Step Climbing Control of Wheeled Robot Based on Slip Ratio Taking Account of Work Load Shift by Anti-Dive Force of Suspensions and Acceleration

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***Abstract*—This paper proposes a method which enables wheeled robot to climb a step that is higher than the radius of the wheel. In conventional method, the velocity at the moment of impact and impact force must be increased in order to climb the step, which causes troubles of wheeled robot. This paper proposes a method utilizing variable loads arised by changing speed and anti-dive force from suspension to climb the step more safely. Effectiveness of the proposed method is verified by simulations and experiments.**

I. Introduction

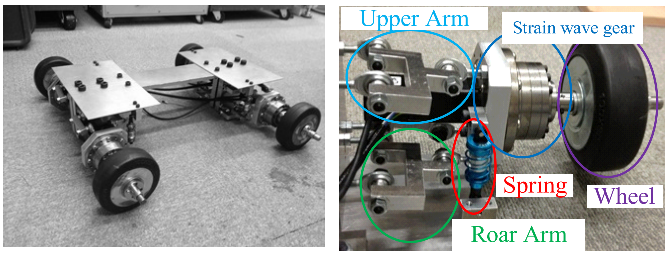
In order to solve challenges raised by aged society and environmental problems, personal mobility vehicles (PMVs) have been intensively studied as alternatives to conventional cars and wheelchairs [1]–[3]. On the other hand, industrial manipulators which are used to convey baggage in the factory and so on are being developed [4]–[6]. In these cases, various robots are researched and developed.

However, these robots have many problems with aspects to safety and operability. One of those problems is to climb a step that is higher than the radius of the wheel [2]. In order to solve this problem, step climbing has been widely researched [6]–[11]. In previous works, the robots can climb the step by moving these legs using some actuators, such as multi-legged walking robot [6][7] and wheel-legged mobile robot [8]–[10]. However, additional actuators result in increasing of cost, consumption of energy, and complexity [10].

On the other hands, it is known that it is easy to climb the step when the rear load of the robot is heavier than that of the front [11]. However, this design has negative influences on other operations such as cornering. S. Nakamura et al [11] invesgitated a robot which can change front and rear loads such that the rear load becomes heavier at the moment of step climbing. However, it has some problems as below:

- It is difficult for the rear tire to climb the step.
- It has negative influences on other operations such as cornering.

Therefore, it is desirable that the front and rear load of robot are the same.



(a)Wheeled robot (b)Wheel

Fig. 1. Experimental wheeled robot

We proposed a method which enables wheeled robot whose front and rear load are same to climb a step that is higher than the radius of the wheel [12]. By controlling the front driving force at the moment when the robot contacts the step, the front driving force is strong enough for the robot to climb the step. In this way, increasing the distance the front wheel surfaces from ground is to increase the velocity of translation at the moment of impact. However, if the velocity of translation at the moment of impact is increased, impact force arising at the moment of impact is increased. This phenomena causes the trouble of robot. Therefore, it is necessary to increase the distance that the front wheel surfaces from the ground, without increasing the velocity of translation at the moment of impact.

This paper verifies that in which case the proposed method is effective, robot contacting by acceleration or deceralation. In the research [12], the static and dynamic conditions of step climbing are examined at a constant speed. Whether the load change because of changing speed is effective for step climbing is verified. Moreover, if suspensions are fitted up with each wheel of wheeled robot, anti-dive force arised at changing speed. This paper verifies the effectiveness which anti-dive force gives step climbing of wheeled robot. Then, the effectiveness of the proposed method is verified by simulations and experiments.

TABLE I
PARAMETERS OF EXPERIMENTAL EQUIPMENT

the radius of the wheel r [m]	0.050
wheelbase l [m]	0.50
wheeled robot height h [m]	0.15
the mass of wheeled robot M [kg]	10

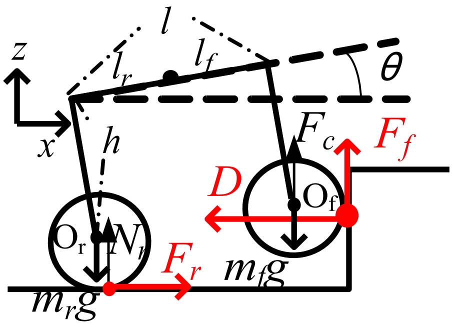


Fig. 2. Diagram of step climbing

II. EXPERIMENTAL WHEELED ROBOT AND MODELING

A. Experimental wheeled robot

An original wheeled robot, designed by the authors' laboratory, is shown in Fig.1. The side-view of the wheeled robot is shown in Fig.1(a), and the enlarged view of wheel is shown in Fig.1(b). Each wheel can be controlled independently by servo motor. Strain wave gear is connected between the motor and the wheel. The reduction ratio of strain wave gear is $1/100$, and the maximum torque of each wheel is 14 Nm.

