# Efficiency Analysis of Powertrain with Toroidal Continuously Variable Transmission for Electric Vehicles

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Abstract—In order to extend electric mileage per charge of Electric Vehicles (EVs), the powertrain efficiency should be further improved. A possible solution is employing transmission which is properly optimized for EV. Continuously variable transmission (CVT) is especially suitable, because it maintains the operating condition of electric motor being closer to the most efficient region even while vehicle speed is changing. In this paper, a toroidal CVT and a single ratio transmission are compared by numerical simulations regarding the overall efficiency. Toroidal CVT has an advantage for high torque-low speed region and low torque-high speed region. However, regarding the rest, the efficiency degrades in contrast. In order to improve the overall efficiency, the best implementation approach is introduced.

#### I. INTRODUCTION

In recent years, there has been growing demands for Electric Vehicles (EVs), because it has large advantages regarding environmental performance, like zero emission and diverseness of energy sources. However, the electric mileage per charge is still much shorter than the mileage of internal combustion engine (ICE) vehicles. It is only from one fifth to one third in practice. So improving overall efficiency of EV is a serious challenge facing.

Permanent magnet synchronous motor (PMSM) is widely used for latest EVs. However, PMSM naturally has a disadvantage for high speed operation because of the back EMF. The inverters for PMSM are imposed to generate higher output voltage while high speed operation. The flux weakening control is often used to overcome the back EMF. But it also degrades the efficiency of the electric motor. Therefore, EV which uses PMSM tends to have narrower speed range than ICE vehicles. Achieving both large drive torque and enough cruising speed is one of a big engineering challenge.

One possible solution is employing a transmission to EV. In generally, internal combustion engines have narrower speed range unlike electric motors. However, the excellent speed range of ICE vehicles are secured by sophisticated transmission. Varieties of transmissions for EV have been proposed by a lot of researchers[1][2][3]. Most of them are only having 2 or 3 speeds. It is enough to secure practical speed range, but for extending the electric mileage.



On the other hand, Continuously Variable Transmission (CVT) is preferable because the reduction ratio continuously varies and it maintains the operating condition of the electric motor being closer to the most efficient region even while vehicle speed is changing. Several efficiency analyses were carried out in the previous study[4][5]. In these previous papers, the efficiencies of CVTs were treated as constants, or the losses of CVTs were ignored. For more precise analysis, the efficiency characteristics should be counted into the simulation of the electric mileage.

In this paper, the efficiency characteristics of a half toroidal CVT is counted into the numerical simulation. The efficiencies vary depending on torque through the transmission and the variator ratio.

## II. SIMULATION MODEL

A. Structure of transmissions

- In this study, we compare two transmissions as following.
- A: Toroidal CVT + gear reducer having fixed gear ratio
- B: Single ratio transmission

A toroidal CVT consists of input disc, output disc, and four power rollers[6]. Each disc contacts with the power rollers through very thin oil film. The oil film transmits tangential force (traction force) between the discs and the rollers. This



Fig. 2. Energy flow diagram of the powertrain.

TABLE I SIMULATION MODEL VEHICLE PARAMETERS

Maximum wheel torque [Nm]	2200
Maximum vehicle speed [km/h]	130
Maximum NET power [kW]	75
Total weight [kg]	1784 or 1771 *
Aerodynamic drag coefficient [-]	0.3
Front drag area [m <sup>2</sup> ]	2.25
Rolling friction coefficient [-]	0.01
Wheel radius [m]	0.305
Battery Rated capacity [kWh]	24
Battery Rated voltage [V]	360
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\* depend on the transmission type

is a typical traction drive. If the inclination angles of the power rollers are changed, the contact points move along with the surfaces on the disc. That means the reduction ratio is controlled by changing the inclination angles of the power rollers. In order to transmit tangential forces, large normal forces are applied on the contact points by a loading device. A mechanical cam or a hydraulic cylinder is often used for the loading device.

