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Equivalent Circuits for Repeater Antennas Used in Wireless Power Transfer via Magnetic Resonance Coupling

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SUMMARY

The demand for wireless power transfer via magnetic resonance coupling is increasing. Magnetic resonance coupling is a new technology that achieves power transfers across a large air gap by using transmitting and receiving antennas. However, repeater antennas can enable power transmission across an even larger distance. These repeater antennas without cross coupling can be expressed as a T-type equivalent circuit. Equivalent circuits that include cross coupling and mutual inductance, which is related to the antenna position, have not been studied. In this paper, a novel way to represent a repeater antenna by an equivalent circuit and a way to determine the mutual inductance are proposed and verified by performing an electromagnetic field analysis and experiment. © 2013 Wiley Periodicals, Inc. Electr Eng Jpn, 183(1): 51–62, 2013; Published online in Wiley Online Library (wileyonlinelibrary.com). DOI 10.1002/eej.22360

Key words: repeater antenna; wireless power transfer; resonance; magnetic coupling.

1. Introduction

Wireless power transfer technology has been attracting attention as a way to overcome the problem of limited power of mobile devices. Users want their mobile devices to be charged automatically by wireless power transfer technology. Wireless power transfer is used not only for small mobile devices but also for electric vehicles with batteries similar to mobile devices. Wireless power transfer for devices without batteries has also been attracting attention. There are several ways of developing wireless power transfer technology. Among these, the most promising is electromagnetic resonance coupling [1–3]. In practical use, electromagnetic resonance coupling provides higher efficiency and larger air gaps than typical wireless power transfer technologies such as electromagnetic induction [4–6], microwave power transmission [7, 8], and laser power transmission [9, 10]. This is why electromagnetic resonance coupling has attracted so much attention. However, because this technology appeared only recently (in 2007), there are many problems yet to be solved. It is possible to address these problems using an equivalent circuit representing a transmitting antenna and a receiving antenna used for power transfer, which also reveals the conditions for achieving the maximum power transfer efficiency [11, 12]. However, a detailed study on repeater antennas has not yet been made.

A repeater antenna is defined as an antenna that is installed between the transmitting antenna and receiving antenna and operates at the same resonant frequency as the transmitting and receiving antennas. The aim of a repeater antenna is to extend the scope of wireless power transfer. This technology can be used to expand the feeding area for wireless charging of electric vehicles [13–16]. In the future, it may be used to feed electric vehicles moving along a highway simply by installing repeater antennas. Naturally, this technology can be adapted for interior use. Repeater antennas can be installed on walls, desks, and floors to realize wireless homes in which electric appliances are charged wirelessly and automatically from any location in the house, and a house robot can work continuously because it does not need to be plugged in for charging. Thus, repeater antennas have huge application potential.

Although there have already been many studies of repeater antennas, they have been limited to electromagnetic field analyses. There has been no detailed theoretical study, equivalent circuit study, or mutual inductance study related to antenna positions [17, 18].

Therefore, in this paper an equivalent circuit for repeater antennas is proposed by considering the signs of the mutual inductance in three dimensions, which represent the planar and vertical positions of a repeater antenna.

2. Fundamental Characteristics of Repeater Antennas

In this section the equivalent circuit of a fundamental repeater antenna is studied. Therefore, in this case the cross coupling between repeater antennas can be ignored. Cross coupling refers to the effect of connection between antennas that are not adjacent to each other. For example, when these antennas are arranged in the order of "transmitting antenna, repeater antenna, and receiving antenna," there is a repeater antenna between the transmitting antenna and the receiving antenna. In this case, cross coupling refers to the connection between the transmitting antenna and the receiving antenna, without considering the repeater antenna.

2.1 One repeater antenna (straight)

Electromagnetic field analysis is performed to determine the fundamental characteristics of wireless power transfer by magnetic resonance coupling. The analysis uses the method of moments model. The antenna parameters of the model are shown in Fig. 1.

All the antennas used in this study have the same parameters: radius r 150 mm, 5.5 turns, turn pitch p_s 5 mm, layer pitch p_h 10 mm, and two-layer configuration. This spiral-type and open-type configuration causes the antenna to resonate by itself, and it operates at 13.56 MHz. The input and output ports of the transmitting and receiving antennas have characteristic impedances of 50 Ω. When an antenna is used as a repeater antenna, the port is shorted. The power transfer efficiency between the transmitting and receiving antenna [19] was evaluated using the experimental setup shown in Fig. 2. From left to right, the photograph shows the transmitting, repeater, and receiving antennas. The antenna parameters were the same as in the electromagnetic field analysis. However, the turns are adjusted to match 13.56 MHz in the experiment. Therefore, 5 turns were used. A vector network analyzer was used in the experiment. The output power from Port 1 at the vector network analyzer became the transmitted power, which passed to Port 3, reflected to Port 1, and was consumed as copper and radiative losses. The amplitude a_1 was input from Port 1, while the reflected amplitude b_1 and the transmitted amplitude b_3 went to Port 3. The power ratio is the square of the amplitude. Therefore, the efficiency η_{31} from Port 1 to Port 3 is calculated by the transmission S_{31}, which is described by Eq. (1). The power reflection ratio η_{11}, which is calculated by S_{11}, is the reflection of Port 1, as described by Eq. (2):

$$\eta_{31} = |S_{31}|^2 = \left|\frac{b_3}{a_1}\right|^2 \quad (1)$$

$$\eta_{11} = |S_{11}|^2 = \left|\frac{b_1}{a_1}\right|^2 \quad (2)$$

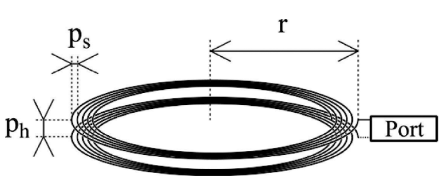
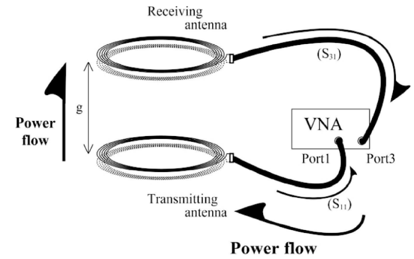
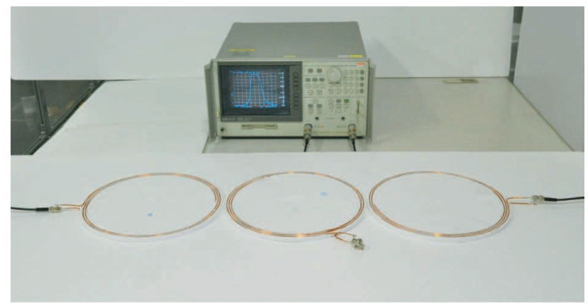


Fig. 1. Parameters of spiral antenna.

