Bi-articular actuators – actuator spanning two joints – play fundamental role in robot arms designed under the human musculoskeletal actuation paradigm. Unlike kinematic redundancy, actuator redundancy resulting from bi-articular actuation brings advantages such as increasing stability, reducing link's inertia, and decreasing non-linearity of the end-effector force with respect to the force direction. The traditional phase different control (PDC) resolves actuator redundancy on the basis of a linearized model derived from measured human muscle activity. Such linear model produces a non-zero error in calculation between a desired output force and necessary inputs. In this paper, the non-linear phase different control (NLPDC) is proposed to resolve actuator redundancy with no error. The maximum end-effector force of BiWi, bi-articularly actuated, and wire-driven arm, is measured using both PDC and NLPDC. When the robot arm moves towards singular configurations, the measured error in output force remains within the modeling error if using NLPDC, while such error increases significantly for PDC. Furthermore, unlike PDC, the proposed NLPDC allows design of joint stiffness and torque independently, reduction of necessary total muscle input force, and precise calculation of maximum output force.

Keywords: actuator redundancy resolution; antagonistic actuators; bi-articular muscles; feedforward control; robot arms

1. Introduction
Humans and animals use bi-articular muscles – muscles spanning two consecutive joints – to regulate stiffness stabilizing unstable dynamics, as running over rough terrain [1], increasing accuracy of movements [2], and transferring power from proximal to distal joints [3]. Moreover, bi-articular muscles appear primarily responsible for controlling external force direction on the ground [4]. For these reasons, robots with bi-articular actuators are recently raising interest. Regarding the hardware design, bi-articularly actuated robots have been realized by means of pneumatic actuators [5,6], and motors with transmissions systems based on pulleys [7,8], planetary gears [9,10], wires [11–13], and passive springs [14,15]. All these robots present more actuators than joints, resulting in actuator redundancy. Actuator redundancy resolution is the first step in the control design for these robots, representing a key aspect for performances as force precision and maximum magnitude [16,17].

A biologically inspired approach widely used to resolve actuator redundancy in robot applications [7,9,18,19] is the phase different control (PDC) [20]. According to PDC, the relationship between the end-effector force and the muscle inputs is determined using a linear model based on muscle activation level patterns, resulting from electromyography activity observation of human arm muscles while applying forces at the end effector under isometric and maximal effort conditions [20]. PDC approach, due to its linearization, presents an error in the calculation process from desired end-effector force to muscle activation level inputs [21]. This error depends on three factors – desired force direction, angle between the links, and on link length ratio – and can increase significantly when the manipulator moves towards singular configurations [16].

In this paper, a new approach – non-linear phase different control (NLPDC) – combining PDC and a geometrically based non-linear model, is proposed. NLPDC:

- Produces no error in calculation from end effector desired force to necessary muscle activation levels.
- Is valid also when the robot arm links have different lengths and the actuators have different maximum joint actuator torques.

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• Allows a precise and direct calculation of the end-effector maximum force given a desired force direction, the manipulator configuration, and the actuator maximum joint torques.

NLPDC is compared with PDC by both calculation and experimental methods. The experimental results are obtained using BiWi, bi-articularly actuated and wire-driven robot arm [11].

In Section 2, modeling of bi-articularly actuated robot arms is described. In Section 3, the actuator redundancy problem is illustrated, and in Section 4, two approaches for its resolution – PDC and NLPDC – are described. In Section 5, BiWi is introduced together with the feedforward control strategy used in the experiment. In Section 6, PDC and NLPDC are compared by both calculation and experimental results. In Section 7, the advantages of NLPDC are summarized. In Appendix A, the geometrical derivation of the non-linear model for NLPDC is provided.

