Smooth Output Reconstruction for Linear Systems
with Quantized Measurements

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ABSTRACT

This paper presents a novel approach to reconstruct the output of linear systems in the case where the measured output is uniformly quantized. By fitting the quantized measurements with polynomials in a moving horizon manner, a reconstruction signal is obtained via solving a convex optimization problem. $\ell_1$-norm regularization method is applied to automatically determine the polynomial with an appropriate degree to obtain a proper fitting. At the same time, some constraint conditions taking advantage of the quantization feature and the system information are applied in order to achieve an accurate and smooth reconstruction. The effectiveness of the proposed approach is verified by experiments using a high-precision linear stage.

Key Words: quantization; moving horizon manner; polynomial fitting approach; convex optimization; observer

I. Introduction

Quantization in I/O signals is an inherent feature in many control systems including digital systems, networked ones, low resolution sensor/actuator systems, and so on. In some cases, the quantization error is substantially small compared to system noise and the desired control accuracy. However, this is not always the case in various systems such as the computer storage systems, NC machine tools, industrial robots and ultra-precision positioning systems where the required accuracy is nano order. In these systems, the quantization error caused by low-resolution sensors would significantly degrade the control performance and may cause limit cycle...
oscillations [1]. In addition, in networked control systems with limited communication capacity, sensor measurement is quantized before transmission. In such case, the quantization error could be too large to be ignored [2, 3]. Therefore, understanding and suppressing the quantization effects have attracted a great deal of attention, see, e.g., [4, 5, 6, 7].

In order to overcome the quantization effects, various methods were proposed in the literature to reconstruct the real system output. For instance, some observer-based methods were proposed to estimate the system state, and a reconstruction output can be obtained by utilizing the estimated state and the model information [8, 9]. These methods have the potential to achieve a precise output reconstruction since they take advantage of the system information. However, the error between the real output and the reconstructed output cannot even be guaranteed to be smaller than the quantization step when the model uncertainties or the input disturbance is too large to be ignored. Furthermore, the highly colored feature of quantization error would also degrade the estimation performance. Besides, a least square method based on plant model was proposed in [10] for estimating the quantization error. The system output can be reconstructed by simply adding the estimated error to the quantized output. Though it is obtained that the estimation algorithm is computationally tractable and does not depend on control methods, the assumption that the input disturbance is constant, however, may be too restrictive in some cases.

There is another line of research on reconstructing the real system output from the noise-corrupted output by curve fitting methods. For instance, polynomial filtering approaches were proposed to recover the non-uniformly sampled signals [11], and position signals obtained from incremental encoders [12]. The methods can work well if the output signal can be locally approximated by low-degree polynomials. However, if this is not the case, higher-degree polynomials are required. Hence, it would be difficult to adopt the methods. Moreover, how to determine the degree of the polynomials was not clear in the literature.

Since the reconstruction methods based on the state estimators share the feature of providing good reconstruction performance and the polynomial fitting methods can achieve smooth reconstruction signals, the purpose of this paper is to propose a method by combining them together to obtain an accurate and smooth output reconstruction. More concretely, we try to achieve more precise information on the current output from a series of past quantized measurements by taking into account of the system dynamics. At the same time, a polynomial fitting approach based on $\ell_1$-norm regularization method is exploited in order to achieve the smooth output reconstruction. Part of this idea is also considered in [13] on estimating the velocity of second order mechanical systems from the position measurements. It was demonstrated that an accurate velocity signal could be obtained even when the quantization step is large. The input disturbance, however, is assumed to be zero-mean, which is not
the case in many practical situations. In this paper, we cope with this problem by estimating the system state with unknown input disturbance to improve the robustness and the practicality of the proposed approach. In addition, we also focus on reducing the limit cycle oscillation caused by the quantized signals since it is also important in many control systems. For example, not only the accurate velocity control but also the precise position control is required in positioning systems where the position signals are measured by optical encoders. These perspectives motivate us to extend the research of [13]. In order to verify the effectiveness of the proposed approach of this study, we perform experimental evaluation using an ultra-precision positioning stage system in this study.