The structure of each is shown in the Fig. 1. The output shaft of the motor is connected to each transmission. The output of each transmission is connected to the differential gear which drives two wheels through the drive shafts. The mechanical losses due to the drive shafts and differential gear are ignored. The inertia of each component is also ignored since all simulation conditions are supposed to be steady-state.

#### B. Vehicle and powertrain

Major parameter listing of the simulation vehicle and powertrain model is shown in TABLE I. Classification of the target vehicle is C-segment medium cars. The vehicle weight is determined by reference to weight of a same classification ICE vehicle, and it includes passengers (five passengers, 55kg each).

The powertrain consists of a battery module, an inverter, an electric motor and a transmission. Overall loss of the powertrain  $W_{\rm all}$  is expressed as follow.

$$W_{\rm all} = W_{\rm Inv} + W_{\rm Motor} + W_{\rm Tm} \tag{1}$$

where  $W_{\text{Inv}}$  is the loss of the inverter,  $W_{\text{Motor}}$  is the loss of the electric motor, and  $W_{\text{Tm}}$  is the loss of the transmission. The energy flow diagram of the powertrain is shown in Fig. 2.

## C. Transmission model

Regarding CVT, the mechanical loss is due to torque loss and speed loss. The torque loss dominantly depends on spin slip of the traction surfaces, and the speed loss depends on



 TABLE II

 COEFFICIENT OF TRANSMISSION EFFICIENCY APPROXIMATION

Transmission type	Toroidal CVT	Single ratio
$A_{H2}$	0.0935	0
$A_{H1}$	-0.1871	0
$A_{H0}$	1.0599	1
$A_{L2}$	0.0068	0
$A_{L1}$	-0.0135	0
$A_{L0}$	0.9731	1
$T_{\rm max}$	250	300
В	20	50
$\eta_{\mathrm{vmax}}$	0.98	1.00
$\eta_{ m gr}$	0.99	0.99

creep of the traction surfaces. The speed loss is ignored since the creep ratio of a traction drive is usually around 0.5% or less while it is running properly. So, the loss is equal to the torque loss. Then, the transmission efficiency  $\eta_{\rm Tm}$  is approximated as follow.

$$\eta_{\rm Tm} = k_n(n)k_T(T_r)\eta_{\rm vmax}\eta_{\rm gr}$$
(2)

where n[-] is the variator ratio,  $k_n$  is variator ratio coefficient,  $k_T$  is input torque coefficient,  $\eta_{vmax}[-]$  is maximum efficiency of the variator, and  $\eta_{gr}[-]$  is the efficiency of the gear reducer.

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It is commonly known that the efficiency of the toroidal CVT is low when the variator ratio is close to 1.0[7]. Variator reduction ratio coefficient  $k_n$  is approximated by following empirical formulas. The first one is for increaser side  $(n \le 1)$ , and the second is for reducer side (n > 1).

$$k_n(n) = \begin{cases} A_{H2}n^2 + A_{H1}n + A_{H0} & (n \le 1) \\ A_{L2}n^2 + A_{L1}n + A_{L0} & (n > 1) \end{cases}$$
(3)

where  $A_{H2}$  to  $A_{L0}$  are determined by corresponding experiments. The derivation of (3) are equal to zero when n = 1.

Transmission efficiency tends to be low while the input torque remains small. This is due to the frictional loss. Then, input torque coefficient is approximately expressed as follows.

$$T_r = \frac{T_{\rm in}}{T_{\rm max}} \tag{4}$$

$$k_T(T_r) = 1 - e^{-BT_r}$$
 (5)

where  $T_{in}[Nm]$  is the input torque of the transmission,  $T_{max}[Nm]$  is rated input torque of the transmission, and B is the coefficient which is determined by experiments.

Each coefficient values are shown in TABLE II. The efficiency characteristics are shown in Fig. 3.