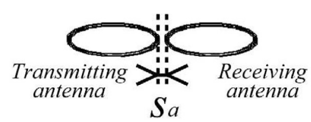


(a) Experimental configuration and parameters

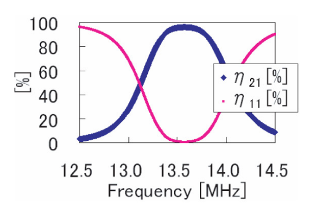


(b) Antennas and VNA

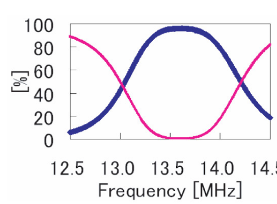
Fig. 2. Experimental configuration. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Electromagnetic field analysis model

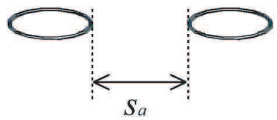


(b) Electromagnetic field analysis result

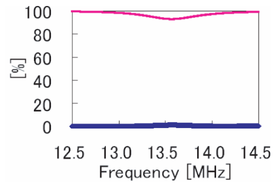


(c) Experimental result

Fig. 3. Model and results for transmitting and receiving antennas without repeater antenna (s_a = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



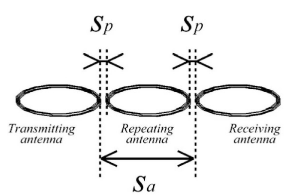
(a)



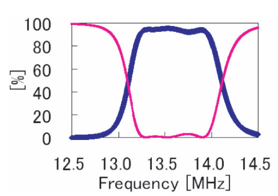
(b)

Fig. 4. Model and results for transmitting and receiving antennas without repeater antenna (s_a = 320 mm). (a) Electromagnetic field analysis model. (b) Electromagnetic field analysis result. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

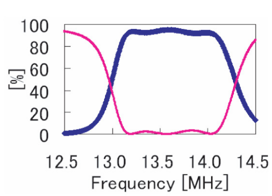
The electromagnetic field analysis and experimental results are shown in Fig. 3, where the air gap between the antennas is s_a = 10 mm. This result indicates that high-efficiency power transfer is possible even if the receiving antenna is placed beside the transmitting antenna. However, as the air gap is increased, as shown in Fig. 4, where the air gap is s_a = 320 mm, the power is not transferred but is almost entirely reflected. Therefore, the efficiency is improved by installing a repeater antenna between the transmitting and receiving antennas with an air gap of s_p = 10 mm, as shown in Fig. 5. The cross coupling between the transmitting and receiving antennas can be ignored when it is calculated in the equivalent circuit because the distance between the transmitting and receiving antennas is s_a = 320 mm, which is sufficiently large. The equivalent circuit should be described as in Fig. 6(a) because the electromagnetic resonance coupling antenna can be treated as an LCR self-resonance, and the coupling is magnetic, which can be treated as a mutual inductance L_m. The self-inductance of the transmitting antenna is L_1, that of the repeater antenna is L_2, and that of the receiving antenna is L_3. The coupling efficiency of the transmitting and repeater antennas is k_{12}, and the mutual inductance is L_{12}. The coupling efficiency of the repeater antenna and receiving antenna is k_{23}, and the mutual inductance is L_{23}. All the antennas' self-inductance parameters are the same, $L_1 = L_2 = L_3$. The equivalent circuit in Fig. 6(a) is also represented as a T-type circuit, as shown in Fig. 6(b). The efficiency and power reflection ratio included in the repeater antenna are calculated. The inductance is expressed by Eq. (3) because the mutual inductance can be ignored, and the impedance of the circuit is given by Eqs. (4) and (5). The S-parameter is expressed by Eq. (6). Therefore, the relation between the impedance and the S-parameter is described by Eq. (7). The parameters are expressed by Eqs. (8) to (11) and are used in Eq. (7). The characteristic impedance, which is written as Z_0, is given in Eq. (8). As explained above, the efficiency is expressed by Eq. (1) from the relation between S_{31} and η_{31}. The power reflection ratio is expressed by Eq. (2) from the relation between S_{11} and η_{11}.



(a) Electromagnetic field analysis model

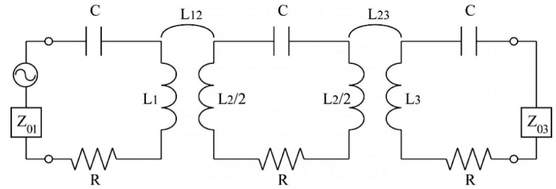


(b) Electromagnetic field analysis result

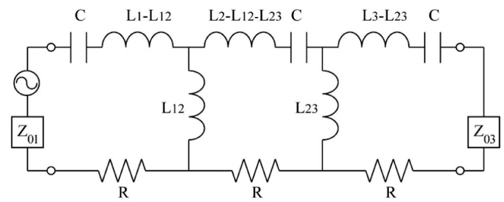


(c) Experimental result

Fig. 5. Model and results for transmitting and receiving antennas with repeater antenna (s_a = 320 mm, s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Equivalent circuit



(b) T-type equivalent circuit

Fig. 6. Equivalent circuit of repeater antenna without cross coupling between transmitting and receiving antennas.

