Proposal of Attitude Control for High-Precision Stage by Compensating Nonlinearity and Coupling of Euler’s Equation and Rotational Kinematics

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Abstract—High-precision stages are used for the fabrication of integrated circuits, liquid crystal displays and so on. Since higher integration density and product quality are continuously required for the development of industry and informatics, not only the stage position but also the stage attitude needs to be controlled rapidly and precisely. The attitude is determined by roll, pitch and yaw motions which are affected by nonlinearity and coupling caused by Euler’s equation and rotational kinematics. These effects deteriorate the attitude control performance. This paper proposes a MIMO nonlinear feedforward attitude controller which compensates such effect. The effectiveness of the proposed approach is verified by simulations and experiments.

I. INTRODUCTION

High-precision stages are used for manufacturing electronic devices such as integrated circuits and liquid crystal displays. Since mass production of these devices requires an increasing demand for high throughput and high quality, the stages come to be larger and larger and also the stages need to be controlled faster and more precisely. Feedback control with high-gain and high-bandwidth, however, cannot be implemented to the large stages because the stages have low resonant modes due to the elasticity and heavy mass of the stages. In this situation, two-degrees-of-freedom (2-DOF) control which contains an accurate feedforward controller is effective [1]. Note that such high-precision stages need accurate control for not only translational motion $x$, $y$, $z$ but also rotational motion $\theta_x$, $\theta_y$, $\theta_z$ [2], [3].

For translational motion, some previous studies have been presented. For instance, Fujimoto et. al have proposed perfect tracking control (PTC) based on multi-rate feedforward control [4] and this method can obtain high performance [1]. This method, however, cannot be applied to rotational motion because of the nonlinearity and coupling caused by Euler’s equation and rotational kinematics.

For rotational motion, typically, the attitude is controlled only by feedback controller [5], [6] or 2-DOF controller which contains linear approximated feedforward controller [7]. Control system with only feedback controller is unsuitable for fast and precise attitude control because of the stages’ low resonant frequencies. In contrast, this 2-DOF control system compensates the nonlinearity and coupling caused by Euler’s equation and rotational kinematics by means of feedback controller. Therefore, this method introduces time delay and may cause overshoot.

This paper proposes a MIMO nonlinear feedforward attitude controller which compensates nonlinearity and coupling caused by both Euler’s equation and rotational kinematics. The effectiveness of the proposed approach is verified by simulations and experiments. Experiments are performed by using 3-DOF $\theta_x$, $\theta_y$, $z$ of our 6+1-DOF high-precision stage.

II. PROBLEM FORMULATION

A. Euler’s equation

Model of the high-precision stage is illustrated in Fig. 1. In this section, nonlinearity and coupling of Euler’s equation is formulated. Rotational dynamics is described by Euler’s equation, which is given by [8]

$$\tau = I \dot{\omega} + \omega \times I \omega, \tag{1}$$

where $\tau = [\tau_x, \tau_y, \tau_z]^T$ denotes the control torque vector, $\omega = [\omega_x, \omega_y, \omega_z]^T$ denotes angular velocity vector, and $I$ denotes
the inertia tensor matrix defined as
\[
I = \begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix}.
\] (2)

Expanding the right-hand side of (1), we have
\[
\begin{align*}
\tau_x &= I_{yx}\omega_y + I_{zx}\omega_z + I_{xx}\omega_x - \beta_1 \omega_x^2 \\
\tau_y &= I_{zx}\omega_z + I_{yy}\omega_y + I_{xy}\omega_x - \beta_1 \omega_y^2 \\
\tau_z &= I_{xz}\omega_z + I_{yz}\omega_y + I_{zz}\omega_z - \beta_1 \omega_z^2
\end{align*}
\] (3)

Equation (3) indicates that Euler’s equation has nonlinearity and coupling caused byproducts of inertia and \( \omega \times I \omega \).

B. Rotational kinematics

Because of coordinate rotation, rotational motion has nonlinearity and coupling [9]. Euler angle \( \theta = [\theta_x, \theta_y, \theta_z]^T \) cannot be calculated by integration of angular velocity \( \omega = [\omega_x, \omega_y, \omega_z]^T \) linearly.

