Proposal of Step Climbing of Wheeled Robot Using Slip Ratio Control

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***Abstract*—This paper proposes a method which enables wheeled robot to climb a step that is higher than the radius of the wheel. In conventional method, this is impossible, because driving force of front wheel is not enough. This paper proposes a method utilizing the moment of impact between the wheel and the step to maximize the normal force, and hence the driving force of the front wheel. Effectiveness of the proposed method is verified by simulations and experiments.**

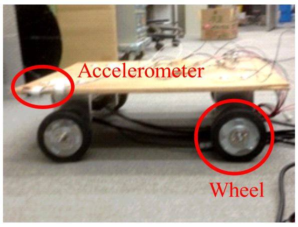
I. Introduction

Aimed at solving challenges raised by aged society and environmental problems, personal mobility vehicles (PMVs) have been intensively studied as alternatives to conventional cars and wheelchairs [1]–[3]. On the other hand, industrial manipulators which are used to convey baggage in the factory and so on are being developed [4]–[6]. In these cases, various robots are researched and developed.

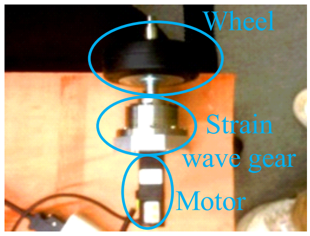
However, these robots have many problems with aspects to safety and operability. One of those problems is to climb a step that is higher than the radius of the wheel [2]. In order to solve this problem, step climbing has been widely researched [6]–[11]. In previous works, the robots can climb the step by moving these legs using some actuators, such as multi-legged walking robot [6][7] and wheel-legged mobile robot [8]–[10]. However, additional actuators result in increased of consumption of energy and complexity [10].

On the other hands, it is known that it is easy to climb the step when the rear load of the robot is heavier than that of front [11]. However, this design has negative influences on other operations such as cornering. S. Nakamura et al [11] invesgitated a robot which can change front and rear loads in order that the rear load becomes heavier at the moment of step climbing. However, it has the same problem as the research [6]–[10].

The purpose of this research is to climb the step using a wheeled robot whose front and rear load are the same and which does not use additional actuators in order to change front and rear load or to move legs. In the case of wheeled robot, as the force exerted on the step is strong, the front driving force is strong, and the robot can climb the step [10]. The force caused by pushing on the step (Reaction force) depends on the rear driving force and on the force caused by the impact on the step (Impact force). The rear driving force is so small that the front driving force is not enough to climb the step. On the other hand, the impact force is so strong that the reaction force is strong. Therefore, by controlling the front driving force at the moment which the robot contacts the step, the front driving force is strong enough for the robot to climb the step.



(a)Wheeled robot



(b)Wheel

Fig. 1. Experimental wheeled robot

This paper proposes the control system that enables wheeled robot climbing the step. The static and dynamic conditions of step climbing are examined, and the control system with reaction force observer (RFO) is proposed. Moreover, effectiveness of the proposed method is verified by simulations and experiments.

II. Experimental Wheeled Robot and Modeling

A. Experimental wheeled robot

An original wheeled robot, designed by the authors' laboratory, is shown in Fig.1. The side-view of the wheeled robot is shown in Fig.1(a), and the enlarged view of wheel is shown in Fig.1(b). Each wheel can be controlled independently by servo motor. Strain wave gear is connected between the motor and the wheel. The reduction ratio of strain wave gear is $1/100$, and the maximum torque of each wheel is 14 Nm. The experimental robot's specification is shown in Table I.

B. Equation of motion of wheeled robot

The diagram of step climbing is shown in Fig.2. Here, F_f [N] and F_r [N] are the front and rear driving force, N_r [N] is the normal reaction force of rear tire, m_f [kg] and m_r [kg]

TABLE I
PARAMETERS OF EXPERIMENTAL EQUIPMENT

the radius of the wheel r [m]	0.050
wheelbase l [m]	0.50
wheeled robot height h [m]	0.15
the mass of wheeled robot M [kg]	10

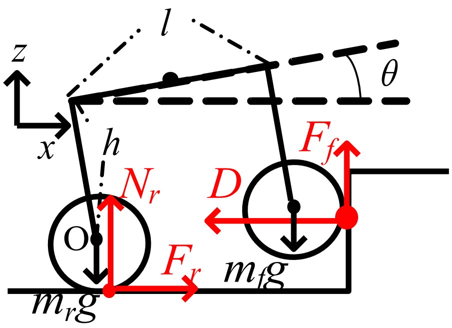


Fig. 2. Diagram of step climbing

are the mass of front and rear tire, g $[\mathrm{m/s^2}]$ is the acceleration of gravity, and θ [rad] is the pitch angle. D [N] is the force caused by pushing the step, which we call "Reaction force" in this paper. The relationship between m_f and m_r is expressed as

$$m_f + m_r = M. \tag{1}$$

In this paper, the mass of the body is ignored.