# D. Electric motor model

The loss of a interior permanent magnet synchronous motor is modeled as an equivalent iron loss resistance model[8]. Motor current is assumed being managed by the the maximum efficiency control method[8], and specific value for each operating condition is obtained by using the Newton-Raphson method. The motor torque  $T_m$  is expressed as follows.

$$i_d = i_{od} - \frac{\omega L_q i_{oq}}{R_c} \tag{6}$$

$$i_q = i_{oq} + \frac{\omega \left(\Psi_a + L_d i_{od}\right)}{R_c} \tag{7}$$

$$T_m = P_n \left( \Psi_a i_{oq} + (L_d - L_q) i_{od} i_{oq} \right) \tag{8}$$

where  $R_a[\Omega]$  is armature phase resistance,  $i_d$ ,  $i_q[A]$  are d and q-axis current respectively,  $L_d$ ,  $L_q[H]$  are d and q-axis inductance respectively,  $\Psi_a[Wb]$  is the magnetic flux due to the permanent magnets, and  $\omega_e[rad/s]$  is electrical angular speed. If the terminal voltage exceeds the limitation, the motor current will be regulated by the flux weakening control method[9]. The motor loss  $W_{Motor}$  is calculated as follows.

$$W_{\text{Motor}} = W_c + W_f$$
 (9)

$$W_c = R_a(i_d^2 + i_q^2)$$
(10)  
$$W_c = v_{od}^2 + v_{og}^2$$
(11)

$$W_f = \frac{c_{oa} + c_{oq}}{R_c'} \tag{11}$$

$$R_c' = \left(1 - \frac{\omega_m}{\omega_{cor}}\right)^{\kappa_{cor}} R_c \tag{12}$$

where  $W_c[W]$  is the copper loss,  $W_f[W]$  is the iron loss, and  $R_c[\Omega]$  is equivalent iron loss resistance. Under the flux weakening control, large difference arise between (11) and corresponding FEM analysis. Then,  $R_c$  is corrected by (12).  $\omega_{cor}$  and  $k_{cor}$  are correction coefficient which are determined from FEM analysis result.

Two electric motor models are prepared for the simulation. The first one which has narrower speed range is applied for the toroidal CVT. The other is applied for the single ratio transmission. Specifications and the parameter listing are shown in TABLE III.

#### E. Inverter model

The inverter is modeled as a voltage-type PWM inverter. The inverter losses are calculated as follows.

$$W_{\text{Inv}} = 6(W_{\text{IGBTsat}} + W_{\text{FWDsat}} + W_{\text{on}} + W_{\text{off}} + W_{\text{rr}})$$
(13)

 TABLE III

 Specifications and parameters of motor models

Speed range	Narrower	Wider
Max torque [Nm]	200	280
Max speed [rpm]	5000	10000
Base speed [rpm]	3772	2626
Max power [kW]	79	77
d axis inductance $L_d$ [mH]	0.429	0.462
q axis inductance $L_q$ [mH]	0.788	0.806
Magnetic flux $\Psi_a$ [Wb]	0.178	0.188
Armature phase resistance $R_a$ [ $\Omega$ ]	0.0264	0.0237
Equivalent iron loss resistance $R_c$ [ $\Omega$ ]	280	280
$\omega_{cor}$ [rad/s]	2094	2094
$k_{cor}$ [-]	2.8	2.8
Estimated weight [kg]	54	62

where  $W_{\rm IGBTsat}[W]$  is steady-state loss of the IGBT,  $W_{\rm FWDsat}[W]$  is steady-state loss of the free wheeling diode (FWD),  $W_{\rm IGBTon}[W]$  and  $W_{\rm IGBToff}[W]$  is switching loss of the IGBT, and  $W_{\rm FWDrr}[W]$  is recovery loss of the FWD. Steady-state losses are estimated by using following formula[10].

$$W_{i\text{sat}} = \frac{V_{i_0}I_e}{2} \left(\frac{1}{\pi} + \frac{m}{4}\cos\phi\right) + \frac{R_iI_e^2}{2} \left(\frac{1}{4} + \frac{2m}{3\pi}\cos\phi\right)$$
(14)

where *i* indicate IGBT or FWD,  $V_{i_0}[V]$  and  $R_i[V/A]$  are coefficient,  $I_e[A]$  is phase current amplitude, m[-] is modulation factor, and  $\cos \phi[-]$  is power factor. Switching and recovery losses are estimated by using following formula.

$$W_j = \frac{k_j I_e f_s}{\pi} \tag{15}$$

where j indicate the three states, IGBTon, IGBToff, or FWDrr,  $k_j$ [J/A/pulse] is the loss coefficient,  $f_s$ [Hz] is switching frequency. Each coefficients are determined by referring to the data sheet of the general purpose IGBT module 2MBI200VA-060-50 (Fuji Electric).