Moreover, double wishbone type suspensions are installed at each wheel. The instantaneous center of rotation angle can be changed from 0 deg to 20 deg by changing upper arm and roar arm.

The experimental robot's specification is shown in Table I.

B. Equation of motion of wheeled robot

The diagram of step climbing is shown in Fig.2. Here, F_f [N] and F_r [N] are the front and rear driving force, N_r [N] is the normal reaction force of rear tire, F_c [N] is the load change from changing speed, m_f [kg] and m_r [kg] are the mass of front and rear tire, g [m/s^2] is the acceleration of gravity, and θ [rad] is the pitch angle. D [N] is the force caused by pushing the step, which we call "Reaction force" in this paper. The relationship between m_f and m_r is expressed as

$$m_f + m_r = M. \tag{1}$$

Lagrangian L is expressed as follow:

$$L = K_f + K_r - U_f - U_r \tag{2}$$

where K_f and K_r are front and rear kinetic energy, and U_f and U_r are front and rear position energy. Equation of motion in the x and θ direction is expressed by Lagrange's equations of motion as follows:

$$\begin{aligned} F_r - D &= \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} \\ &= M\ddot{x} - m_f l\ddot{\theta}\sin\theta - m_f l(\dot{\theta})^2\cos\theta, \end{aligned} \tag{3}$$

$$\begin{aligned} &F_f(l\cos\theta + r) + F_c l\cos\theta + F_r r + Dl\sin\theta \\ &\qquad = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} \\ &= m_f l^2\ddot{\theta} - m_f l\ddot{x}\sin\theta + m_f gl\cos\theta. \end{aligned} \tag{4}$$

Equation of motion in the x direction is expressed as follows by linearizing (3) as

$$M\dot{v_x} = F_r - D, \tag{5}$$

where v_x [m/s^2] is the robot velocity in the x direction. On the other hand, equation of motion in the z direction is expressed as follows by linearizing (4) (Assumed that $l \gg r$):

$$(F_f + F_c - m_f g)l = m_f l\dot{v_z} - m_f\dot{v_x}v_z, \tag{6}$$

where v_z [m/s^2] is the robot velocity in the z direction.

III. THE REQUIREMENTS OF STEP CLIMBING FROM EQUATION OF MOMENT

In this section, the requirements of step climbing are analyzed. In our research[12], they are analyzed from the equation of static condition. In this paper, they are analyzed from the equation of moment arised at the impact.

A. Requirment of Step Climbing

In this section, the requirment of step climbing is considered from the equation of moment arised at the impact.

Right after the front tire leaves the ground, the moment can be expressed as

$$\begin{aligned} I\ddot{\theta} &= F_f(l\cos\theta + r) + F_c l\cos\theta \\ &-m_f gl\cos\theta + F_r r + Dl\sin\theta, \end{aligned} \tag{7}$$

where I [kg $\cdot$ m^2] is the inertia of wheeled robot($I = m_f l^2$).

If the front tire climbs the step, $theta$ must be greater than 0. $theta$ equals 0 before the robot ckimbs the step because the front tire contacts the ground. Therefore, the condition that θ is greater than 0 is as follow in the case that the front tire contacts the ground,

$$\ddot{\theta} > 0. \tag{8}$$

From (7) and (8), the requirement that the front tire leaves the ground is expressed as (assumed that r is much smaller than l),

$$\theta > \arctan\frac{m_f g - F_c - F_f}{D} = \theta_{th}, \tag{9}$$

where θ_{th} is the threshold of the body angle. $theta$ equals 0 before the robot ckimbs the step. Therefore, θ_{th} must be less than 0 at the moment of impact. From (9), the requirement that the front tire leaves the ground is expressed as (10),

$$F_f + F_c > m_f g. \tag{10}$$

B. *Consideration of step climbing in static case ($\dot{v_x} = 0$ and $\theta \simeq 0$)*

In this section, the condition of step climbing in static case($\dot{v_x} = 0$ and $\theta \simeq 0$) is considered. Now, in this case, F_c equals 0.