2. Modeling and advantages of robot arms with bi-articular actuators

Conventional manipulators present one actuator for every joint. On the contrary, animal present a complex musculoskeletal structure based on two types of muscles:

1. Mono-articular muscles producing a torque about one joint.
2. Multi-articular muscles spanning more joints.

The simplified model of the complex animal musculoskeletal system shown in Figure 1(a) is widely used for robot arms [7,18,20,22,23]. This model is based on six contractile actuators – extensors ($e_1$, $e_2$, and $e_3$) and flexors ($f_1$, $f_2$, and $f_3$) – coupled in three antagonistic pairs:

- $e_1$–$f_1$ and $e_2$–$f_2$: pairs of mono-articular actuators producing torques about joints 1 and 2, respectively.
- $e_3$–$f_3$: pair of bi-articular actuators producing torque about joints 1 and 2 contemporaneously.

Each of the six muscles has a muscle activation level $-e_1$, $e_2$, $e_3$, $f_1$, $f_2$, and $f_3$ – that ranges from 0, which means no muscle contractile force produced, to 100% corresponding to the maximum muscle contractile force $-e_1^m$, $e_2^m$, $e_3^m$, $f_1^m$, $f_2^m$, and $f_3^m$.

Bi-articularly actuated robot arms present numerous advantages.

Bi-articular actuators dramatically increase the range of end-effector impedance which can be achieved without feedback [24,25], improving balance control for jumping robots that do not use force sensors [26]. Bi-articular muscles transfer mechanical energy from proximal to distal joints [3]. Such aspect is fundamental for legged robots in hopping [6], jumping [27], running [5], as well as for power assist robots [28,29].

Bi-articular actuators significantly improve the learning speed in robot locomotion control [30].

Moreover, multijoints actuators such as tri-articular actuators increase the efficiency in output force production [7].

Another advantage of arm equipped with bi-articular actuators is the ability to produce an end-effector maximum end-effector force in a more homogeneously distributed way [31]. In Figure 1(b), the maximum end-effector force for a two-link conventional manipulator and a arm equipped with bi-articular actuators is shown for comparison. In the case of conventional manipulator,
two actuators with maximum joint actuator torque \( T_1 = T_2 = 10 \text{ Nm} \) are considered. On the other hand, for the bi-articularly actuated robot arm three actuators with maximum joint actuator torque \( \tau_1 = \tau_2 = \tau_3 = 6.66 \text{ Nm} \) are taken into account. Therefore, the total maximum actuator torque in the two cases is the same (i.e. 20 Nm).

The conventional quadrilateral shape becomes an hexagon for arms driven by bi-articular actuators, which therefore produce a maximum end-effector force with a more homogeneous distribution in respect to force direction. This aspect is peculiar for application interacting with humans such as rehabilitation robots [28,29], or for jumping and walking robots [14,26]. There are researches in which rehabilitation robots are pneumatically actuated increasing user safety [32,33]. Peak end-effector forces such as the one in point \( M \) in Figure 1(b) cannot be produced by the human arm, therefore are unnecessary and dangerous in case of controller failure. Consequently, the use of bi-articular actuators further increases safety in rehabilitation applications [28,29,34].

3. Actuator redundancy problem

A robot arm modeled as in Figure 1(a) is redundant in actuation: has six actuators and two DOF. The resulting statics are shown in Figure 2 where \( F \) is a general force at the end effector; \( T_1 \) and \( T_2 \) are total torques about joints 1 and 2, respectively; \( \tau_1 \) and \( \tau_2 \) are torques produced by mono-articular actuators about joints 1 and 2, respectively; \( \tau_3 \) is the bi-articular torque produced about both joints simultaneously.

The joint torques \( T \) are:

\[
T = [T_1 \ T_2] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} [\tau_1 \ \tau_2 \ \tau_3] = [\tau_1 + \tau_3 \ \tau_2 + \tau_3] \tag{1}
\]

The problem represented by (1) is referred as the redundancy actuator problem. Given \( \tau = [\tau_1, \tau_2, \tau_3]^T \), it is possible to determine \( T \) by using (1), and \( F \) by:

\[
F = (J^T)^{-1} T
\]

where

\[
J = \begin{bmatrix} -l_1 \sin(\theta_1) - l_2 \sin(\theta_1 + \theta_2) & -l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) & l_2 \cos(\theta_1 + \theta_2) \end{bmatrix}
\]

\[
= \begin{bmatrix} a & b \\ c & d \end{bmatrix}
\]

(3)

On the other hand, given \( F \), and therefore \( T \), it is generally not possible to determine uniquely \( \tau \) due to the presence of the bi-articular actuator (see (1)).