The efficiency chart of the motor and the inverter are shown in Fig. 4. Values on contour lines are efficiency value in percent.

#### F. Battery model

The battery parameters are used to determine the electric mileage, vehicle weight, and voltage limitation of the motor. The efficiency due to charging and discharging is ignored.





Fig. 5. Optimal variator ratio distribution chart.

#### III. SIMULATION

# A. Optimal variator ratio

A driving performance diagram shows the relation between wheel torque and vehicle speed. By using the numerical models which are introduced in chapter 2, optimal variator ratio which minimize  $W_{\rm all}$  can be determined for all operating point in the driving performance chart. An iterative calculation is used to find the optimal.

Fig. 5 shows the distribution of the optimal variator ratio in the driving performance diagram. Contour lines indicate variator ratio. From the result, high variator ratio (n < 0.6, increaser side) and low variator ratio (n > 2, reducer side) occupy large region. On the low torque region which is less than 500Nm, incleaser side is selected in order to reduce the iron loss of the motor. On the other hand, on the high torque region, reducer side is selected in order to reduce the copper loss of the motor.

#### B. Overall efficiency

Overall efficiency of each cases are simulated by supposing variator ratio tracks the optimal. Results are shown in Fig. 6. Results indicate the followings.

- The distribution of overall efficiency is completely different for each cases.
- In the case of the toroidal CVT, over 90% efficiency occupy large region.
- The single ratio transmission records the highest efficiency.







The comparison of overall efficiency between the toroidal CVT and the single ratio transmission is shown in Fig. 6(c). Contour lines indicate efficiency difference from the single ratio transmission in percent. In Fig. 6(c), positive value means efficiency is improved than the single ratio transmission. The toroidal CVT has advantages over the single ratio transmission in two regions. The first one is high torque - low speed region and the second is low torque - high speed region. For these regions, operating condition could be closer to the optimal by shifting. Regarding the rest, the efficiency degrades in contrast.

In order to analyze the above simulation result, breakdown of the overall losses of two operating conditions are shown in Fig. 7. Operating conditions (vehicle speed and wheel torque) are follows.

- Condition A: 100km/h, 200Nm
- Condition B: 60km/h, 1000Nm



Fig. 8. Simulation result of the five speed transmission.

In the case of the condition A, loss of the motor become less than one-fourth compare to the single ratio transmission. The amount of the motor loss difference is bigger than the loss addition of the CVT. Then, the overall efficiency improves by employing the CVT in this condition.

On the other hand, in the case of the condition B, motor loss increase in spite of the operating condition of the motor is optimized regarding the efficiency. The reason is because required motor output power become bigger by employing the CVT due to the mechanical loss of the CVT. In addition, the loss of the CVT is added into the overall loss. Then, the overall efficiency degrades in this condition.

This phenomenon is unavoidable because transmission efficiency is certainly less than 100%.

## C. Best implementation approach

From the above simulation results, the best implementation of the transmission is considered to have characteristics as follows.

- A transmission has especially high efficiency on the middle speed/torque region. Preferably it directly output the motor input.
- A transmission has more than three speed gear steps. It means reduction gear step, direct gear step, and increaser gear step.
- Transmisson has wide range on increaser side because increaser side is frequently used in actual driving condition.

A candidate of the best implementation is five speed transmission. Parameter listing of the five speed transmission is shown in TABLE IV. This transmission has special gear step set. Gear ratio of the highest (5th) gear step of a conventional five speed transmission for ICE vehicle is generally over 0.6. On the other hand, suggested transmissions 5th gear ratio is 0.25. It is considered that such low gear ratio is effective to efficiency improvement on the low torque region which is less than 500Nm.