This paper is organized as follows. In Section II, the system model is introduced, and the problem setting is described. Section III presents the polynomial fitting approach based on convex optimization that consists of the $\ell_1$-norm regularization method and some constraint conditions. Sections IV demonstrates the effectiveness of the proposed approach by experiments using a high-precision positioning stage. The conclusions are summarized in Section V.

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II. System description

Consider the linear time-invariant SISO system with quantized output given by

$$x[k + 1] = Ax[k] + Bu[k] + Fw[k], \quad (1)$$
$$y[k] = Cx[k], \quad (2)$$
$$y_c[k] = y[k] + v[k], \quad (3)$$
$$y_q[k] = Q(y_c[k]), \quad (4)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, $y_c \in \mathbb{R}$ and $y_q \in \mathbb{R}$ are the system state, the system output, the corrupted output and the quantized output, respectively. $v \in \mathbb{R}$ is the measurement noise and $w \in \mathbb{R}$ is the unknown input disturbance. $A, B, F, C$ are constant system matrices of suitable dimensions. $Q(\cdot)$ denotes the quantization. $v$ is assumed to be zero-mean and bounded as $\|v\|_{\infty} \leq \delta$ with known $\delta$. In many practical situations, $w$ could be expressed by the following state equation

$$\zeta[k + 1] = \Gamma\zeta[k], \quad (5)$$
$$w[k] = H\zeta[k], \quad (6)$$

where $\zeta$ denotes the disturbance state, $\Gamma$ and $H$ are real matrices of suitable dimensions. Define the augmented state $x_e$ as $x_e := [x^T \ \zeta^T]^T$, the augmented system of (1), (2), (3), (4) can be expressed by:

$$x_e[k + 1] = A_e x_e[k] + B_e u[k], \quad (7)$$
$$y_e[k] = C_e x_e[k] + v[k], \quad (8)$$
$$y_q[k] = Q(y_e[k]), \quad (9)$$
where

\[
A_e = \begin{bmatrix}
A & FH \\
0 & \Gamma
\end{bmatrix}, \quad B_e = \begin{bmatrix}
B \\
0
\end{bmatrix}, \quad C_e = \begin{bmatrix}
C & 0
\end{bmatrix}.
\]

In addition, the function \(Q(\cdot)\) in (4) is assumed as the uniform quantization defined by

\[
Q(y_v) = i \cdot \Delta, \quad y_v \in ((i - 0.5)\Delta, (i + 0.5)\Delta]\quad (10)
\]

where \(i \in \mathbb{Z}, \Delta > 0\) denotes the quantization step. The relationship between \(y_v\) and \(y_q\) is shown in Figure 1. In practical systems, the quantizer (10) can be adopted to model optical encoders or analog-to-digital converters where \(\Delta\) is referred to as the resolution. Due to the measurement noise \(v\), the difference between the system output \(y\) and the measured quantized output \(y_q\), denoted by \(\xi := y - y_q\), is bounded by

\[
|\xi| \leq \frac{\Delta}{2} + \delta. \quad (11)
\]

In output-quantized control systems, the step-type output would cause limit cycle oscillation if it is applied to feedback directly. In many situations, the system output \(y\) should be smooth and band limited due to the system dynamics, and it is therefore possible to get more precise information on \(y[k]\) from the past quantized measurement series \(y_q[i] \ (i \leq k)\). In the following sections, we present a curve fitting method combining the moving horizon polynomial fitting approach [13] with an augmented observer to reconstruct the true output from the quantized output, and verify its effectiveness using an ultra-precision positioning stage.

III. Algorithm for output reconstruction

In this section, an output reconstruction approach is proposed. Firstly, the quantized measurements are locally fitted by a polynomial based on \(\ell_1\)-norm regularization method. Then, the moving horizon manner is applied to achieve the real-time output reconstruction. Finally, some constraint conditions taking advantage of the quantization feature and observer techniques are adopted to achieve a smooth reconstruction signals.