For both PDC and NLPDC approaches, the three joint actuator torques \( \tau \) are functions of the six muscle forces. They are widely modeled in static conditions as [20]:

\[
\tau = \begin{bmatrix} (f_1 - e_1) r - K r^2 (f_1 + e_1) \Delta \theta_1 \\ (f_2 - e_2) r - K r^2 (f_2 + e_2) \Delta \theta_2 \\ (f_3 - e_3) r - K r^2 (f_3 + e_3) (\Delta \theta_1 + \Delta \theta_2) \end{bmatrix} \tag{4}
\]

where \( r \) is distance between the joint and the point where the muscle force is applied, considered to be the same for all the muscles; \( K \) is a elastic constant considered to be independent from joint angles (\( \theta \)), and the same for all muscles; \( \Delta \theta \) is the difference between the actual position and the muscle natural length position. The difference between the activation level of two antagonistic muscles \( (f_i - e_i, \text{ for } i = 1, 2, 3) \) is referred in the following as the difference mode. The sum of the activation level of two antagonistic muscles \( (f_i + e_i, \text{ for } i = 1, 2, 3) \) is the sum mode. The sum mode is related to the joint stiffness \( s \) as:

\[
s = \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} K r^2 (f_1 + e_1) \Delta \theta_1 \\ K r^2 (f_2 + e_2) \Delta \theta_2 \\ K r^2 (f_3 + e_3) (\Delta \theta_1 + \Delta \theta_2) \end{bmatrix} \tag{5}
\]

In the following, \( \Delta \theta \) is considered to be null, that is the actuators are always at natural length, therefore:

\[
\tau = \begin{bmatrix} (f_1 - e_1) r \\ (f_2 - e_2) r \\ (f_3 - e_3) r \end{bmatrix} \tag{6}
\]

Hence the statics of the arm in Figure 1(a) are:

\[
T = \begin{bmatrix} (f_1 - e_1) r + (f_1 - e_1) r \\ (f_2 - e_2) r + (f_2 - e_2) r \\ (f_3 - e_3) r \end{bmatrix} \tag{7}
\]

4. Actuator redundancy resolution methods

4.1. PDC

The muscle activation level patterns on which PDC is based are shown in Figure 3(b). These patterns are the linearization of the patterns resulting from electromyog-
The six muscle activation level patterns expressed in respect to the force direction $\theta_f$. According to PDC approach, the end-effector maximum force $F^m$ with angle $\theta_f$ in Figure 3(a) is obtained using the muscle activation level patterns in Figure 3(b) as inputs. $F^m$ in direction $\theta_f$ is the sum of the six forces produced by the six muscles:

$$F^m = F_{e_1} + F_{e_2} + F_{e_3} + F_{e_4} + F_{e_5} + F_{e_6}$$  

For the particular case in Figure 3(b), where $l_1 = l_2 = 1 \text{ m}$, $\theta_1 = -45^\circ$, $\theta_2 = 90^\circ$, $e^m_i r = f^m_i r = 1 \text{ Nm}$ for $i = (1, 2, 3)$, and $\theta_f = 33.75^\circ$ result:

$$F^m = F_{e_1} + F_{e_2} + 0.25 F_{e_3} + 0.75 F_{e_6}$$

The six muscle activation levels producing $F^m$ in direction $\theta_f$ are calculated as follows:

1. On the basis of the Jacobian (3), the actual configuration ($\theta_1$ and $\theta_2$) and the maximum muscle forces, calculate the angles $\alpha$, $\beta$, $\gamma$, $\delta$, $\epsilon$, and $\zeta$, defined as the angles between the x-axis and the line passing through the center O and points A, B, X, A, B, and Z in Figure 3(b), respectively.

