Optimal Analog-to-Digital Converter Resolution on Current Control for Precise Positioning Control

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Abstract
Current control for precise positioning control requires accurate current signals. However, current measurement error including current metering error and quantization error is unavoidable in digital control. In this paper, an approach of determining the optimal resolution of A/D converter to reduce the quantization effects is presented by taking into account the statistical properties of the current metering error. The effectiveness of the proposed method is verified by experiments using a high-precision stage.

Key words: current measurement, quantization, dither, analog-to-digital converter, current control

1. Introduction

Precise current measurement is essential for current control and sensorless control to produce high static and dynamic performance of ac motor drive systems. As the current measurement error is introduced into the control system by current sensors and analog-to-digital converters, the real current signals of a motor are not guaranteed to follow the reference values even when the measured current can follow them exactly. This would cause torque ripple and deteriorate the control performance especially at low load [2][3]. Therefore, understanding and suppressing the current measurement error have attracted a great deal of attention [1][4][5].

Current measurement error can be classified into the metering error and the quantization error. The metering error, which is caused by thermal drift, dc offset and scaling inaccuracy, is introduced into control loop by current sensors and other analog devices in the measuring system. The quantization error arises because the analog current signal may assume any value within the input range of the A/D converter while the output data is a sequence of finite precision samples. Quantization error behaves as highly colored noise and cannot be ignored in many cases. For example, it is reported that quantization error can affect the accuracy of the motor position estimation for sensorless control when high frequency signal injection method is used [4].

It is known that white noise is easier to deal with than other colored noise because it contains equal power at any center frequency. Many methods, such as Kalman filter and LQG controller design methods, are studied based on the assumption that the noises introduced to the system are white. Motivated by this perspective, in this paper, dithering techniques are applied to analyze the effects of current measurement error. Dither, which is an intentionally applied form noise, can decorrelate signal-dependent noise, has been actively researched to suppress the quantization effects on audio or video signal processing [6][7]. It is indicated that by treating the current metering error as a sort of dither, the quantization error can be whitened if a proper resolution of A/D converter is applied.

\[ Q(x) = \Delta \left[ \frac{x}{\Delta} + \frac{1}{2} \right], \quad \Delta = \frac{2\Delta}{x} \]

(1)

where \( Q(\cdot) \) is denoted as the quantizing operation. The quantization error is defined by

\[ q = y - x \]
\[ e = y - x \]
\[ = Q(x + \nu) - (x + \nu) \]
\[ = q(x + \nu), \quad \text{................................. (6)} \]

where \( \nu \) denotes the dither. As reported in [6], the statistical properties of the dither \( \nu \) can be chosen to control the properties of the total error \( e \) without increasing its variance. However, the requirement of synchronous dither subtraction at the system output cannot always be implementable in practical situations. It is for such reason, nonsubtractivly dithered technique introduced in the following is of interest.

In the nonsubtractivly dithered system, the total error \( e \) can be expressed by

\[ e = y - x \]
\[ = Q(x + \nu) - x \]
\[ = q(x + \nu) + \nu, \quad \text{................................. (7)} \]

The total error is not simply the quantization error, but also involves the dither. It is shown that the following result can be obtained [7]:

**Theorem:** In an nonsubtractivly dithering system, \( E[\nu' | x] \) is independent of the distribution of the input \( x \) for \( i = 1, 2, ..., M \) if and only if the characteristic function of the dither, \( \Phi_x \), satisfies the condition that

\[ \Phi_x(k) \equiv \int_{-\infty}^{\infty} p_x(e)e^{-j2\pi ke} \, de = 0 \quad \text{for} \quad k = \pm 1, \pm 2, \pm 3, \ldots, \quad \text{........ (8)} \]

where \( i = 0, 1, 2, ..., M - 1 \). The characteristic function of a random variable is the Fourier transform of its probability density function, which is defined by

\[ P_x(u) \equiv \int_{-\infty}^{\infty} p_x(e)e^{-j2\pi ke} \, de \quad \text{................................. (9)} \]

Based on this theorem, the moments of the total error is shown as follows:

\[ E[\nu] = E[\nu], \quad \text{................................. (10)} \]
\[ E[\nu'^2] = E[\nu'^2] + \frac{\Delta^2}{12}, \quad \text{................................. (11)} \]
\[ E[\nu'^m] = \sum_{l=0}^{m/2} \binom{m}{2l} \left( \frac{\Delta}{2} \right)^{2l} E[\nu'^{m-2l}] \frac{(2l+1)!}{2l!}, \quad \text{................................. (12)} \]

where \( m = 1, 2, ..., M \).

It is shown in [6] that triangular-pdf dither of 2-LSB peak-to-peak amplitude
is unique and optimal in the sense that it renders the total error be spectrally white while minimizing its second moment error \( E[e^2] \). According to calculation, the mean and the variance of a dither with the triangular-pdf (13) is

\[
E[v] = \frac{1}{3} \Delta, \\
E[v^2] = \frac{\Delta^2}{6}.
\]

Therefore, the mean and the variance of the total error \( e \) are

\[
E[e] = 0, \\
E[e^2] = \frac{\Delta^2}{4}.
\]

Fig. 4 shows the results of applying nonsubtractive dithering technique. The input signal and the quantization step are the same as the signal shown in Fig. 2(a). The aforementioned triangular-pdf dither (13) is used. From the figure of frequency characteristic, surprising as it may seem, the output signal sounds like to be composed of the ideal input signal and a constant white noise. The results should be compared with those in Fig. 2.