Lagrangian L is expressed as follow:

$$L = K_f + K_r - U_f - U_r \tag{2}$$

where K_f and K_r are front and rear kinetic energy, and U_f and U_r are front and rear position energy. Equation of motion in the x and θ direction is expressed by Lagrange's equations of motion as follows:

$$\begin{aligned} F_r - D &= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} \\ &= M\ddot{x} - m_f l\ddot{\theta}\sin\theta - m_f l(\dot{\theta})^2\cos\theta, \end{aligned} \tag{3}$$

$$\begin{aligned} F_f(l\cos\theta + r) + F_r r + Dl\sin\theta &= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \\ &= m_f l^2\ddot{\theta} - m_f l\ddot{x}\sin\theta \\ &\quad + m_f g l\cos\theta. \end{aligned} \tag{4}$$

Equation of motion in the x direction is expressed as follows by linearizing (3) as

$$M\dot{v_x} = F_r - D, \tag{5}$$

where v_x $[\mathrm{m/s^2}]$ is the robot velocity in the x direction. On the other hand, equation of motion in the z direction is expressed as follows by linearizing (4) (Assumed that $l \gg r$):

$$(F_f - m_f g)l = m_f l\dot{v_z} - m_f\dot{v_x}v_z, \tag{6}$$

where v_z $[\mathrm{m/s^2}]$ is the robot velocity in the z direction.

III. THE REQUIREMENTS OF STEP CLIMBING

A. Consideration of step climbing in static case (in the case of $\dot{v_x} = 0$ and $\theta \simeq 0$)

In this section, the condition of step climbing in static case($\dot{v_x} = 0$ and $\theta \simeq 0$)) is considered.

In static case, (6) becomes (7),

$$m_f\dot{v_z} = F_f - m_f g. \tag{7}$$

If the front tire climbs the step, v_z must be greater than 0. However, v_z equals 0 before the robot ckimbs the step because the front tire contacts the ground. Therefore, the condition that v_z is greater than 0 is as follow in the case that the front tire contacts the ground,

$$\dot{v_z} > 0. \tag{8}$$

From (7) and (8), the requirement that the front tire leaves the ground is expressed as (9),

$$F_f > m_f g. \tag{9}$$

The relationship between F_f and D is expressed as (10),

$$F_f = \mu_f D, \tag{10}$$

where μ_f is the friction coefficient between the front tire and the step. The maximum coefficient of static friction is μ_{f0}, so the maximum of F_f is expressed as

$$F_f \leqq \mu_{f0}D. \tag{11}$$

Therefore, μ_f or D must be increased in order to increase F_f. In this section, the way of increasing D is considered.

From (5), D is described as

$$D = F_r - M\dot{v_x}. \tag{12}$$

In static case($v_x = 0$), D equalls F_r.

Moreover, the maximum of F_r is expressed as

$$F_r = \mu_r m_r g \leqq \mu_{r0} m_r g, \tag{13}$$

where μ_r is the friction coefficient between the rear tire and the road and μ_{r0} is the biggest coefficient of static friction. From (10) to (13), (14) is expressed,

$$F_f \leqq \mu_{f0}\mu_{r0} m_r g = k\mu_f\mu_r m_f g, \tag{14}$$

where $k(=\frac{m_r}{m_f})$ is the ratio of front and rear tire mass. From (9) and (14), k must satisfy (15) in order to climb the step,

$$k > \frac{1}{\mu_{f0}\mu_{r0}}. \tag{15}$$

In static case, m_r must become bigger than m_f because μ_{f0} and μ_{r0} are smaller than 1. In the case of $k > 1$, however, the problems are occurred as below,

- It is difficult for the rear tire to climb the step.
- It has negative influences on other operations such as cornering.

Therefore, it is desirable that m_f and m_r are the same. In that case, however, F_f is under $m_f g$ from (14), so it does not satisfy (9).