2. Calculate the muscle activation level $\epsilon_1$, $\epsilon_2$, $\epsilon_3$, $f_1$, $f_2$, $f_3$ using the graph in Figure 3(a). The graphical illustration of the muscle activation levels for PDC (Figure 3(a)), is mathematically represented by a set of six piecewise linear functions, one for every muscle. Every piecewise function is defined on six intervals, as shown in Table 1 in the PDC rows.

The $\tau$ producing $F^m$ are calculated using (6). In the case illustrated in Figure 3(b):

$$\tau = \begin{bmatrix} (f_1 - e_1) r \\ (f_2 - e_2) r \\ (f_3 - e_3) r \end{bmatrix}$$

By using PDC, the sum modes of the three antagonistic muscle pairs cannot be designed independently from the difference modes:

$$s = \begin{bmatrix} \frac{Kr^2(f_1 + e_1)\Delta \theta_1}{Kr^2(f_2 + e_2)\Delta \theta_2} \\ \frac{Kr^2(f_3 + e_3)(\Delta \theta_1 + \Delta \theta_2)}{Kr^2(0.75f_3 + 0.25e_3)(\Delta \theta_1 + \Delta \theta_2)} \end{bmatrix}$$

### 4.2. NLPDC

The muscle activation levels that produce $F^m$ on direction $\theta_f$ using the proposed NLPDC approach are shown in Table 1 in the NLPDC rows. NLPDC is based on PDC as it uses the same muscle activation levels when they are null (0) or maximum ($e^m_1$ and $f^m_i$ for $i = (1, 2, 3)$). Elsewhere, the muscle activation levels are derived on the basis of the robot arm geometry (as shown in Appendix A), resulting in:

$$m^s_1 = \frac{-e^m_1 - e^m_2(a + ctan(\theta_f))}{d tan(\theta_f) + b} + e^m_3$$

$$m^s_2 = \frac{-f^m_1 - e^m_2(a + ctan(\theta_f))}{d - e tan(\theta_f) + b - a} - f^m_3$$

$$m^s_3 = \frac{(f^m_1 + f^m_2)(b + dtan(\theta_f))}{ctan(\theta_f) + a} - f^m_3$$
Table 1. Muscle activation levels for PDC and NLPDC. $(\alpha^+) = \max(0, x)$ and $(\alpha^-) = \max(-x, 0)$.

<table>
<thead>
<tr>
<th>$\zeta \leq \theta_j^* &lt; \alpha$</th>
<th>$\alpha \leq \theta_j^* &lt; \beta$</th>
<th>$\beta \leq \theta_j^* &lt; \gamma$</th>
<th>$\gamma \leq \theta_j^* &lt; \delta$</th>
<th>$\zeta \leq \theta_j^* &lt; \alpha$</th>
<th>$\zeta \leq \theta_j^* &lt; \alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_1$ PDC</td>
<td>$e_1^m - \frac{\theta_j^* - \alpha}{\alpha - \beta}e_1^m$</td>
<td>0</td>
<td>0</td>
<td>$\frac{\theta_j^* - \gamma}{\gamma - \delta}e_1^m$</td>
<td>$e_1^m$</td>
</tr>
<tr>
<td>NLPDC</td>
<td>$(m_1^{\phi})^- + \sigma 1$</td>
<td>0</td>
<td>0</td>
<td>$(m_1^\phi)^+ - \sigma 1$</td>
<td>$(m_1^\phi)^+ - \sigma 1$</td>
</tr>
<tr>
<td>$f_1$ PDC</td>
<td>$f_1^m$</td>
<td>$f_1^m$</td>
<td>$f_1^m - \frac{\theta_j^* - \gamma}{\gamma - \delta}f_1^m$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLPDC</td>
<td>$(m_1^\phi)^+ + \sigma 1$</td>
<td>$(m_1^\phi)^+ + \sigma 1$</td>
<td>$(m_1^\phi)^+ + \sigma 2$</td>
<td>$(m_1^\phi)^+ + \sigma 2$</td>
<td>$(m_1^\phi)^+ + \sigma 2$</td>
</tr>
<tr>
<td>$e_2$ PDC</td>
<td>$e_2^m$</td>
<td>$e_2^m$</td>
<td>$e_2^m - \frac{\theta_j^* - \beta}{\beta - \alpha}e_2^m$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NLPDC</td>
<td>$(m_2^\phi)^- + \sigma 2$</td>
<td>$(m_2^\phi)^- + \sigma 2$</td>
<td>$(m_2^\phi)^- + \sigma 3$</td>
<td>$(m_2^\phi)^- + \sigma 3$</td>
<td>$(m_2^\phi)^- + \sigma 3$</td>
</tr>
<tr>
<td>$f_2$ PDC</td>
<td>0</td>
<td>0</td>
<td>$f_2^m$</td>
<td>$f_2^m$</td>
<td>$f_2^m - \frac{\theta_j^* - \gamma}{\gamma - \delta}f_2^m$</td>
</tr>
<tr>
<td>NLPDC</td>
<td>$(m_2^\phi)^+ + \sigma 2$</td>
<td>$(m_2^\phi)^+ + \sigma 2$</td>
<td>$(m_2^\phi)^+ + \sigma 3$</td>
<td>$(m_2^\phi)^+ + \sigma 3$</td>
<td>$(m_2^\phi)^+ + \sigma 3$</td>
</tr>
</tbody>
</table>