In the case of step gear transmission, torque variation occurs between shift change and it detracts driving comfort. But in this study, we only focus on the efficiency.

The simulation result is shown in Fig. 8. Fig. 8(a) shows optimal gear step select. From the result, each gear is equally

 TABLE IV

 Parameters of the five speed transmission

	1st: 4.0
	2nd: 1.8
	3rd: 1.0
	4th: 0.6
Gear reduction ratio [-]	5th: 0.25
Efficiency [-]	0.99 (every gear step)
Rated input torque [Nm]	250
B of the equation (5)	20

used. The envelope of the driving performance has discontinuous parts. This is due to the limited motor performance. These high power operating condition is only used in the limited situation, then it is not serious problem.

Fig. 8(b) shows overall efficiency of the powertrain with five speed transmission, and Fig. 8(c) shows comparison to the single ratio transmission. More than 93% efficiency is achieved on large region. On the low torque region which is less than 200Nm, efficiency imprivement is over 10%. In addition, efficiency degrade on the middle region is less than 2% in most region. It is smaller than in the case of the toroidal CVT.

# D. Electric mileage per charge

The toroidal CVT, the five speed transmission, and the single ratio transmission are compared by simulation at three different driving cycles, Japanese JC08 mode, New European Driving Cycle (NEDC), and U.S. EPA Highway Fuel Economy cycle (HWFET). Time step of the simulation is one second. It is assumed that variator ratio completely track the optimal. Driving resistance consists of acceleration resistance, rolling friction resistance, and aerodynamic resistance. It is assumed that 50% of the regenerating energy is charged to the battery. Parameter listing of the resistance coefficient is shown in TABLE I.

Fig. 9 and TABLE V show the results. The best option is five speed transmission at every driving cycle. The toroidal CVT improve overall efficiency compare to the single ratio transmission in the case of the NEDC and the HWFET, but in the case of the JC08 mode.

Improvement of electric mileage per charge is especially bigger in the case of high speed driving. In the case of the





Fig. 9. Electric mileage simulation result.

TABLE V Electric mileage simulation result

Driving	Average	Electric mileage per charge [km]		
cycle	speed [km/h]	Toroidal CVT	Five speed	Single ratio
JC08 mode 34.8	24.9	204.0	211.6	211.0
	54.0	(-7.0)	(+0.6)	(-)
NEDC	44.4	188.3	194.5	181.3
NEDC	44.4	(+7.0)	(+13.2)	(-)
HWEET	77 7	186.9	190.2	166.4
11 11 12 1	//./	(+20.5)	(+23.8)	(-)

 $\cdot$  (value [km]) are compare with single ratio transmission.

· Average speed excludes idle time.

HWFET, electric mileage per charge improve 14.3% by employing the five speed transmission. From this result, suggested five speed transmission has preferred characteristics for EVs.

#### IV. CONCLUSION

In this paper, the toroidal CVT and the single ratio transmission are compared regarding the overall efficiency and electric mileage by using 3 driving cycles by numerical simulation. Employing CVT to EV have advantage over the single ratio transmission for high torque - low speed region and low torque - high speed region. Regarding the rest, the efficiency degrades in contrast. Five speed transmission is one of the best implementation approach. Electric mileage per charge improve from city driving to fast speed driving condition by employing five speed transmission. Especially, electric mileage improve 14.3% in the case of the HWFET by employing five speed transmission compare to the single ratio transmission. Transmission for EVs is required different reduction ratio setting compare to the transmission for ICE vehicles.

Further works are follows.

- Finding solutions (transmission structure implementation) for spreading out the region where the CVT have advantage.
- Counting the losses due to bearings, clutches and oil pumps in to the simulation.
- Dynamic simulation (counting acceleration resistance) and optimize torque-speed locus regarding efficiency when accelerating or decelerating.
- Optimizing electric motor for combining with the CVT regarding efficiency, size, and weight.

• Making other comparisons for fair assessment regarding different performance.

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