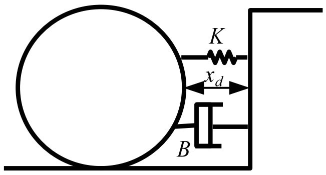


Fig. 3. Model diagram of reaction force D

Thus, in static case, the wheeled robot which does not use actuators in order to change the front/rear mass cannot climb the step if m_f and m_r are the same.

B. Consideration of step climbing in dynamic case ($\dot{v_x} \neq 0$)

In this section, the condition of step climbing in dynamic case (The robot driving at a constant speed impacts the step) is considered.

1) Analysis of the moment of impact: At the moment the robot impacts the step, D increases because the force F_I (expressed as (16)) is occurred,

$$F_I = -M\dot{v_x}. \tag{16}$$

In this case, the maximum of F_f is increased. Therefore, it is considered that the front tire leaves the ground by controlling F_f at the moment of impact.

2) Analysis after the front tire leaves the ground: Right after the front tire leaves the ground, the moment can be expressed as

$$\begin{aligned} I\ddot{\theta} &= F_f(l\cos\theta + r) - m_f gl\cos\theta \\ &\quad + m_r gr + Dl\sin\theta + m_f l\ddot{x}\sin\theta, \end{aligned} \tag{17}$$

where I $[\mathrm{kg}\cdot\mathrm{m}^2]$ is the inertia of wheeled robot($I = m_f l^2$). Thanks to the above moments, θ is increased, and the distance from front tire to the ground is increased.

IV. Control System for Step Climbing

From considerations in section Ⅲ, increasing F_f at the moment of impact enables the front tire leave the ground. If μ_f or D is increased, F_f is increased because F_f is expressed as (10). As is said in section Ⅲ, D is increased at the moment of impact. Therefore, if μ_f can be controlled at the moment of impact, F_f is increased. In this section, the control system which is based on the slip ratio control method researched studied in the field of electric vehicle [12] is proposed.

A. Modeling of Reaction Force D

The model diagram of reaction force D is shown as Fig.3. As shown in (18), D is the sum of elastic force and friction force assumed that spring and damper are inserted into tire,

$$D = Kx_d + B\dot{x_d}, \tag{18}$$

where K $[\mathrm{kg/s^2}]$ is the elasticity of spring, B $[\mathrm{kg/s}]$ is the coefficient of viscosity, and x_d $[\mathrm{m}]$ is the magnitude of tire.

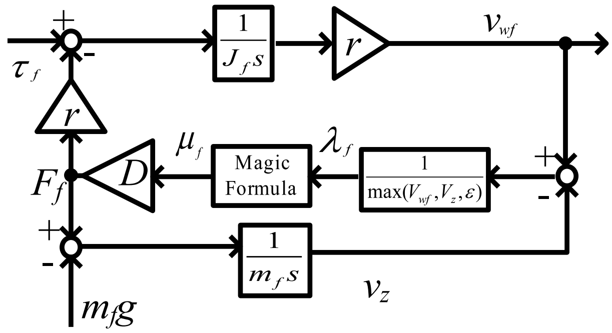


Fig. 4. Block diagram of front wheel in the z direction

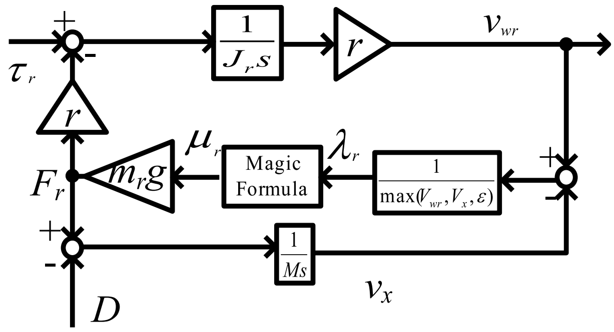


Fig. 5. Block diagram of rear wheel in the x direction

B. Modeling of front and rear tire

The block diagram of front wheel in the z direction is shown as Fig.4, and the block diagram of rear wheel in the x direction is shown as Fig.5. The equations of the wheeled robot are expressed as

$$J_i\dot{\omega}_i = \tau_i - rF_i, \tag{19}$$

$$m_f\dot{v_f} = F_f - m_f g, \tag{20}$$

$$M\dot{v_x} = F_r - D, \tag{21}$$

$$v_{\omega_i} = r\omega_i, \tag{22}$$

where ω_i $[\mathrm{rad/s}]$ is the wheel angular velocity, J_i $[\mathrm{kg}\cdot\mathrm{m}^2]$ is the wheel inertia, τ_i $[\mathrm{N}\cdot\mathrm{m}]$ is the motor torque, and $v_{\omega i}$ $[\mathrm{m/s}]$ is the wheel velocity. Also, i is indices for f/r (front/rear).