$$[\boldsymbol{L}] = \begin{bmatrix} L_1 & L_{12} & 0 \\ L_{12} & L_2 & L_{23} \\ 0 & L_{23} & L_3 \end{bmatrix} \tag{3}$$

$$[\boldsymbol{Z}] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \tag{4}$$

$$[\boldsymbol{Z}] = \begin{bmatrix} R+j\left(\omega L-\dfrac{1}{\omega C}\right) & j\omega L_{12} & 0 \\ j\omega L_{12} & R+j\left(\omega L-\dfrac{1}{\omega C}\right) & j\omega L_{23} \\ 0 & j\omega L_{23} & R+j\left(\omega L-\dfrac{1}{\omega C}\right) \end{bmatrix} \tag{5}$$

$$[\boldsymbol{S}] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \tag{6}$$

$$[\boldsymbol{S}] = \left\{[\hat{\boldsymbol{Z}}] + [\boldsymbol{I}]\right\}^{-1} \left\{[\hat{\boldsymbol{Z}}] - [\boldsymbol{I}]\right\} \tag{7}$$

$$[\boldsymbol{Z}_0] = \begin{bmatrix} Z_{01} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_{03} \end{bmatrix} \tag{8}$$

$$[\boldsymbol{Y}_0] = [\boldsymbol{Z}_0]^{-1} \tag{9}$$

$$\left[\hat{\boldsymbol{Z}}\right] = \left[\sqrt{\boldsymbol{Y}_0}\right][\boldsymbol{Z}]\left[\sqrt{\boldsymbol{Y}_0}\right] \tag{10}$$

$$[\boldsymbol{I}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

The parameters are calculated by electromagnetic field analysis. The parameters of the transmitting, receiving, and repeater antennas in Fig. 5 are as follows: self-inductance L = 11.0 μH, capacitance C = 12.5 pF, internal resistance R = 0.77 Ω, mutual inductance $L_{12} = L_{23}$ = 0.542 μH, coupling coefficient $k_{12} = L_{12}/L$ = 0.049, $k_{23} = L_{23}/L$ = 0.049, and $Z_{01} = Z_{03}$ = 50 Ω [12]. The equivalent circuit calculation results are shown in Fig. 7. Comparing these results with Fig. 5 shows that the equivalent circuit results agree with the electromagnetic field analysis results. Thus, the equivalent circuit of the repeater antenna, where the cross coupling may be ignored, can be expressed as shown in Fig. 6.

2.2 Multiple repeater antennas (straight)

In the previous section only one repeater antenna was examined. However, a single repeater antenna is not always

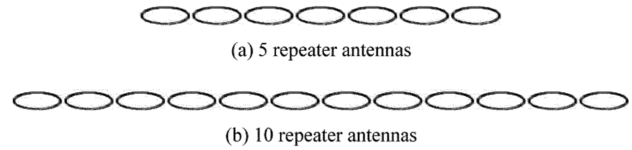


Fig. 8. Electromagnetic field analysis models of multiple repeater antennas (s_p = 10 mm).

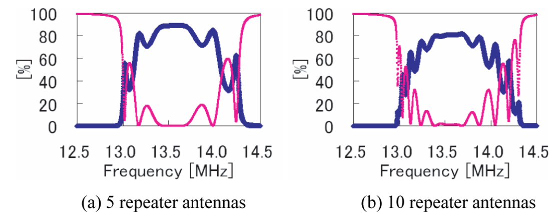


Fig. 9. Electromagnetic field analyses of multiple repeater antennas, s_p = 10 mm. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

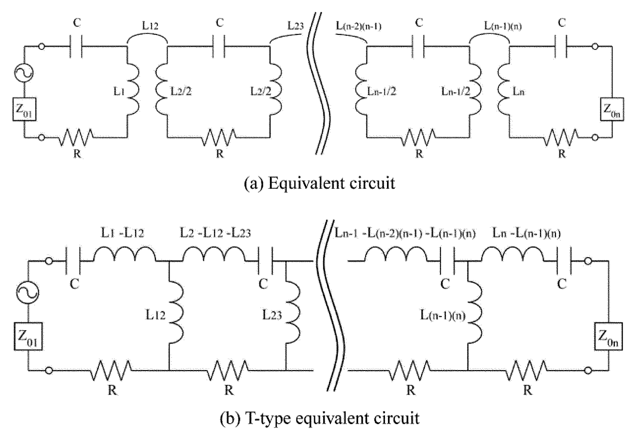


Fig. 10. Equivalent circuits for multiple repeater antennas without cross couplings between antennas.

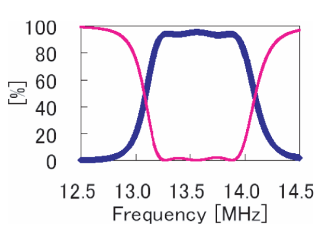


Fig. 7. Efficiency calculation of equivalent circuit with repeater antenna (s_a = 320 mm, s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

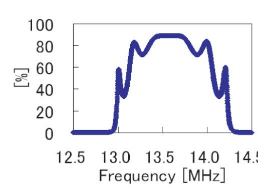


Fig. 11. Calculation result of equivalent circuit with 5 repeater antennas (s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

sufficient. Multiple repeater antennas can be used, and it should be possible to express these using an equivalent circuit. Therefore, configurations of 5 and 10 repeater antennas are examined using electromagnetic field analyses and equivalent circuits. The diagrams of the electromagnetic field analysis models are shown in Fig. 8, and the results are shown in Fig. 9. The equivalent circuit is shown in Fig. 10. The transmitting antenna is at the far left, the receiving antenna is at the far right, and 5 or 10 repeater antennas are in between them in the electromagnetic field analyses. In the equivalent circuit, the number of repeater antennas is denoted by n. The equivalent circuit result when 5 repeater antennas are used is shown in Fig. 11. The mutual inductance is 0.542 μH, the coupling coefficient is 0.049, and $Z_{01} = Z_{0n} = 50\ \Omega$. This result agrees with that in Fig. 9(a), which means that cases using more than 2 repeater antennas can be calculated by using an equivalent circuit.

3. Cross Coupling and Mutual Inductance

When the distance between the transmitting antenna and receiving antenna is large, the coupling coefficient is close to 0, which means that the effect of cross coupling may be ignored. Therefore, the equivalent circuit can be expressed by a T-type equivalent circuit, as shown in Figs. 6 and 10. However, it is not suitable to use a T-type equivalent circuit in a case where cross coupling must be considered. Thus, in this section, an equivalent circuit that considers the effect of cross coupling is studied. The mutual inductance is discussed when the antennas are arranged vertically and horizontally.

3.1 Equivalent circuit considering cross coupling

An equivalent circuit that considers cross coupling between the transmitting antenna and receiving antenna is shown in Fig. 12. Because cross coupling between the transmitting and receiving antennas is considered in this case, the coupling coefficient of the transmitting and receiving antennas is k_{13}, the mutual inductance is L_{13}, the inductance of this circuit is expressed by Eq. (12), and the impedance is expressed by Eq. (13). The antenna parameters are the same as in the previous section: self-inductance $L = 11.0$ μH, capacitance $C = 12.5$ pF, internal resistance $R = 0.77\ \Omega$, and $Z_{01} = Z_{03} = 50\ \Omega$.