Equations (12)–(17) are derived considering the geometry of the robot arm as described in Appendix A. The values $\sigma_1$, $\sigma_2$, and $\sigma_3$ represent the minimum activation level between two muscles of every antagonistic pair, which influences the sum modes. They depend on the maximum muscle forces as in the following:

$$\sigma_i \in \left[ 0, \min \left( e_i^m - (m_i^\phi)^-, f_i^m - (m_i^\phi)^+ \right) \right]$$

where $i = (1, 2, 3)$ and $(k, l) = ((\alpha, \beta), (\beta, \gamma), (\gamma, \delta), (\delta, \epsilon), (\epsilon, \zeta), (\zeta, \alpha))$.

A graphical example of the use of NLPDC is illustrated in Figure 4(b) for $l_1 = l_2 = 1 \text{ m}$, $\theta_1 = -45^\circ$, $\theta_2 = 90^\circ$, $e_1^0r = f_1^m = e_2^3r = f_2^m = e_3^3r = f_3^m = 1 \text{ Nm}$, and $\sigma_1 = \sigma_2 = \sigma_3 = 0$. $F_m$ for $\theta_j^* = 33.75^\circ$ is:

$$F_m = F_{l_1}^m + F_{l_2}^m + 0.6F_{l_3}^m$$

Using Table 1 results:

$$\tau = \begin{bmatrix} (f_1 - e_1)\tau \\ (f_2 - e_2)\tau \\ (f_3 - e_3)\tau \end{bmatrix}$$

$$= \begin{bmatrix} (f_1^m - 0)r = f_1^m \tau \\ (0 - e_2^3)r = -e_2^3 \tau \\ ((m_3^\phi)^+ + \sigma_3) - ((m_3^\phi)^+ + \sigma_3) \tau = 0.6f_3^m \tau \end{bmatrix}$$
By using NLPDC, the sum modes are designed independently from the difference modes:

\[
s = \begin{bmatrix}
kr_1^2(f_1 + e_1 + \sigma_1)\Delta\theta_1 \\
kr_2^2(f_2 + e_2 + \sigma_2)\Delta\theta_2 \\
kr_3^2(f_3 + e_3 + \sigma_3)(\Delta\theta_1 + \Delta\theta_2)
\end{bmatrix}
= \begin{bmatrix}
kr_1^2f_1^m\Delta\theta_1 \\
kr_2^2e_2^m\Delta\theta_2 \\
kr_3^2(0.6f_3^m + [0, \min(e_3^m, 0.4f_3^m)])(\Delta\theta_1 + \Delta\theta_2)
\end{bmatrix}
\]

where \(\sigma_1 = \sigma_2 = 0\) because \(m_1^\alpha = f_1^m\) and \(m_2^\alpha = -e_2^m\). If an end-effector force lower than the maximum one is required, then also \(\sigma_1 \geq 0\) and \(\sigma_2 \geq 0\). By choosing the minimum \(\sigma\), the desired force is produced with the minimum sum modes, resulting in smaller maximum total muscle force with respect to PDC.