In this paper, the attitude of the body-fixed frame is expressed using Z-Y-X Euler angle \( \theta = [\theta_x, \theta_y, \theta_z]^T \) and rotation matrix \( R \).Rotation matrix \( R \) can be written using Euler angle \( \theta \) as follows:
\[
R(\theta_x, \theta_y, \theta_z) = 
\begin{bmatrix}
\cos \theta_x \cos \theta_y & \cos \theta_x \sin \theta_y & -\sin \theta_x \\
\cos \theta_z \sin \theta_x \cos \theta_y - \sin \theta_z \sin \theta_y & 
\cos \theta_z \sin \theta_x \sin \theta_y + \cos \theta_z \cos \theta_y & 
\cos \theta_z \cos \theta_x \\
\cos \theta_z \sin \theta_x \sin \theta_y - \cos \theta_z \cos \theta_y & 
\cos \theta_z \sin \theta_x \sin \theta_y + \cos \theta_z \cos \theta_y & 
\cos \theta_z \cos \theta_x
\end{bmatrix}.
\] (4)

Here, \( s \) and \( c \) are the abbreviation of \( \sin \) and \( \cos \), respectively. The derivative of rotation matrix \( \dot{R} \) is given by [8]
\[
\dot{R} = [\omega] \times R.
\] (5)

The notation \([\omega] \times \) is the skew symmetric matrix formed from angular velocity \( \omega = [\omega_x, \omega_y, \omega_z]^T \),
\[
[\omega] \times \equiv \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}.
\] (6)

To solve (5), we obtain
\[
R(t + \Delta t) = e^{[\omega] \times \Delta t} R(t),
\] (7)
where \( \Delta t \) denotes time updating period and \( e^{[\omega] \times \Delta t} \) denotes matrix exponential. By Rodrigues’ rotation formula, \( e^{[\omega] \times \Delta t} \) can be written as
\[
e^{[\omega] \times \Delta t} = E + [\alpha \times] \sin(\omega \Delta t) + [\alpha \times]^2 (1 - \cos(\omega \Delta t)),
\] (8)
where \( E \) denotes the 3 \( \times \) 3 identity matrix. Here, \( \alpha \) and \( \omega \) are defined as
\[
\omega = a \omega, \quad \omega \equiv ||\omega||_2, \quad ||a||_2 = 1.
\] (9)

Equation (7) shows that rotational kinematics has nonlinearity and coupling.

III. CONVENTIONAL FEEDFORWARD CONTROLLER DESIGN

A. Linearization for rotational dynamics

Since principal axes of inertia do not correspond with control axes, products of inertia are not zero generally. Then, for linearization, it is usually assumed that products of inertia and angular velocity are small and neglectable:
\[
\begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix}
\approx
\begin{bmatrix}
0 & 0 & 0 \\
0 & I_{yy} & 0 \\
0 & 0 & I_{zz}
\end{bmatrix}.
\] (10)

Applying the above approximations, (3) becomes
\[
\begin{align*}
\tau_x &= I_{yx}\omega_y + I_{zx}\omega_z - \beta_1 \omega_x^2 \\
\tau_y &= I_{zx}\omega_z + I_{yy}\omega_y - \beta_1 \omega_y^2 \\
\tau_z &= I_{xz}\omega_z + I_{yz}\omega_y - \beta_1 \omega_z^2
\end{align*}
\] (12)

Equation (12) shows that the relationship between \( \tau \) and \( \dot{\omega} \) is linearized.

B. Linearization for rotational kinematics

Because of coordinate rotation, rotational kinematics has nonlinearity and coupling. Here, ignoring coordinate rotation, the relationship between \( \theta \) and \( \dot{\theta} \) is simplified as follows
\[
\begin{align*}
[\theta_x] &= [\omega_x] dt \\
[\theta_y] &= [\omega_y] dt \\
[\theta_z] &= [\omega_z] dt
\end{align*}
\] (13)

Equation (13) shows that the relationship between \( \theta \) and \( \omega \) is linearized.

C. Design of conventional feedforward controller

Applying linearization for rotational dynamics and kinematics, which is described as (10), (11) and (13), Euler’s equation (3) becomes
\[
\begin{align*}
\tau_x &= I_{yx}\dot{\omega}_y + I_{zx}\dot{\omega}_z \\
\tau_y &= I_{zy}\dot{\omega}_y + I_{yy}\dot{\omega}_y \\
\tau_z &= I_{zy}\dot{\omega}_y + I_{zz}\dot{\omega}_z
\end{align*}
\] (14)

Using Laplace transform for (14), we obtain conventional feedforward controller.
\[
\begin{bmatrix}
\tau_x(s) \\
\theta_x(s) \\
\theta_y(s) \\
\theta_z(s)
\end{bmatrix} = 
\begin{bmatrix}
I_{xx}s^2 + I_{xy} \\
I_{xy} \\
I_{yz} \\
I_{zz}s^2
\end{bmatrix}.
\] (15)

Block diagram of the conventional feedforward controller is shown in Fig. 2.
IV. PROPOSED FEEDFORWARD CONTROLLER DESIGN

In this section, the proposed feedforward controller is formulated. This novel feedforward controller can compensate the nonlinearity and coupling caused by both Euler’s equation and rotational kinematics. Block diagram of the proposed feedforward controller is shown in Fig. 3 and also the timing chart is illustrated in Fig. 4.