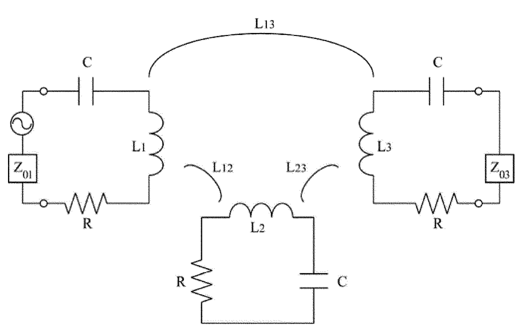


Fig. 12. Equivalent circuit of repeater antenna when effect of cross coupling is taken into consideration.

$$[\boldsymbol{L}] = \begin{bmatrix} L_1 & L_{12} & L_{13} \\ L_{12} & L_2 & L_{23} \\ L_{13} & L_{23} & L_3 \end{bmatrix} \tag{12}$$

$$[\boldsymbol{Z}] = \begin{bmatrix} R+j\left(\omega L-\dfrac{1}{\omega C}\right) & j\omega L_{12} & j\omega L_{13} \\ j\omega L_{12} & R+j\left(\omega L-\dfrac{1}{\omega C}\right) & j\omega L_{23} \\ j\omega L_{13} & j\omega L_{23} & R+j\left(\omega L-\dfrac{1}{\omega C}\right) \end{bmatrix} \tag{13}$$

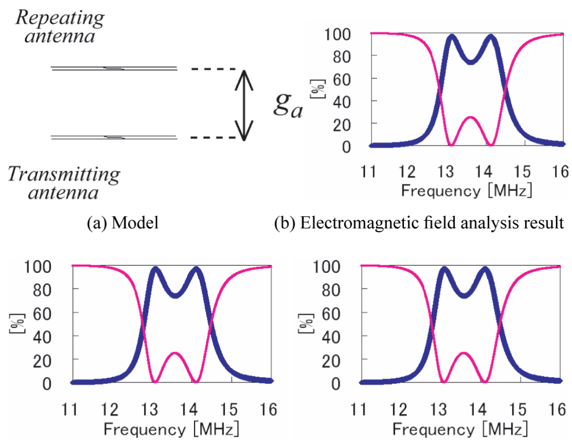
3.2 Sign of mutual inductance

Information about the sign of the mutual inductance L_m is not shown because the coupling coefficient is calculated with two resonant angle frequencies ($\omega_m < \omega_e$) using Eq. (14) [11]:

$$k = \frac{L_m}{L} = \frac{\omega_e^2 - \omega_m^2}{\omega_e^2 + \omega_m^2} \tag{14}$$

It is not necessary to consider the sign of the mutual inductance because it has no effect on the characteristics of efficient power transfer when only transmitting and receiving antennas are used, without a repeater antenna. This phenomenon is studied for cases in which the transmitting and receiving antenna positions are arranged vertically and horizontally.

Figure 13 shows the electromagnetic field analysis model and results, along with the equivalent circuit results with negative and positive mutual inductance values, for a vertical alignment of the transmitting and receiving antenna positions, and with no repeater antenna used. The coupling coefficient k is 0.092. The mutual inductance L_m is 1.019 μH in Fig. 13(c), and is –1.019 μH in Fig. 13(d). Similarly, Fig. 14 shows the equivalent circuit results for transmitting and receiving antennas arranged horizontally without a repeater antenna, with positive and negative signs for the mutual inductance. The electromagnetic field analysis and experimental results are shown in Fig. 3. The coupling coefficient k is 0.049. The mutual inductance is $L_m = 0.542$ μH in Fig. 14(a), and $L_m = -0.542$ μH in Fig. 14(b). According to these results, the sign of the mutual inductance can be ignored when only the transmitting and receiving antennas are used. However, when the repeater antenna is used, the sign of this parameter cannot be ignored.



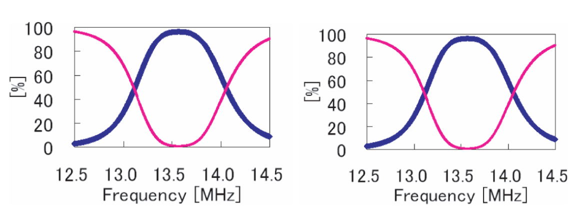
(a) Model (b) Electromagnetic field analysis result

(c) Equivalent circuit ($+L_m$) (d) Equivalent circuit ($-L_m$)

Fig. 13. Model and results of wireless power transfer in vertical direction (g_a = 150 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Therefore, the magnetic field connections around the antennas are discussed next.

The parameters of the transmitting and receiving antennas are shown in Fig. 15, and the efficiency distribution around the antennas is shown in Fig. 16. The results in Fig. 16 are obtained when the operation frequency matches the resonance frequency in order to determine the peak efficiency at each position. High-efficiency wireless power transfer is possible when the receiving antenna is in the vertical position as in Fig. 16 (A) or the horizontal position as in Fig. 16 (B). However, in the null position shown in Fig. 16 (C), between the vertical and horizontal directions, the efficiency worsens. Thus, the crossing direction of the magnetic fields is determined geometrically. A magnetic field illustration is shown in Fig. 17. There are two configurations in which the efficiency is high. In the first configuration, the magnetic fields across the transmitting antenna and receiving antenna are aligned in the same direction, as shown in Fig. 17(a), with the magnetic field H_g moving across the antenna along loop C_g. In the second configuration, the magnetic fields across the transmitting and receiving antennas are moving in opposite directions, with one magnetic field coming from over the first antenna and the other coming from under the second antenna, as shown in Fig. 17(b), where the magnetic field H_s moves across the antenna along loop C_s. Of course, these directions change multiple times every second; however, the combination of the same direction or reverse direction does not change. The condition in Fig. 17(a) corresponds to Fig. 16 (A) and that in Fig. 17(b) corresponds to Fig. 16 (B), with high efficiency for either condition. On the other hand, in the null direction, the portion of the magnetic field moving in the same direction (loop C_g) cancels the magnetic field moving in the reverse direction (loop C_s). Therefore, the efficiency should be worse, as shown in Fig. 16 (C). If only the transmitting and receiving antennas are used, the magnetic fields move in either the same direction or opposite directions, which can be expressed by using a positive value. However, if there is a repeater antenna, it is possible for the magnetic fields to cancel each other. Therefore, in the next