The six piecewise non-linear functions in Table 1 are defined and continuous on the domain \(\theta_j \in [0, 360^\circ]\).

### 4.3. Simplified NLPDC

If the antagonistic muscles of every pair produce the same maximum joint actuator torque, that is \(e_i^m = f_i^m\) for \(i = 1, 2, 3\), then the actuator redundancy is resolved with the simplified NLPDC by using Table 2, where:

**Table 2.** Joint actuator torque for NLPDC.

<table>
<thead>
<tr>
<th>(\zeta \leq \theta_j^* &lt; \alpha)</th>
<th>(\alpha \leq \theta_j^* &lt; \beta)</th>
<th>(\beta \leq \theta_j^* &lt; \gamma)</th>
<th>(\gamma \leq \theta_j^* &lt; \delta)</th>
<th>(\delta \leq \theta_j^* &lt; \epsilon)</th>
<th>(\epsilon \leq \theta_j^* &lt; \zeta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_1)</td>
<td>(\tau_1^{\text{up}})</td>
<td>(\tau_1^m)</td>
<td>(\tau_1^{\text{up}})</td>
<td>(\tau_1^m)</td>
<td>(\tau_1^m)</td>
</tr>
<tr>
<td>(\tau_2)</td>
<td>(-\tau_2^m)</td>
<td>(-\tau_2^{\text{up}})</td>
<td>(\tau_2^m)</td>
<td>(\tau_2^{\text{up}})</td>
<td>(-\tau_2^{\text{up}})</td>
</tr>
<tr>
<td>(\tau_3)</td>
<td>(-\tau_3^m)</td>
<td>(-\tau_3^{\text{up}})</td>
<td>(\tau_3^m)</td>
<td>(\tau_3^{\text{up}})</td>
<td>(-\tau_3^{\text{up}})</td>
</tr>
</tbody>
</table>

Figure 4. NLPDC approach: muscle activation level patterns and end-effector maximum force.

Figure 5. BiWi, bi-articularly actuated and wire-driven robot arm.
\[
\tau_2^{\text{inp}} = \frac{(\tau_1^m + \tau_3^m)(b + d\tan(\theta_j^f))}{c\tan(\theta_j^f) + a} - \tau_3^m \quad (23)
\]

\[
\tau_3^{\text{inp}} = \frac{(- \tau_1^m - \tau_3^m)(a + c\tan(\theta_j^f))}{(d - c)\tan(\theta_j^f) + b - a} - \tau_3^m \quad (24)
\]

The simplified NLPDC is suitable when:

1. The antagonistic muscles of every pair produce the same maximum joint actuator torques.
2. A two-link robot arm is actuated by two mono- and one bi-articular actuator as in [35,36].

5. Experimental set-up

5.1. BiWi: Bi-articularly actuated, and Wire-driven robot arm

BiWi, bi-articularly actuated, and wire-driven robot arm are shown in Figure 5(a). BiWi, a two-link planar manipulator actuated by six motors, each representing one of the muscles in Figure 1(a). The power is transmitted to the joints through pulleys and polyethylene wires as shown in Figure 5(b):

- A pair of antagonistic mono-articular motors \((e_1-f_1)\) is connected to two pulleys fixed on joint 1.
- A pair of antagonistic mono-articular motors \((e_2-f_2)\) is connected by thrust wires to two pulleys fixed on joint 2.
- A pair of antagonistic bi-articular motors \((e_3-f_3)\) is connected to pulleys fixed on joint 2, and to free pulleys about joint 1.