A. Reference attitude trajectory \( \theta^*[k] \) and reference rotation matrix \( R^*[k] \)

First, \( \theta^*[k] \) is generated arbitrarily such as five-order polynomials. Then, \( \theta^*[k] \) is converted into \( R^*[k] \) by (4).

B. Reference angular velocity \( \omega^*[k] \) and \( \omega^*[k + 1] \)

Expressing (7) into discrete-time system, we obtain

\[
e^{[\omega^*[k] \times T_s]} = R^*[k + 1]R^{-1}[k],
\]

where \( T_s \) denotes sampling period. For simplicity, the right-hand side of (16) is denoted by

\[
R^*[k + 1]R^{-1}[k] = \begin{bmatrix}
R_{11} & R_{12} & R_{13} \\
R_{21} & R_{22} & R_{23} \\
R_{31} & R_{32} & R_{33}
\end{bmatrix}.
\]

Then, according to equation (8) and (16), \( \omega^*[k] \) is given by

\[
\omega^*[k] = \begin{cases}
[0 \ 0 \ 0]^T, & \text{if } R = E, \\
\frac{\phi}{2T_s \sin \phi} \begin{bmatrix}
R_{32} - R_{23} \\
R_{13} - R_{31} \\
R_{21} - R_{12}
\end{bmatrix}, & \text{if } R \neq E,
\end{cases}
\]

where \( \phi \) is given by

\[
\phi = \cos^{-1} \left( \frac{R_{11} + R_{22} + R_{33} - 1}{2} \right).
\]

In this way, the nonlinearity and coupling of rotational kinematics can be compensated by \( \omega^*[k] \). In the same way, we can obtain \( \omega^*[k + 1] \) from \( R^*[k + 2] \) and \( R^*[k + 1] \).

C. Feedforward reference torque \( \tau^*[k] \)

Euler integral is given by

\[
\omega[k + 1] = \omega[k] + \dot{\omega}[k]T_s.
\]

According to (1) and (20), we obtain

\[
\tau^*[k] = \frac{1}{T_s} I (\omega^*[k + 1] - \omega^*[k]) + \omega^*[k] \times I \omega^*[k].
\]

From the above equations, we obtain the feedforward reference torque \( \tau^*[k] \), which can compensate the nonlinearity and coupling caused by both Euler’s equation and rotational kinematics.

V. FEEDFORWARD CONTROL SIMULATIONS

A. Simulation conditions

In this section, we show that the proposed feedforward controller can compensate the nonlinearity and coupling.

Initial attitude is set as

\[
\theta[0] = \begin{bmatrix} 300 & -300 & 0 \end{bmatrix}^T [\mu rad],
\]

and the reference attitude trajectories are given by 5-order polynomials, which are shown in Fig. 5. In these simulations, time updating period \( \Delta t \) and sampling period \( T_s \) are set as 100 \( \mu s \), and inertia tensor \( I \) is set as

\[
I = 0.16 \begin{bmatrix} 0.40 & 0.024 & 0.0019 \\
0.024 & 0.62 & 0.00090 \\
0.0019 & 0.00090 & 1.0 \end{bmatrix} [kgm^2],
\]

which is the analyzed value of high-precision stage illustrated in Fig. 6.
B. Simulation results

Simulation results are shown in Fig. 7 and Tab. I. Fig. 7(a), (d) and (g) illustrate that trajectories using the conventional feedforward controller exceed the reference trajectories. These results indicate that the trajectories are affected by the nonlinearity and coupling from other axes. In contrast, the trajectories using the proposed feedforward controller can follow the reference trajectories, and according to Tab. I, the tracking performances are improved 100 times for $\theta_x$, 68 times for $\theta_y$ and 7600 times for $\theta_z$. These results demonstrate that the proposed feedforward controller can compensate the nonlinearity and coupling from other axes.

Fig. 7(c), (f) and (i) show that $\tau^*$ of the proposed feedforward controller is less than $\tau^*$ of conventional feedforward controller because of considering the nonlinearity and coupling.