Also, the slip ratio of front and rear tire λ_i is expressed as eq(23),

$$\lambda_i = \frac{v_{\omega_i} - v_j}{\max(v_{\omega_i}, v_j, \epsilon)}, \tag{23}$$

where ϵ is a tiny value to prevent division by zero. j is indices for x/z. If i is f, j is z, and if i is r, j is x.

The relationship between μ_i and λ_i depends on the road condition, and is known to be Magic Formula [13]. F_f and F_r are expressed as (10) and (13).

C. Reaction Force Observer(RFO)

The block diagram of Reaction Force Observer(RFO) is shown in Fig.6. Nominal plant $P_{1n}(s)$ and $P_{2n}(s)$ are ex-

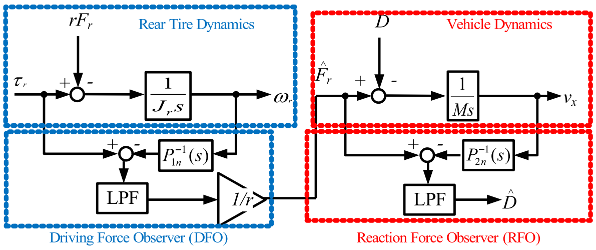


Fig. 6. Block diagram of Reaction Force Observer(RFO)

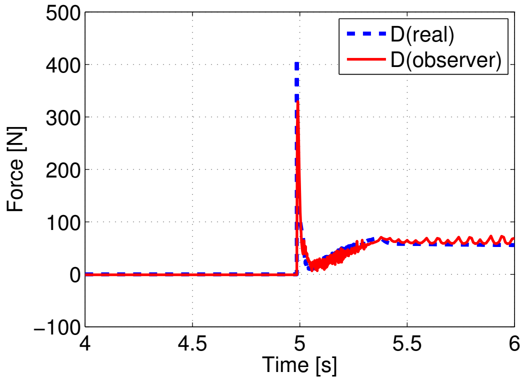


Fig. 7. Simulation result(Estimation of Reaction Force)

pressed as

$$P_{1n}(s) = \frac{1}{J_{rn}s}, \tag{24}$$

$$P_{2n}(s) = \frac{1}{M_n s}. \tag{25}$$

F_r can be estimated by Driving Force Observer (DFO)[12]. By using F_r estimated by DFO and v_x measured by the accelerometer, D can be estimated by disturbance observer (RFO).

The precision of estimation D using RFO is verified by simulation. Fig.7 shows the result. The estimated value $\hat{D}$ and the real value D are almost the same although they are a little different at the moment of impact.

D. Proposed control system for step climbing based on slip ratio control

The block diagram of proposed control system is shown as Fig.8. From (23), the wheel angular velocity reference ω_i^* [rad/s] is calculated as

$$\omega_i^* = \frac{v_j}{r(1-\lambda_i^*)}, \tag{26}$$

where λ_i^* is the slip ratio reference. Each controller is designed as PI law using pole placement method.

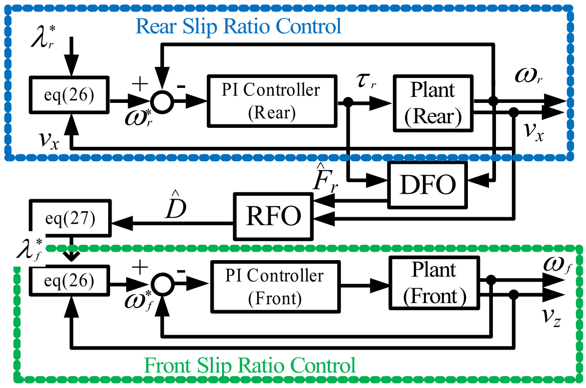


Fig. 8. Block diagram of front/rear slip ratio control

The front slip ratio reference is determined as (27), using $\hat{D}$ estimated by RFO,

$$\lambda_f^* = \begin{cases} 0 & (\hat{D} \leqq D_{th}) \\ \lambda_{peak} & (\hat{D} > D_{th}) \end{cases}, \tag{27}$$

where D_{th} [N] is threshold determined at one's will. By controlling the front slip ratio and maximizing μ_f, F_f is increased because of (5).

V. SIMULATIONS

In this section, effectiveness of the proposed method is verified by simulations.