Basic characteristics of BiWi and of the actuator and sensor systems are shown in Table 3 and Table 4, respectively.
Further characteristics of BiWi are in [11].

5.2. Feedforward control strategy

The feedforward control block diagram used in the experiment is shown in Figure 6, where \( h/C_3 \) is the desired output force direction; \( \tau \) is the actuator joint torque as in Figure 2; \( \text{mot}_i \) and \( \text{mot}_f \) for \( i = 1, 2, 3 \) are the motor reference torques calculated as in Table 5 where \( K_{sl} = 1.33 \) is a coefficient used to compensate for the inevitable transmission loss in the thrust wires. A force sensor is used to measure the end-effector force \( F \), and its steady state value is considered. Maximum joint actuator torques are \( \tau_1 = \tau_3 = 1.84 \text{ Nm} \) and \( \tau_2 = 1.38 \text{ Nm} \).

The six muscle activation levels \( -e_1, f_1, e_2, f_2, e_3, \) and \( f_3 \) are not sent directly as motor torque inputs due to the fact that the common mode of the antagonistic actuators is not taken into account in this work.

6. Results

PDC and NLPDC are compared in two configurations:

- Configuration I: \( \theta_1 = -60^\circ \) and \( \theta_2 = 120^\circ \)
- Configuration II: \( \theta_1 = -25^\circ \) and \( \theta_2 = 50^\circ \)

The joint actuator torque input patterns calculated using PDC and NLPDC are shown in Figure 7. In both

![Diagram of desired force and measured force comparison](image-url)
the approaches, the joint actuator input torque patterns are continuous in respect to \( h_f \). Therefore, the six switching conditions do not cause torque reference discontinuities, which could cause instability to the system.

The calculated \( F_m \) of BiWi calculated using the \( s \) of Figure 7 is shown in Figure 8, where \( h_f/C_3 \) varies from 0 to \( 360/C_14 \) every \( 5/C_14 \). The \( s \) obtained with NLPDC produces no error in calculation of \( F_m \). On the other hand, for PDC, the error in calculation of \( F_m \) is small in configuration I, but is significant in configuration II.

The measured \( F_m \) of BiWi is shown in Figure 9 for both configurations.

The experimental results show a greater error in measured \( F_m \) for PDC. In Figure 10, it is shown that the relative error of end-effector force magnitude for the calculation and experimental measurement, respectively defined as

\[
F_{err\,cal} = \left| \frac{F_{calculated}}{F^*} - 1 \right|
\]

and

\[
F_{err\,mea} = \left| \frac{F_{measured}}{F^*} - 1 \right|
\]

where \( F_{calculated} \) is derived from (1), (2), and Table 1; and \( F^* \) is the desired end-effector maximum force. \( F_{err\,mea} \) is not significantly different in configuration I, where \( \text{Avg}(F_{err\,mea}) \) for PDC and NLPDC are 0.0188 and 0.0128, respectively (Table 6). Such error, however, for PDC increases when the arm moves towards singular configurations. In configuration II, \( \text{Avg}(F_{err\,mea}) \) for PDC is 0.0953, which is almost four times the one of NLPDC (0.0259). Moreover, \( F_{err\,mea} \) has a high error peak of 0.25 in configuration II for PDC.

### 7. Conclusions

A new approach to resolve actuators redundancy for bi-articularly actuated robot arms – the NLPDC – is proposed. NLPDC combines the traditional PDC and a geometrically based non-linear model. In contrast with PDC, NLPDC:

- Produces no error in calculation between the end-effector desired force, and necessary muscle force inputs.
- Precisely calculates the maximum output force at the end effector using a closed form solution given the force direction, the manipulator configuration, and the maximum muscle forces.
- Precisely calculates the necessary muscle force inputs for any desired end-effector force by multiplying the muscle activation levels by the ratio between desired and maximum force.
- Allows design of the sum mode (which determines the joint stiffness) independently from the difference mode.
- Allows generation of desired force with a smaller sum mode, resulting in a smaller total muscle force input.