(a) Equivalent circuit ($+L_m$) (b) Equivalent circuit ($-L_m$)

Fig. 14. Analysis results of wireless power transfer in horizontal direction (s_a = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

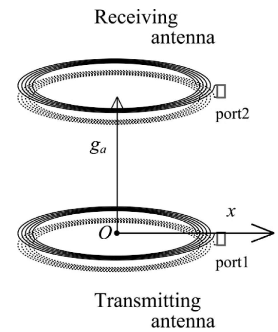


Fig. 15. Parameters of transmitting and receiving antennas. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

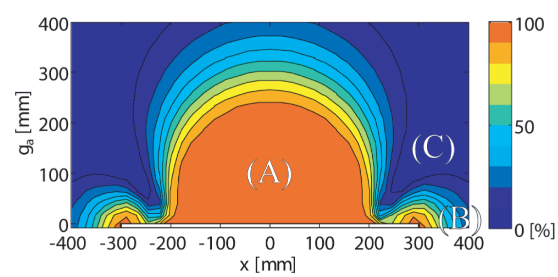


Fig. 16. Efficiency in x and g directions. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

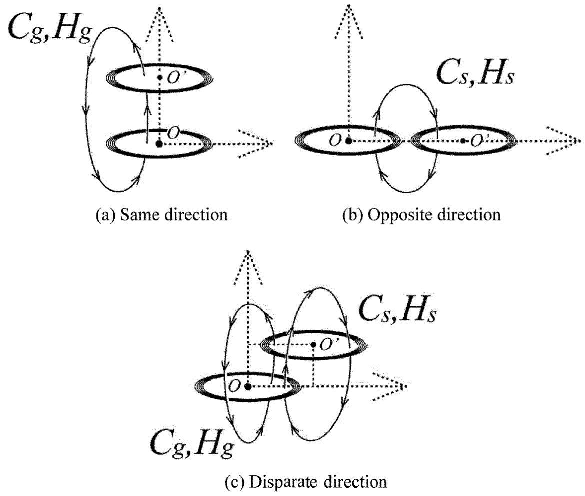


Fig. 17. Magnetic field and coupling for antennas positioned horizontally and vertically.

section, the effect of the sign of the mutual inductance L_m is discussed for the cases in which the antennas are arranged vertically or horizontally for an equivalent circuit.

3.3 Vertical direction and $+L_m$

The equivalent circuit of a repeater antenna positioned in the vertical direction is next discussed. First, the effect of such a repeater antenna is confirmed. The efficiency with and without a repeater antenna is shown in Fig. 18. When the air gap between the transmitting antenna and receiving antenna is $g_a = 610$ mm, the efficiency is very low, as shown in Fig. 18(a). However, when a repeater antenna is installed between the transmitting and receiving antennas and the distance between the transmitting antenna and repeater antenna and also between the repeater antenna and receiving antenna is g_p = 300 mm, the wireless power transfer efficiency is improved and the distance between the transmitting and receiving antennas is extended. In this way, the good effect of the repeater antenna is observed for the vertical direction. Next, the case in which a cross coupling effect between the transmitting and receiving antennas appears is evaluated. In this study, the air gap was small enough for cross coupling to appear between the transmitting and receiving antennas for g_p = 150 mm and $g_a = 310$ mm. The electromagnetic field analysis model, an illustration of the magnetic field coupling, and the electromagnetic field analysis and experimental results are presented in Fig. 19. The experimental and electromagnetic field analysis results agree well, with only a small difference between them. The characteristics of the transmitting and receiving antennas are the same as when only the transmitting and receiving antennas are used and the air gap

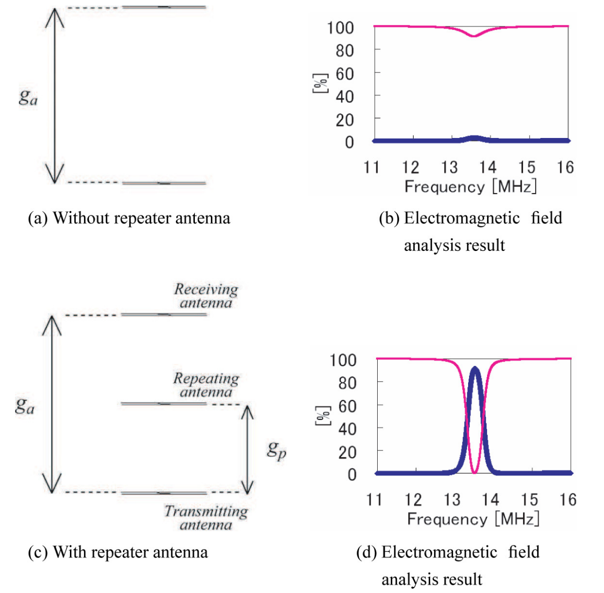


Fig. 18. Effect of repeater antenna in vertical direction (g_a = 610 mm, g_p = 300 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

is g_a = 150 mm, which is shown in Fig. 13. Thus, the formulation of the equivalent circuit is examined.

The parameters related to the transmitting and receiving antennas are L_{13} = 0.271 μH and k_{13} = 0.025 at an air

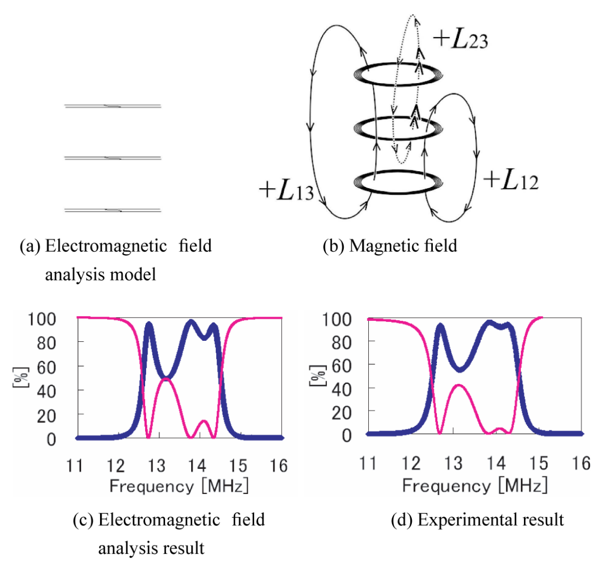


Fig. 19. Electromagnetic field analysis and experimental results with repeater antenna in vertical direction (g_p = 150 mm, g_a = 310 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