From the above results, the advantage of the proposed feedforward controller is verified by simulations.

VI. EXPERIMENTS

A. Structure of the high-precision stage

In this section, we perform experiments to verify the advantage of the proposed feedforward controller. We fabricated a high-precision stage which consists of a fine stage and a coarse stage, where the fine stage has 6-DOF ($\theta_x, \theta_y, \theta_z, x, y, z$) and the coarse stage has 1-DOF ($X$). In this paper, we use 3-DOF ($\theta_x, \theta_y, z$) of the fine stage. The height $z$, the rolling $\theta_x$ and the pitching $\theta_y$ are measured by four linear encoders and controlled by three voice coil motors (VCMs) which are illustrated in Fig. 6. The fine stage is supported by fulcrum which is located at the fine stage’s center of gravity.

Plant structure is shown in Fig. 8, where $f_1, f_2, f_3$ denote references of VCMs’ driving forces, $f_1, f_2, f_3$ denote
c) the viscoelasticity by

\[
T_2 = \begin{bmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{2(z_2 - z_3)} & \frac{1}{2(z_2 - z_3)} & \frac{1}{2(z_2 - z_3)} & \frac{1}{2(z_2 - z_3)}
\end{bmatrix}
\]  

B. Controller design

The conventional controller and the proposed controller are shown in Fig. 10 and Fig. 11, respectively. Note that both 2-DOF controllers have the same feedback controllers. Feedback controllers are designed by pole assignment approach. The bandwidths of position loops are 20 Hz, 7 Hz and 7 Hz for the translation \( z \), the rotation \( \theta_x \) and \( \theta_y \), respectively. Fig. 9 shows that the plants have viscoelasticity. Therefore, we compensate the viscoelasticity by

\[
\begin{align*}
[\hat{\theta}_x, \hat{\theta}_y]^T &= T_2 [z_1, z_2, z_3, z_4]^T, \\
\hat{\theta}_x &= c_x \hat{\theta}_x[k] + k_x (\theta_x[k] - \theta_x[0]) \\
\hat{\theta}_y &= c_y \hat{\theta}_y[k] + k_y (\theta_y[k] - \theta_y[0])
\end{align*}
\]  

in both the conventional controller and the proposed controller. Note that \( c_x \) and \( c_y \) denote coefficients of viscosity, and \( k_x \) and \( k_y \) denote coefficients of elasticity.

C. Experimental results

The reference trajectories are shown in Fig. 5, which are the same as the trajectories used in simulations. Experimental results are illustrated in Fig. 12 and Tab. II. Fig. 12(a) and
TABLE II
EXPERIMENTAL RESULTS (FF + FB CONTROL).
THE MAXIMUM VALUES OF TRACKING ERRORS.

<table>
<thead>
<tr>
<th></th>
<th>$\theta_x$ [\u03b8 rad]</th>
<th>$\theta_y$ [\u03b8 rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional FF + FB</td>
<td>20</td>
<td>8.9</td>
</tr>
<tr>
<td>Proposed FF + FB</td>
<td>8.2</td>
<td>5.3</td>
</tr>
</tbody>
</table>

(d) show that the trajectories using the conventional controller exceed the reference trajectories. This result indicates that the trajectories are affected by the nonlinearity and coupling from other axes. Fig. 12(b), (e) and Tab. II demonstrate that the use of the proposed feedforward controller improves the tracking performances 2.4 times for $\theta_x$ and 1.7 times for $\theta_y$. Fig. 12(c) and (f) shows that $\tau^*$ of the proposed controller is less than $\tau^*$ of the conventional controller considering the nonlinearity and coupling.

From the above, the effectiveness of the proposed controller is verified by experiments. The difference between simulations and experiments can be attributed to the modeling error of vibration modes and coefficients of viscoelasticity showed in Fig. 9.

VII. CONCLUSION

Since the high-precision stages have low resonant frequencies, the high-bandwidth feedback controller cannot be implemented to the stages. Therefore, 2-DOF control system which has an accurate feedforward controller is effective. Rotational motion, however, is affected by the nonlinearity and coupling caused by Euler’s equation and rotational kinematics.

In this paper, we proposed a novel feedforward controller which compensates the nonlinearity and coupling caused by both Euler’s equation and rotational kinematics. The effectiveness of this method was verified by simulations and experiments.

A possible subject of future work is modeling of discrete-time plant considering nonlinearity and coupling.

REFERENCES