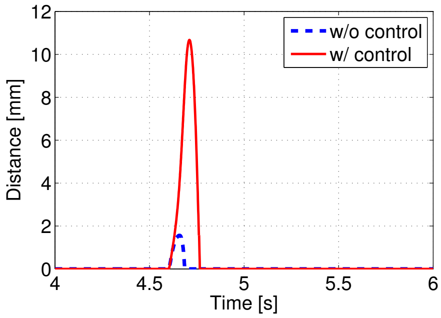
Assumed the experimental wheeled robot, simulation model is established using the parameters in Table.I. In this simulations, m_f and m_r are same. Also, μ_{f0} and μ_{r0} are 0.5. The gain of controller is determined by pole placement method. And the pole of feedback control of front tire is -30 rad/s, and that of rear tire is the same. The front slip ratio reference is calculated by (27), which D_{th} is 10 N. The rear slip ratio reference is determined as v_x is 0.15 m/s.

A. Comparison of the distance from the front tire to the ground

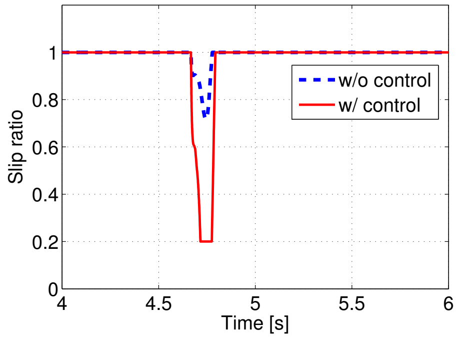
Assumed that wheeled robot impacts the wall which height is infinite at a constant speed (0.15 m/s), the distance from the front tire to the ground z is verified. The results are shown as Fig.9. Compared to the method that without proposed control (w/o control), the distance from the front tire to the ground when proposed control is used (w control/) increases. These results show that effectiveness of proposed control system is verified by this simulation.

B. Verifying of step climbing

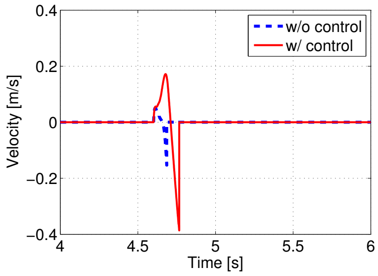
Whether the wheeled robot can climb the step which is 0.060 m height or not is verified by simulation. A constant impacting speed (0.15 m/s) is assumed. The results are shown as Fig.10. As Fig.10(a) shows, the front tire climbs the step. These results show that effectiveness of step climbing using proposed control method is verified.



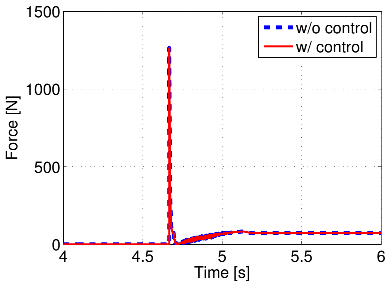
(a)Distance from ground z



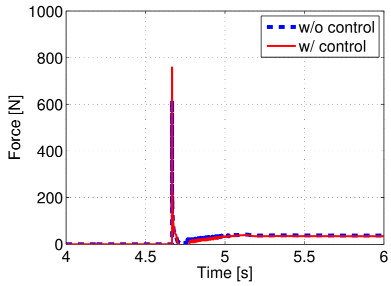
(b)Slip ratio λ_f



(c)Velocity v_z

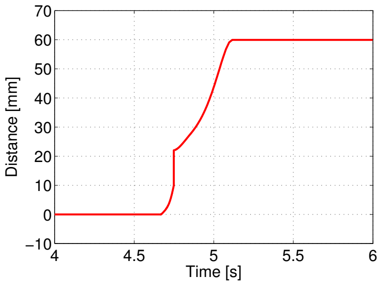


(d)Reaction force D

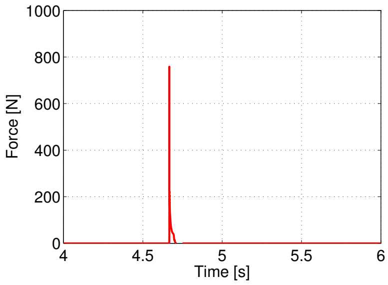


(e)Driving force F_f

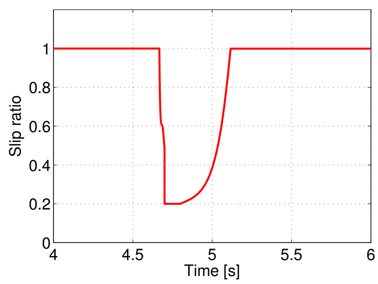
Fig. 9. Simulation results (Comparison of distance from ground)