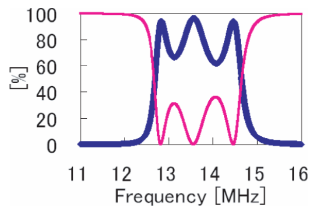


Fig. 20. Calculation of equivalent circuit for repeater antenna at vertical position without considering cross coupling (g_p = 150 mm, g_a = 310 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

gap g_a = 310 mm. The parameters related to the coupling between the transmitting and repeating antennas and between the repeating and receiving antennas are $L_{12} = L_{23}$ = 1.019 μH and $k_{12} = k_{23}$ = 0.092 at an air gap g_p = 150 mm. The equivalent circuit result, ignoring cross coupling, using Eqs. (3) and (5), is shown in Fig. 20. This result indicates that there is cross coupling between the transmitting antenna and receiving antenna, which cannot be ignored, and thus the results do not agree. Therefore, the result for an equivalent circuit that considers cross coupling using Eqs. (12) and (13) is shown in Fig. 21. In this case, the sign of the mutual inductance is examined. The case of positive mutual inductance is shown in Fig. 21(a), and that of a negative one is shown in Fig. 21(b). In this vertical direction case, the mutual inductance should become positive because the direction of crossing of the magnetic field is the same at all positions. Thus, Fig. 19 should match Fig. 21(a), which is confirmed.

3.4 Horizontal direction and $-L_m$

The equivalent circuit of a repeater antenna aligned in the horizontal direction is discussed next. To examine the effect of cross coupling between the transmitting and receiving antennas, this study was performed with a small air gap between the transmitting and receiving antennas. The electromagnetic field analysis model, an illustration of the magnetic field coupling, and the electromagnetic field analysis and experimental results are shown in Fig. 22, with s_a = 10 mm and s_p = 10 mm. The transmitting antenna is shown on the lower left, with the receiving antenna on the lower right, and the repeater antenna at the top. The experimental and electromagnetic field analysis results do not agree perfectly, but are close. The parameters when considering cross coupling in Fig. 22 are $k_{12} = k_{13} = k_{23}$ = 0.049, $L_{12} = L_{13}$, and $-L_{23}$ = −0.542 μH. The result of ignoring cross coupling between the transmitting and receiving antennas, which is calculated by the equivalent circuit, is shown in Fig. 23, where $k_{13} = L_{13}$ = 0. However, in this kind of

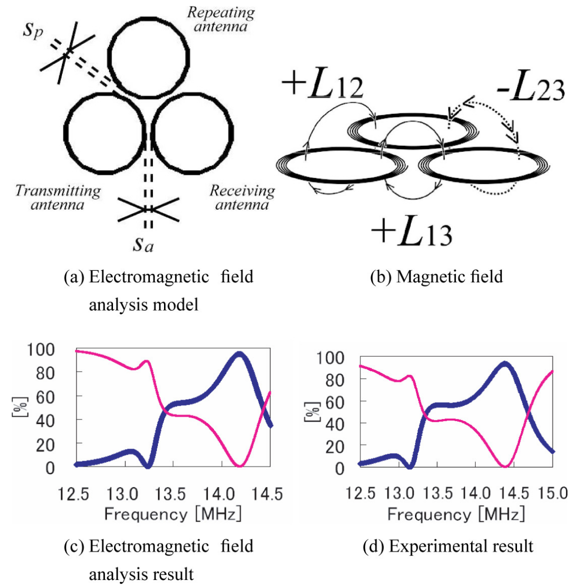


Fig. 22. Electromagnetic field analysis and experimental results for repeater antenna positioned horizontally (s_a = 10 mm, s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

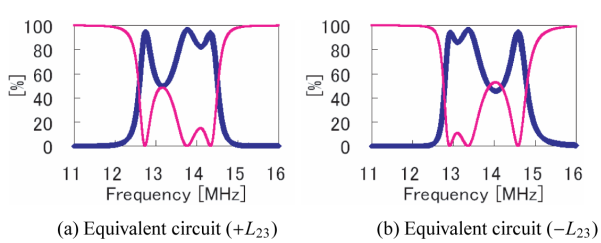


Fig. 21. Calculation of equivalent circuit for repeater antenna at vertical position (g_p = 150 mm, g_a = 310 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

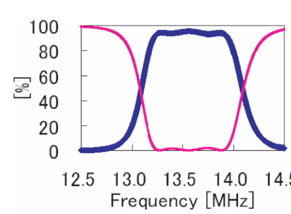
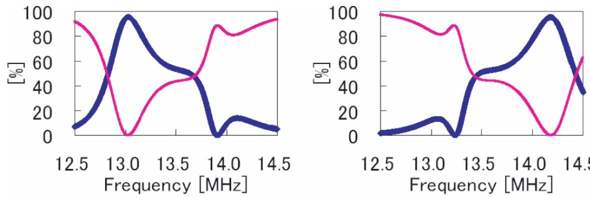
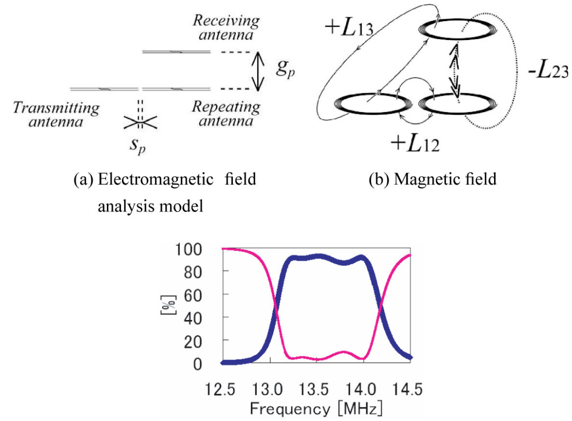


Fig. 23. Calculation of equivalent circuit for repeater antenna positioned horizontally without considering cross coupling (s_a = 10 mm, s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Equivalent circuit ($+L_{23}$) (b) Equivalent circuit ($-L_{23}$)