3. Analysis of Current Measurement

In this section, firstly, the current measurement for digital current control is analyzed. Then, the dithering techniques presented in the previous section are applied to determine the optimal resolution of A/D converters.

Fig. 5 shows the typical path of the current measurement (12). Currents, transduced to the voltage signal by current sensors, are transformed into digital values via A/D converters after amplitude amplification and noise filtering. During this procedure, the metering error and the quantization error are introduced to the control system. The block diagram of current measurement is shown in Fig. 6. The metering error, denoted by \( \eta \), is caused by thermal drift, dc offset and scaling inaccuracy of the analog devices. The quantization error arises because of A/D converters is applied as converting analog signal to digital. Without loss of generality, the real current value is assumed to be within the measuring range of current sensors, which means that the metering accuracy is guaranteed. The quantization step \( \Delta \) can be defined by

\[
\Delta = \frac{I_0}{2^{N_b}}.
\]

where the measurement range of current sensor is set as \( \pm I_0 \) and \( N_b \) is the resolution of an A/D converter. Based on the nonsubtractive dither technique, it is easy to come out that the metering error should have positive effects on reducing the quantization effects caused by A/D converters. The following Proposition is achieved:

**Proposition**: Suppose that the metering error \( \eta \) is zero-mean and its probability density function is a triangular-pdf with the variance of \( V \), the optimal resolution of the corresponding A/D converter in the sense of achieving independent measurement error is

\[
N_b = \left[ \log_2 \frac{I_0}{\sqrt{6}V} + 1 \right],
\]

where \([\cdot]\) means the operation of round-off.

This is easy to be proved from the perspective of adjusting the quantization step \( \Delta \) to "hit" the metering error \( \eta \) so that the quantization noise can be whitened.

It will be illustrated in next section that the distribution of the metering error \( \eta \) can be approximated to triangular-pdf noise in practical situation. In addition, it is reported in (10) that a noise with Gaussian distribution is also effective on suppressing the quantization effects.

4. Experiments

In this section, the method of determining optimal A/D
converters from the viewpoint of dither techniques are verified via experiments by using a high-precision stage. The experimental setup is shown in Fig. 7, whose parameters are shown in Table 1. The stage is driven by linear motors and the position information is measured by linear encoder with the resolution of 1 nm. U-phase and W-phase currents are measured by FA-050PV with the range of ±f0 = ±50A. The A/D converters whose resolution is 14-bit is exploited. VSPTC with SRC method is used to design position/speed controllers to generate current reference. Vector control method is applied for current control and pole-zero cancellation controller is used. The block diagram of control system is shown in Fig. 8.

Firstly, the metering error is measured by the 14-bit A/D converter. Its histogram plot is shown in Fig. 9. It is observed that the metering error can be approximated with triangular-pdf noise. Based on the data, the variance of the current metering error is \( V = 1.1218 \times 10^{-4} \). According to (19), the optimal resolution of A/D converter is

\[
N_0 = \left[ \log_{10} \frac{f_0}{\sqrt{6}V} + 1 \right] = [11.9123] = 12.
\]

For comparison, the resolution of the A/D converter is also dropped to 12-bit and 10-bit by software in the following experiments.

In the setup, the target trajectory of motion is shown in Fig. 10. A/D converters with resolution of 10-bit, 12-bit and 14-bit are implemented, respectively. In order to achieve real current signals, two high-precision current probes (Tektronix A622) are also used. The experiment results are shown in Fig. 11~15. In all these figure, the dashed line shows the results obtained by 10-bit ADC, the solid line shows the results obtained by 12-bit ADC, and the dot-dashed line shows the results obtained by 14-bit one. Fig. 11 shows the comparison of U-phase measured current signals: the signals after the A/D conversion. These signals are used as feedback signals, respectively. Fig. 12 shows the comparison of real current signals (upper part) and the current tracking errors (lower part). The fast-Fourier-transform analysis of the current tracking error is shown in Fig. 13. It is observed that the amplitude of current tracking error is almost the same over whole range of frequencies, which means that the error is whitened. The fast Fourier transform analysis of d,q-axis current are shown in Fig. 14~15. The position tracking errors are shown in Fig. 16. It is observed that the A/D converter with resolution of 12-bit obtained at least the same performance with 14-bit ADC. The comparison of average tracking error is shown in Table 2. These results also indicate that white noise has smaller effects on control performance compared with other colored noise.

5. Conclusion

In this paper, some significant theorems on quantization and dithering techniques are reviewed. The current measurement for digital current control is analyzed. It is illustrated that by the use of A/D converters having a suitably chosen resolution, the quantization noise can be whitened by the current metering error as well as the variance of total
measurement error is minimized. Based on the view of this point, the optimal resolution of A/D converters can be determined. The effectiveness of this perspective is verified by experiments using a high-precision stage.

Fig. 13. The FFT analysis of the U-phase currents tracking error.

Fig. 14. FFT analysis of the d-axis currents error.

Fig. 15. FFT analysis of the q-axis currents error.

Fig. 16. Comparison of the position tracking error.

Table 2. The RMS of tracking error

<table>
<thead>
<tr>
<th>Tracking error (μm)</th>
<th>14-bit</th>
<th>12-bit</th>
<th>10-bit</th>
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<tbody>
<tr>
<td></td>
<td>2.1634</td>
<td>2.1485</td>
<td>2.5365</td>
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References


