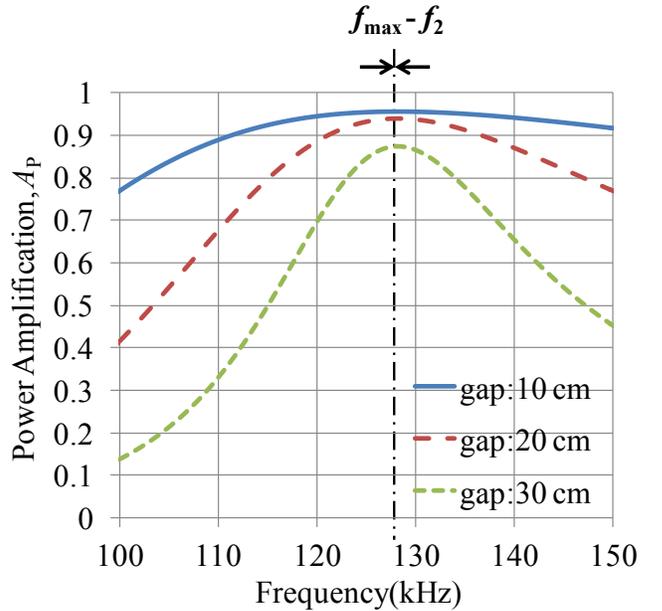


Fig. 5. Simulation result in Case 2.

#### IV. EXPERIMENT

The proposed method is confirmed by experiments. Fig. 6 shows the resonators used in the experiment and the specifications are shown in Case 2 in Table 1. The coils are made from litz wire wound into 60 turns with 46cm outer diameter. The coil is connected to a capacitor in series in both transmitting and receiving side. The experimental setup is illustrated in Fig. 7. The reflection coefficient,  $S_{21}$ , and transmission coefficient,  $S_{11}$ , are measured with the vector network analyzer (VNA). The efficiency was determined using (9) which consider power reflection as loss.

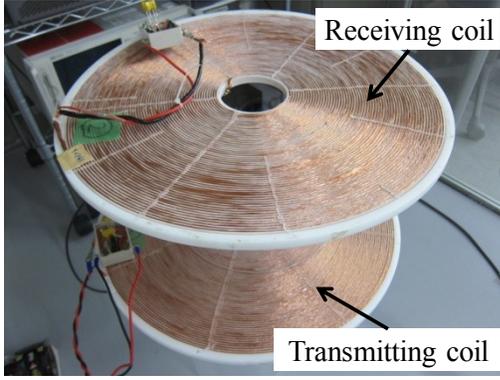


Fig. 6. Resonator used in experiment.

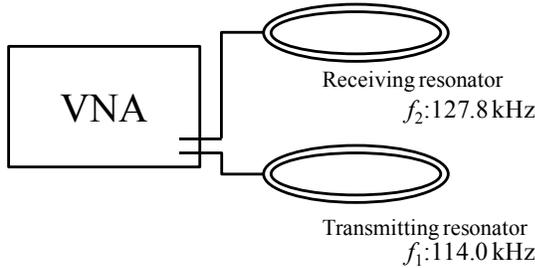


Fig. 7. Experiment configuration.

The experiment results are shown in Fig. 8. The result in Fig. 8 shows that the frequency of maximum efficiency does not depend on the transmitting gap. In the air-gap is equal to or less than 10 cm, the change in efficiency when changing transmitting frequency is small. But, in the case of 30 cm, or when air gap is larger, the changes in efficiency become more significant. Thus, transmitting frequency control is important to achieve high-efficiency with larger air gap. The frequency of maximum efficiency is almost the same as the resonant frequency of the receiving resonator which is set to 127.8 kHz. The simulation result in Fig. 5 and experiment result in Fig. 8 are consistent. They confirmed that the resonator can be designed in a way that the frequency of maximum efficiency and the resonant frequency of receiving resonator are the same.

$$A_p = \frac{\text{Re}[V_2 \cdot \bar{I}_2]}{\text{Re}[V_1 \cdot \bar{I}_1]} = \frac{|S_{21}|^2}{1 - |S_{11}|^2} \quad (9)$$

## V. CONCLUSION

A design method for resonator to match the frequency of maximum efficiency to self-resonant frequency of the receiving resonator is proposed. By selecting appropriate values for the inductance and capacitance values of the resonator, the point of maximum efficiency can be successfully adjusted to the same as self-resonant frequency of the receiving resonator.

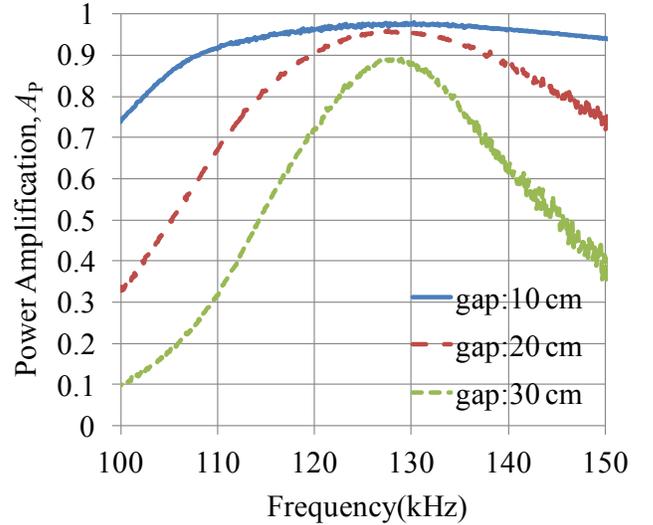


Fig. 8. Experiment result in Case 2.

Two ways for achieving maximum efficiency using the proposed method are mentioned. The instant maximization of efficiency can be realized by using this method. The proposed method is verified by simulations and experiments. For future work, it is necessary to study the relationship between the maximum efficiency and  $C_2/L_2$ , and the way to estimate the self-resonant frequency of the receiving resonator from the transmitting side in order to achieve high efficiency by frequency control from the transmitting side only.

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