Fig. 24. Calculation with equivalent circuit for repeater antenna positioned horizontally (s_a = 10 mm, s_p = 10 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Electromagnetic field analysis model (b) Magnetic field

(c) Electromagnetic field analysis result

Fig. 25. Electromagnetic field analysis result of repeater antenna at vertical and horizontal position (s_p = 10 mm, g_p = 200 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

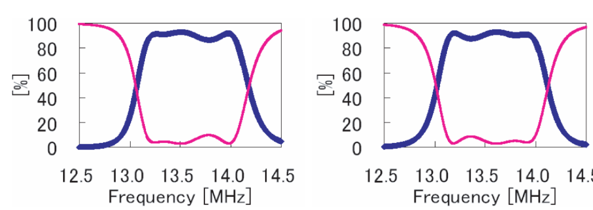
configuration, the result does not agree with the equivalent circuit if cross coupling is ignored. Thus, the result for an equivalent circuit that considers cross coupling at the respective parameter values is shown in Fig. 24. In this case, the sign of the mutual inductance is examined. A positive mutual inductance is shown in Fig. 24(a), and a negative one is shown in Fig. 24(b). In this horizontal direction case, the mutual inductance should become negative because the directions of the crossing magnetic fields are reversed between the repeater and receiving antennas. Thus, Fig. 22 should agree with Fig. 24(b), which is confirmed.

4. Range of Adaptation of Equivalent Circuit and Antenna Arrangement

As discussed in the previous section, the sign of the mutual inductance is determined by the directions of the crossing magnetic fields. Therefore, in this section, we examine the adaptive possibilities when the repeater antenna arrangement is changed.

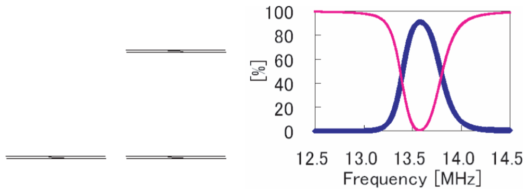
4.1 Case of vertical and horizontal arrangement

In this case, a mixed vertical and horizontal arrangement is examined. The electromagnetic field analysis model, an illustration of the magnetic field coupling, and the power transfer efficiency are shown in Fig. 25, where the transmitting and repeater antennas are arranged horizontally at a distance s_p = 10 mm and the receiving antenna is arranged vertically above the repeater antenna at a distance g_p = 200 mm. The mutual inductance should be reversed because the directions of the magnetic fields at the repeater antenna and receiving antenna are the opposite of that formed by the transmitting antenna. First, the electromagnetic field analysis and experimental results agree well. The sign of the mutual inductance is discussed using the results of electromagnetic field analysis. The equivalent circuit result is shown in Fig. 26. The coupling coefficient and inductances are k_{12} = 0.049, k_{13} = 0.0044, k_{23} = 0.058, L_{12} = 0.542 μH, L_{13} = 0.0480 μH, and $-L_{23}$ = –0.643 μH. Figure 26(a) shows the results obtained when the mutual inductance between the repeater and receiving antennas is negative, and Fig. 26(b) shows the results obtained when all of the signs for the mutual inductance are reversed. In this way, the equivalent circuit of a repeater can be calculated by determining the sign of the mutual inductance with consideration of the magnetic field direction, even if a mixed horizontal and vertical arrangement is used. If the coupling coefficient becomes too weak and there is only one peak, the sign of the mutual inductance makes no difference and can be ignored when simply calculating the



(a) (b)

Fig. 26. Calculation with equivalent circuit for repeater antenna in planar and vertical direction (s_p = 10 mm, g_p = 200 mm). (a) Same signs for mutual inductance. (b) Opposite signs for mutual inductance. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

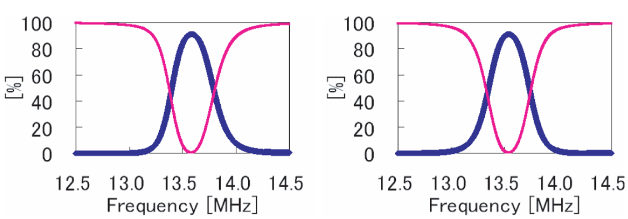


(a) Model (b) Electromagnetic field analysis result

Fig. 27. Electromagnetic field analysis result for repeater antenna at vertical and horizontal direction (s_p = 50 mm, g_p = 300 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

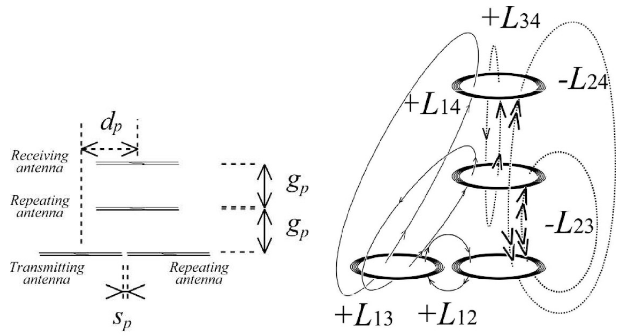
efficiency. These results are shown in Figs. 27 and 28, where $s_p = 50$ mm, $g_p = 300$ mm, $k_{12} = 0.025$, $k_{13} = 0.0036$, $k_{23} = 0.026$, $L_{12} = 0.280$ µH, $L_{13} = 0.0398$ µH, and $-L_{23} = -0.290$ µH. When only one repeater antenna is used, the waveform variance is small. Therefore, an additional repeater antenna is added to increase the waveform variance. The electromagnetic field analysis model, an illustration of the magnetic field connections, and the electromagnetic field analysis results are shown in Fig. 29.

As in the previous method, the signs of the mutual inductances are determined from the figure, and the equivalent circuit results are shown in Fig. 30. When the number of antennas is increased to four, the results are calculated by expanding the three-dimensional square matrix to a four-dimensional square matrix. Even though the number of repeater antennas is increased, the cross coupling and mutual inductance sign can be discussed in the same way, where $L_{12} = 0.542$ µH, $L_{13} = 0.306$ µH, $L_{14} = 0.147$ µH, $-L_{23} = -0.711$ µH, $-L_{24} = -0.223$ µH, and $L_{34} = 1.019$ µH.