3.1. Polynomial fitting formulation

A polynomial for fitting \(p + 1\) quantized measurements (\(\{y_q[k - i]\}_{i=0,1,\ldots,p}\)) is considered. Here, \(k\) denotes the current time instant. In order to do so, first, choose the time interval \([a, b]\) in advance, and introduce the virtual time indices \(\{\tau_i\}_{i=0,1,\ldots,p}\), which are defined by

\[
\tau_0 = a, \tau_1 = a + h, \ldots, \tau_i = a + ih, \ldots, \tau_p = b,
\]
where \( h = (b - a)/p \), to equally divide \([ a \ b] \). Then the data \((\tau_i, y_q[k - p + i])\) for \( i = 0, 1, \ldots, p \) are fitted in Euclidean space with the polynomial:

\[
g_k(t) = \alpha_0 + \alpha_1 \tau + \alpha_2 \tau^2 + \cdots + \alpha_m \tau^m, \quad \tau \in [a \ b] \tag{12}
\]

where \( \alpha_0, \alpha_1, \ldots, \alpha_m \) are the polynomial coefficients, and \( m \) is the degree of the polynomial. Without loss of generality, \( m \) is assumed to be \( m \leq p + 1 \). The fitting problem is formulated by

\[
\min_{\alpha} : \left\| \begin{bmatrix} g_k(\tau_0) - y_q[k - p] \\ \vdots \\ g_k(\tau_i) - y_q[k - p + i] \\ \vdots \\ g_k(\tau_p) - y_q[k] \end{bmatrix} \right\|_2^2 + \eta\|\alpha\|_1, \tag{13}
\]

with variable \( \alpha := [\alpha_0 \ \alpha_1 \ \cdots \ \alpha_m]^T \). Here, \( \eta \) is the weighting factor. This is an \( \ell_1 \)-norm regularization problem, and can be expressed by

\[
\min_{\alpha} : \|T\alpha - \beta\|_2^2 + \eta\|\alpha\|_1, \tag{14}
\]

where

\[
T = \begin{bmatrix} 1 & \tau_0 & \cdots & \tau_0^m \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \tau_i & \cdots & \tau_i^m \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \tau_p & \cdots & \tau_p^m \end{bmatrix}, \quad \beta = \begin{bmatrix} y_q[k - p] \\ \vdots \\ y_q[k - p + i] \\ \vdots \\ y_q[k] \end{bmatrix}.
\]

Figure 2 demonstrates the fitting strategy. \( \ell_1 \)-norm introduced in (14) is used for reducing the number of non-zero elements of \( \alpha \) [13]. In this way, the unnecessary terms of the polynomial (12) can be removed automatically if a high-degree polynomial is used to fit a simple signal. In practical situation, \( m \) can be assigned to be relative large so that simple signals as well as complex signals could be properly fitted.

In general, the time indices \( \{\tau_i\}_{i=0, 1, \ldots, p} \) is chosen as the real time instants. However, in such case, the Vandermonde matrix \( T \) is time-varying and would become ill-condition so that the fitting accuracy might be degraded. In order to cope with the problem, a fixed symmetric interval with respect to the origin is chosen to make \( T \) well-condition [14]. Without loss of generality, \([a \ b]\) is chosen as \([-1 \ 1]\) in this study.

### 3.2. Moving horizon manner

The polynomial (12) can be determined when the convex optimization problem (14) is solved. In this
case, the quantity \( \bar{y}[k] \) given by

\[
\bar{y}[k] := g_k(\tau_p) = \tau_p^T \alpha, \tag{15}
\]

where \( \tau_p = [1 \ \tau_p \ \tau_p^2 \ \cdots \ \tau_p^m]^T \), is regarded as the reconstruction value of the true output \( y[k] \). When the time instant \( k \) is updated, say, from \( k \) to \( k + 1 \), a new quantized measurement \( y_q[k+1] \) is sampled. Then, a new polynomial \( g_{k+1}(\tau) \) can be determined by fitting the new data \( \{y_q[k-p+1], y_q[k-p+2], \cdots, y_q[k+1]\} \). The updated quantity \( \bar{y}[k+1] \) calculated by

\[
\bar{y}[k+1] = g_{k+1}(\tau_p)
\]

is regarded as the reconstruction value of the true output \( y[k+1] \). Figure 3 shows the updating strategy. For the sake of convenience, the procedure for calculating \( \bar{y}[k] \) is denoted by \( \Pi[\bar{y}[k] | y_q(k - p : k)] \). In the case of \( k \leq p \), \( y_q[k - i] \) \( (i = k, k + 1, \cdots, p) \) is set as \( y_q[k - i] := 0 \), and the procedure is denoted by \( \Pi[\bar{y}[k] | y_q(1 : k)] \). The signal \( \bar{y} \) can be obtained by successively repeating the procedures. Figure 4 demonstrates the moving horizon strategy.