(a)Distance from ground z



(b)Driving force F_f



(c)Slip ratio λ_f

Fig. 10. Simulation results (60 mm step climbing)

VI. Experiment of Step Climbing Using Wheeled Robot

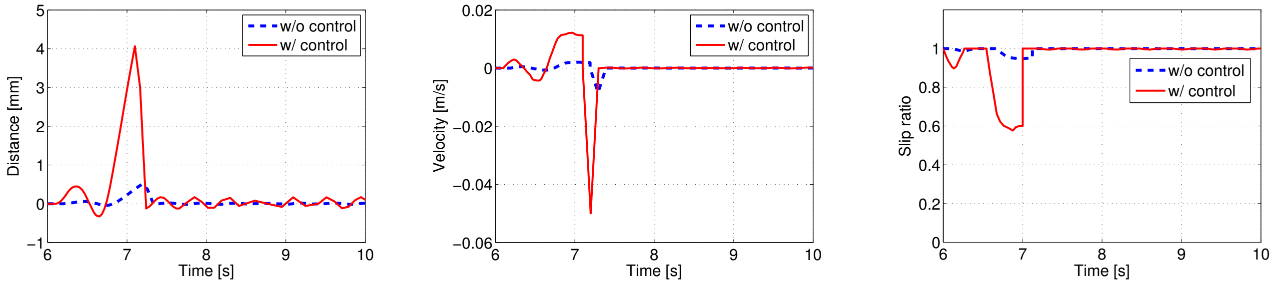
In this section, effectiveness of the proposed method is verified by experiments using experimental wheeled robot.

A. *Verifying step climbing*

Whether the robot can climbed the 0.060 m height step when the robot drives and impacts the step at a constant speed (0.15 m/s) is verified. Experimental results is shown as Fig.11. Compared to the method that without proposed control (w/o control), the distance from the front tire to the ground when proposed control is used (w control/) increases. However, the robot cannot climb the step. As Fig.11(c) shows, the front slip does not track the front slip ratio reference (0.2). The reason why step climbing is not realized is that slip ratio control does not work well. This is because F_f satisfies 9 only few time.

B. *Verifying the relashinship between elocity v_x and the distance z*

Same experiments are done under varying velocity v_x (0.10 m/s, 0.20 m/s, and 0.30 m/s). Fig.12 shows the results about the relationship between velocity v_x and the distance x.



(a)Distance from ground z (b)Velocity v_z (c)Slip ratio λ_f

Fig. 11. Experimental results (Step climbing)

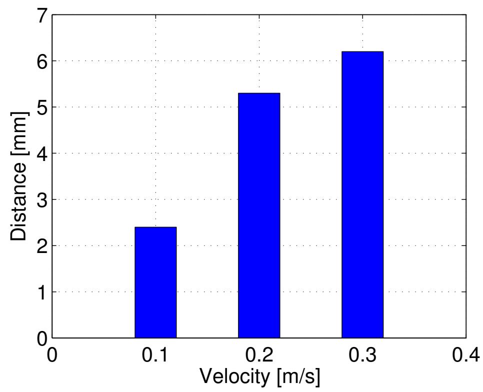


Fig. 12. Experimental results (Relationship between v_x and z)

As the velocity is larger, the distance is larger. This is because F_I and D increase because of the increasing v_z.

VII. Conclusion

In this paper, a control method for the wheeled robot whose front and rear load are same and which does not use actuators for changing front and rear mass to climb the step is proposed. Although it is hopeful that the mass of front tire and that of rear tire are same considering other operation such as cornering, the front driving force is not enough for the wheeled robot to climb the step when the robot begins working at the condition of contacting with step. Therefore, we verify whether or not the robot can climb the step by controlling front tire at the moment it impacts the step. The control method based on slip ratio and reaction force observer used in proposed method are proposed, and step climbing by proposed method is verified by simulations and experiments. It is verified that the front tire leaves the ground when proposed method is used at the moment of impact, although the robot cannot climb the step.

The future work is to increase the distance from the front tire to the ground. When velocity is changed, the loads of front and rear tire are changing because of acceleration and deceleration. It will be verified whether the robot can climb the step as the velocity is being changed.

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