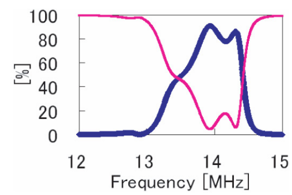


(a) (b)

Fig. 28. Calculation of equivalent circuit for repeater antenna at plane and vertical direction (s_p = 50 mm, g_p = 300 mm). (a) Same signs for mutual inductance. (b) Opposite signs for mutual inductance. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Electromagnetic field analysis model (b) Magnetic field

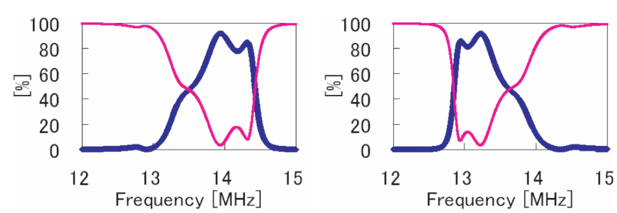


(c) Electromagnetic field analysis result

Fig. 29. Electromagnetic field analysis and experimental results for 2 repeater antennas at vertical and horizontal directions (s_a = 10 mm, d_p = 200 mm, g_p = 150 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

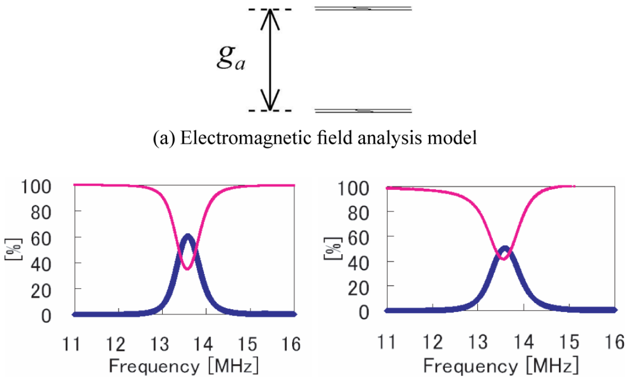
4.2 90° arrangement as wall hanging and the like

The repeater antenna can be arranged between the transmitting and receiving antennas as shown in Fig. 19. However, it is not always possible to place an antenna in the direction in which the power needs to be transmitted. In particular, if it must be hung on a wall, the repeater antenna angle may be rotated by 90°. First, the electromagnetic field analysis model and the electromagnetic field analysis and



(a) (b)

Fig. 30. Calculation of equivalent circuit for 2 repeater antennas at vertical and horizontal directions (s_a = 10 mm, d_p = 200 mm, g_p = 150 mm). (a) Corresponding signs of mutual inductance. (b) Opposite signs of mutual inductance. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



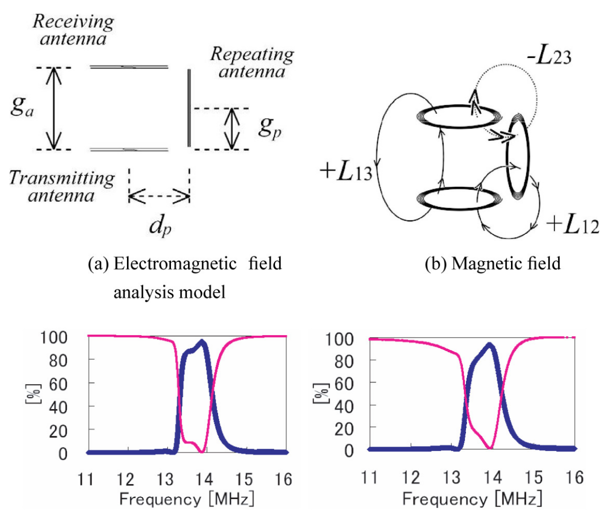
(a) Electromagnetic field analysis model

(b) Electromagnetic field analysis result

(c) Experimental result

Fig. 31. Electromagnetic field analysis and experimental results without repeater antenna (g_a = 300 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

experimental results without a repeater antenna are shown in Fig. 31. The electromagnetic field analysis model, the results of electromagnetic field analysis, and the experimental results with an installed repeater antenna are shown in Fig. 32. The experimental and electromagnetic field analysis results do not agree perfectly, but are close. In these results, the wireless power transfer distance extension effect



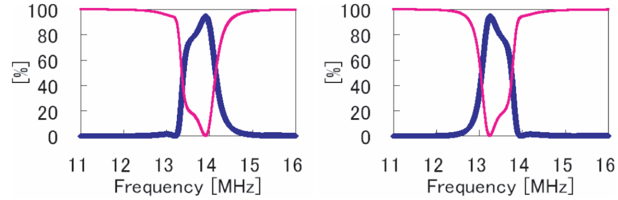
(a) Electromagnetic field analysis model

(b) Magnetic field

(c) Electromagnetic field analysis result

(d) Experimental result

Fig. 32. Electromagnetic field analysis and experimental results of repeater antenna in perpendicular arrangement (g_a = 300 mm, g_p = 150 mm, d_p = 225 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]



(a) Corresponding sign of L_m

(b) Opposite sign of L_m

Fig. 33. Calculation with equivalent circuit for repeater antenna in perpendicular arrangement (g_p = 150 mm, d_p = 225 mm). [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

is confirmed, even if the antenna is rotated by 90°. Considering the transmitting antenna as a reference, the connection directions between the transmitting and receiving antennas and between the transmitting and repeating antennas are the same. However, these directions become opposite at the receiving antenna and repeating antenna. Therefore, the mutual inductance between the transmitting and receiving antennas and that between the transmitting and repeater antennas become positive. The mutual inductance between the repeater and receiving antennas is negative. To examine the effect of the sign, all of the signs are reversed and checked. The equivalent circuit results using these corresponding signs and opposite signs are shown in Fig. 33, where $k_{12} = k_{23} = 0.038$, $k_{13} = 0.026$, $L_{12} = 0.423$ µH, $-L_{23} = -0.423$ µH, and $L_{13} = 0.291$ µH. As discussed previously, the repeater antenna can be treated correctly as an equivalent circuit by determining the sign of the mutual inductance by considering the magnetic field relationship.

5. Conclusions

In this study, the cross coupling effect was confirmed by proposing equivalent circuits with and without cross coupling. The equivalent circuit with the sign of the mutual inductance, which was defined by the geometric antenna arrangement, was presented. The mutual inductance can be ignored when only two antennas are used: a transmitting antenna and a receiving antenna. In this paper, the aim was only to formulate repeater antennas, and therefore the results were not optimized. To allow high-efficiency wireless power transfer, it will be necessary to optimize the impedance matching in future work.

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