In this way, however, \( \bar{y} \) cannot be guaranteed to be smooth since \( \bar{y}[k] \) and \( \bar{y}[k+1] \) belongs to different polynomials, say, \( \bar{y}[k] \) belongs to \( g_k(\tau) \) and \( \bar{y}[k+1] \) belongs to \( g_{k+1}(\tau) \). In the following subsection, some constraint conditions are added to the optimization problem (14) to improve the accuracy and the smoothness of \( \bar{y} \).

### 3.3. Constraint conditions

In order to improve the reconstruction accuracy, some constraint conditions taking advantage of the model information are considered. As described in Section 2, the system output relative to the quantized output is always bounded by (11). Therefore, the quantity \( \bar{y}[k] \) need also be bounded by

\[
|\bar{y}[k] - y_q[k]| \leq \frac{\Delta}{2} + \delta. \tag{16}
\]

According to (12) and (15), this condition can be expressed by

\[
|\tau_p^T \alpha - y_q[k]| \leq \frac{\Delta}{2} + \delta. \tag{17}
\]

Note that the left hand side of (17) is convex to \( \alpha \).

In addition, the reconstruction signal \( \bar{y} \) is preferred to be smooth since the true system output \( y \) is smooth due to the system dynamics. Therefore, the following two constraint conditions are firstly considered:

\[
g_k(\tau_{p-1}) = \bar{y}[k - 1], \tag{18}
\]

\[
\dot{g}_k(\tau_{p-1}) = \frac{1}{h}(\bar{y}[k] - \bar{y}[k - 1]), \tag{19}
\]

where \( \dot{g}_k(\tau) \) is the first derivative of the polynomial (12), which is given by

\[
\dot{g}_k(\tau) = \alpha_1 + 2\alpha_2 \tau + \cdots + m\alpha_m \tau^{m-1}. \tag{20}
\]

Equation (18) indicates that both \( \bar{y}[k - 1] \) and \( \bar{y}[k] \) belong to \( g_k(\tau) \), and equation (19) is a slope condition. Therefore, these conditions imply that \( \bar{y}[k - 1] \) and \( \bar{y}[k] \)
Fig. 3. Updating strategy. A new polynomial is calculated when the time instant is updated.

Fig. 4. Moving horizon output reconstruction strategy.

According to the augmented system (7) (8), an augmented observer can be designed as

$$\hat{x}[k+1] = A_e \hat{x}[k] + B_e u[k] + L_e (\bar{y}[k] - \hat{y}[k]),$$

(21)

$$\hat{y}[k] = C_e \hat{x}[k],$$

(22)

where $L_e \in \mathbb{R}^{(n+1) \times 1}$ is the observer gain designed to stabilize $A_e - L_e C_e$. Then, the conditions (18) (19) are

One problem of applying conditions (18) and (19) is that the reconstruction error at last instant $k - 1$ may be transmitted to next fitting procedure to deteriorate the reconstruction accuracy at the instant $k$. In order to cope with the problem, the system model is exploited in the following to produce new quantities to use instead of $\bar{y}[k - 1]$ and $\hat{y}[k]$ in (18) and (19).

are smoothly connected.
replaced by
\[
g_k(\tau_{p-1}) = \hat{y}[k - 1], \tag{23}
\]
\[
\hat{g}_k(\tau_{p-1}) = \frac{1}{h}(\hat{y}[k] - \hat{y}[k - 1]). \tag{24}
\]

For the implementation of real-time calculation, \(\hat{y}[k]\) is computed via
\[
\hat{y}[k] = C_e[A_e\hat{x}[k - 1] + B_eu[k - 1] + L_e(\bar{y}[k - 1] - \hat{y}[k - 1])]. \tag{25}
\]

Note that the left hand sides of (23) and (24) are also affine to \(\alpha\).