Experimental results obtained using BiWi, bi-articularly actuated, and wire-driven robot arm, and a feedforward control strategy, show that the error in end-effector force is kept within the modeling error when using NLPDC. On the other hand, such error increases when the arm moves towards singular configurations if using PDC.

### Table 6. Average of relative error of end-effector force magnitude.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Approach</th>
<th>( \text{Avg}(F_{err,cal}) )</th>
<th>( \text{Avg}(F_{err,exp}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>PDC</td>
<td>0.0055</td>
<td>0.0188</td>
</tr>
<tr>
<td></td>
<td>NLPDC</td>
<td>0</td>
<td>0.0128</td>
</tr>
<tr>
<td>II</td>
<td>PDC</td>
<td>0.0809</td>
<td>0.0953</td>
</tr>
<tr>
<td></td>
<td>NLPDC</td>
<td>0</td>
<td>0.0259</td>
</tr>
</tbody>
</table>

![Figure 10](image_url) Relative error of end-effector force magnitude \( F_{err\,cal} \) and \( F_{err\,mea} \) using PDC and NLPDC.
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References


Appendix

A. Geometrical derivation of NLPDC

Coordinates of points $A$ and $B$ in Figure 11 are:

$$\begin{align*}
AX &= \frac{1}{JU} \left[ \begin{array}{c}
d - c \\ -b + a
\end{array} \right] \left[ \begin{array}{c}
f^m_1 + \frac{r_m}{f_m} \\ 0
\end{array} \right] \\
AY &= \frac{1}{JU} \left[ \begin{array}{c}
d - c \\ -b + a
\end{array} \right] \left[ \begin{array}{c}
f^m_2 + \frac{r_m}{f_m} \\ 0
\end{array} \right]
\end{align*}$$

(27)

$$\begin{align*}
BX &= \frac{1}{JU} \left[ \begin{array}{c}
d - c \\ -b + a
\end{array} \right] \left[ \begin{array}{c}
f^m_1 + \frac{r_m}{f_m} \\ f^m_3 + \frac{r_m}{f_m}
\end{array} \right] \\
BY &= \frac{1}{JU} \left[ \begin{array}{c}
d - c \\ -b + a
\end{array} \right] \left[ \begin{array}{c}
f^m_2 + \frac{r_m}{f_m} \\ f^m_3 + \frac{r_m}{f_m}
\end{array} \right]
\end{align*}$$

(28)

Given $\theta^*_f$, $F^m = [F^m_x, F^m_y]^T$ is on the line represented by:

$$F^m_y = F^m_x \tan(\theta^*_f)$$

(29)

By using the equation of a line though points $A$ and $B$ together with (29):

$$(B_x - A_x)(F^m_x \tan(\theta^*_f) - A_x) + (A_y - B_y)(F^m_y - A_y) = 0$$

(30)

Resolving (30) in respect to $F^m_y$, using (27), (28), with the opportune simplifications follows:

$$(d - c) \tan(\theta^*_f) + b - a$$

(31)

The torques required about joints 1 and 2 producing $F^m$ is calculated using (2), (29), and (31):

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = J^T \begin{bmatrix} F^m_x \\ F^m_y \tan(\theta^*_f) \end{bmatrix} = \begin{bmatrix} (-f^m_1 - f^m_2) \frac{r_m}{f_m} (a + \tan(\theta^*_f)) \\ (d - c) \tan(\theta^*_f) + b - a \end{bmatrix}$$

(32)

The muscle activation level $m^i_3$ is found subtracting $f^m$ to $T_1$:

$$m^i_3 = \frac{(-f^m_1 - f^m_2) \frac{r_m}{f_m} (a + \tan(\theta^*_f))}{(d - c) \tan(\theta^*_f) + b - a}$$

(33)

Using the same method $m^i_2$, $m^i_1$, $m^i_2$, $m^i_3$, $m^i_4$ are derived.