3.4. System output reconstruction

The quantized output reconstruction algorithm is summarized as follows:

Algorithm: Output reconstruction

1. **Choose** proper values for \(p, m, \eta\); set initial value for \(\beta\) as \(\beta = 0\), assign the values for \(\Delta, \delta\);

2. **repeat**

   (a) Sample the position measurement \(y_q[k]\), update the vector \(\beta\);

(b) **solve the problem**

\[
\min_{\alpha} : \|T\alpha - \beta\|^2_2 + \eta\|\alpha\|_1 \tag{26}
\]

subject to : (17), (23), (24)

to obtain \(\alpha\);

(c) **calculate** \(\bar{y}[k], \hat{x}[k]\) by (15), (21) and (22);

(d) set \(k \leftarrow k + 1\).

The optimization problem (26) is a convex optimization problem and can be solved efficiently. Note that the three constraint conditions are independent with each other, the problem is therefore feasible if the polynomial (12) has a degree not less than 2 \((m \geq 2)\). The block diagram of the proposed approach is shown in Figure 5. Since the reconstruction error is bounded by (17), it is guaranteed that the closed-loop system is BIBO (bounded-input-bounded-output) if \(\bar{y}\) is used to feedback and a controller is designed to stabilize the nominal system.

IV. Experiments

In this section, the proposed approach is applied to a high-precision stage. Figure 6 shows the experimental
setup. The stage is a model of the scanner in exposure systems used for the fabrication of integrated circuits. In real exposure systems, successful scanning requires extremely precise motion control. The configuration of experimental system is shown in Fig. 7. Two linear motors located at the both sides of the carriage are applied to drive the stage. An air guide system is also introduced to reduce the friction between the stator and the slider of the motors. A linear encoder with the resolution of 1nm is exploited for the position measurement. DSP(TMS320C6713, 225MHz) is used as the processor to implement the controllers and the proposed fitting approach. Fig. 8 shows the frequency characteristics of the Stage from the current to the position. The nominal model is expressed as

\[ P(s) = \frac{26.5}{s(14.7s + 24)}, \]  

where \( s \) is the Laplace operator.

A 2-DoF controller is exploited to control the stage, and the block diagram of the control system is shown in Figure 9. The feed forward controller, designed by perfect tracking control (PTC) method [15], is the stable inverse system of the nominal plant so that the perfect tracking at every sampling instant can be theoretically guaranteed. The feedback controller is designed as the PID compensator given by

\[ K = k_p + \frac{k_d}{0.004s + 1} + k_i \frac{1}{s}, \]  

where \( k_p = 2065.6, \ k_i = 34624, \ k_d = 43.45 \). The bandwidth of the close-loop system is 10Hz. The control system is discretized with the sampling period \( T_s = 5 \text{ ms} \). Denote \( \mathbf{x} \) as \( \mathbf{x} := [y \ y]^{T} \), where \( y, \dot{y} \) are the position and the velocity of the stage, respectively, a
discrete-time state space expression of (27) using zero-order-hold method is

\[ x[k+1] = Ax[k] + Bu[k] + w[k], \]
\[ y[k] = Cx[k], \]

where

\[ A = \begin{bmatrix} 1 & 0.0050 \\ 0 & -0.9919 \end{bmatrix}, \quad B = \begin{bmatrix} 2.247e^{-5} \\ 8.977e^{-3} \end{bmatrix}, \]
\[ C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \]

and \( w \) is the input disturbance. According to the real situation, \( w \) is treated as a sort of step-type disturbance. Therefore, \( \Gamma \) and \( H \) in the disturbance state equation (5) (6) can be set as \( \Gamma = 1 \) and \( H = 1 \).

In order to evaluate the effectiveness of the proposed method, a software quantizer is introduced to quantize the position signal measured by the linear encoder. In this way, the measurements from the linear encoder is regarded as the actual output \( y \), and the signal from the software quantizer is treated as the quantized output \( y_q \). For comparison, the quantized output, the output estimated by the observer (21) (22), and the output obtained by the proposed fitting method are used as the feedback signals, respectively. A switch is used to select the feedback signal, as shown in Figure 9. In order to evaluate the control performance and the reconstruction performance, the following errors are defined:

\[ e_q = y_q - y, \quad (29) \]
\[ e_t = y_d - y, \quad (30) \]
\[ e_r = \bar{y} - y. \quad (31) \]

\( e_q \) denotes the quantization error, \( e_t \) denotes the control error and \( e_r \) denotes the reconstruction/estimation error.

In the setup, the quantization step is set as \( \Delta = 20\mu \text{m} \). The number of data used for polynomial fitting is set as \( p + 1 = 15 \), and the degree of polynomial is set as \( m = 3 \). The weight factor \( \eta \) is properly chosen as \( \eta = 2 \times 10^{-3} \). \( \delta \) is set as \( 2 \times 10^{-9} \) by trial and error. The C-code of solving the convex optimization problem (26) is generated by CVXGEN [16]. The gain \( L_e \) of the state estimator is determined by placing the poles of the observer at \([0.5335, 0.6242, 0.7304]\), which results in the observer bandwidth as 20Hz. By this setting, the average computational time of solving the convex optimization problem (26) is about 3 ms, which implies the problem can be solved during the sampling period.

In order to examine the transient and the settling performance of the stage, the trajectory reference including acceleration, uniform motion, deceleration and settling motion is given, as shown in Figure 10.
Fig. 11. Experimental results in the case that the quantized measurements are used for feedback. (a) shows the comparison of the desired reference \( y_d \) and real output \( y \). (b) shows the position tracking error. (c) shows the quantization error, and (d) shows the control error.

Fig. 12. Experimental results in the case that the observer (21) (22) is used. (a) shows the comparison of the desired reference \( y_d \), the real output \( y \) and the estimated output \( \tilde{y} \). (b) shows the position tracking error and the estimation error. (c) shows the control input.

The experimental results are shown in Figure 11, Figure 12 and Figure 13. Figure 11 shows the results when the quantized measurements are used as feedback. Figure 11(a) shows the comparison of the desired reference \( y_d \) and the real output \( y \). Figure 11(b) shows the control error. It is observed that the high-order vibration is excited not only at the transient process, but also in the settling motion area after \( t = 1.5 \)s. The
quantization error is shown in Figure 11(c), and the control input is shown in Figure 11(d). It is shown that the control input behaves wildly even when the stage is in the settling motion area. Figure 12 shows the results when the observer (21) (22) is applied. Figure 12(a) shows the comparison of the desired output, the real output and the estimated output, and Figure 12(b) shows the comparison of the control error and the estimated error. It is observed that the estimated error cannot be guaranteed to be less than quantization step, which further degrades the control performance, as compared to Figure 11(b). The results of the proposed method are shown in Figure 13. Figure 13(a) shows the comparison of the desired output $y_d$, the controlled output $y$ and the reconstructed output $\hat{y}$. The control error, the quantization error and the reconstructed error are shown in Figure 13(b), (c). It is observed that the reconstruction error is much smaller than the quantization error, which implies that the accuracy of the feedback signal is improved. Moreover, compared with Figure 11(b) and Figure 12(b), it is observed that the high-order vibration excited by the quantization error is suppressed by the proposed method, especially in the settling motion area. Figure 13(d) shows the control input and the estimated disturbance. The control input behaves much softer compared to Figure 11(d). In addition, it can be shown that the residual control input $u$ during the settling motion area is correctly estimated by the augmented observer. Therefore, the effectiveness of the proposed method is verified.

Fig. 13. Experimental results when the quantized measurements are fitted. (a) shows the comparison of the desired reference $y_d$, real output $y$ and the reconstructed output $\hat{y}$. (b) shows the position tracking error. (c) shows the quantization error and the reconstruction error, and (d) shows the control input and the estimated disturbance.
V. Conclusion

In this paper, a novel output reconstruction approach consists of polynomial fitting approach and augmented observer for quantized linear system is proposed. In order to fit the quantized measurements by an appropriate polynomial, $\ell_1$-norm regularization method is exploited to automatically determine the degree of polynomial. In addition, some constraint conditions deduced from the feature of quantization and the system model are utilized to improve the accuracy of reconstruction. The approach is based on the observer techniques and the convex optimization technique, and therefore it is computationally tractable. The experimental results also verify that the proposed approach can reconstruct the real output accurately from the quantized output.

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