The relationship between F_f and D is expressed as (11),

$$F_f = \mu_f D, \tag{11}$$

where μ_f is the friction coefficient between the front tire and the step. The maximum coefficient of static friction is μ_{f0}, so the maximum of F_f is expressed as

$$F_f \leqq \mu_{f0} D. \tag{12}$$

Therefore, μ_f or D must be increased in order to increase F_f. In this section, the way of increasing D is considered.

From (5), D is described as

$$D = F_r - M\dot{v_x}. \tag{13}$$

In static case($v_x = 0$), D equalls F_r. F_r is expressed as

$$F_r = \mu_r N_r = \mu_r (m_r g + \frac{h}{l} Ma), \tag{14}$$

where μ_r is the friction coefficient between the rear tire and the road. In static case($\dot{v_x} = 0$), F_r equalls $\mu_r m_r g$. Therefore, the maximum of F_r is expressed as

$$F_r \leqq \mu_{r0} m_r g, \tag{15}$$

where μ_{r0} is the biggest coefficient of static friction. From (11) to (15), (16) is expressed,

$$F_f \leqq \mu_{f0} \mu_{r0} m_r g = k \mu_f \mu_r m_f g, \tag{16}$$

where $k(= \frac{m_r}{m_f})$ is the ratio of front and rear tire mass. From (10) and (16), k must satisfy (17) in order to climb the step,

$$k > \frac{1}{\mu_{f0} \mu_{r0}}. \tag{17}$$

In static case, m_r must become bigger than m_f because μ_{f0} and μ_{r0} are smaller than 1. However, in case of experimental wheeled robot, m_r equalls m_f because of the functionality as is said in Section I. Therefore, (11) is not satisfied.

Thus,in static case, the wheeled robot which does not use actuators in order to change the front/rear mass cannot climb the step if m_f and m_r are the same.

C. *Consideration of step climbing in dynamic case ($\dot{v_x} \neq 0$)*

In this section, the condition of step climbing in dynamic case (The robot driving at a constant speed impacts the step) is considered.

At the moment the robot impacts the step, D increases because the force F_I (expressed as (18)) is occurred,

$$F_I = -M\dot{v_x}. \tag{18}$$

From (13), (14), and (18), D is expressed as

$$D = \mu_r m_r g + F_I. \tag{19}$$

In this case, D is increased because F_I is increased at the moment of impact. From (11) and (eq:401) the maximum of F_f is increased. Therefore, it is considered that the front tire leaves the ground by controlling F_f at the moment of impact.

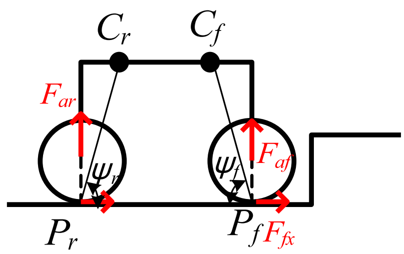


Fig. 3. Suspension geometry

IV. Effect of Load Change by Speed to Step Climbing

In this section, the effect that variable load by speed changing gives step climbing is considered.

A. *Effect of front load change to step climbing by acceleration and deceleration*

When the wheeled robot runs on the level, the vertical load of front tire N_f $[rmN]$ and rear tire N_r $[rmN]$ is expressed as follows:

$$N_f = m_f g - F_n, \tag{20}$$

$$N_r = m_r g + F_n, \tag{21}$$

where F_n [N] is expressed as follow

$$F_n = \frac{h}{l} Ma, \tag{22}$$

where a $[\mathrm{m/s^2}]$ is the acceleration of wheeled robot in direction of x. In case of wheeled robot without suspensions, F_c equals F_n.

In case of running at a constant speed ($a = 0$), F_n equals 0 and N_f doesn't change. In case of running at decelating ($a < 0$), F_n (F_c) is less than 0, and v_z becomes smaller from (6). On the other hand, in case of running at accelerating ($a > 0$), F_n (F_c) is more than 0, and F_f becomes larger from (6). Therefore it is easier for wheeled robot to climb the step when the robot runs at accerelating than when the robot runs at a constant speed.

B. *Effect of anti-dive force to step climbing*

1) Suspension geometry [?]: A description will be given for the anti-dive force by each driving force. Suspension geometry is shown as Fig.3. Anti-dive force is determined from the angle to the horizontal plane and the line connecting the instantaneous center of rotation of the suspension from the ground point. Front and rear anti-dive force F_{af} [N] and F_{ar} [N] are expressed as follows:

$$F_{af} = -F_{fx} \tan \psi_f, \tag{23}$$

$$F_{ar} = F_r \tan \psi_r, \tag{24}$$

where ψ_f and ψ_r are the instantaneous center of rotation angle.

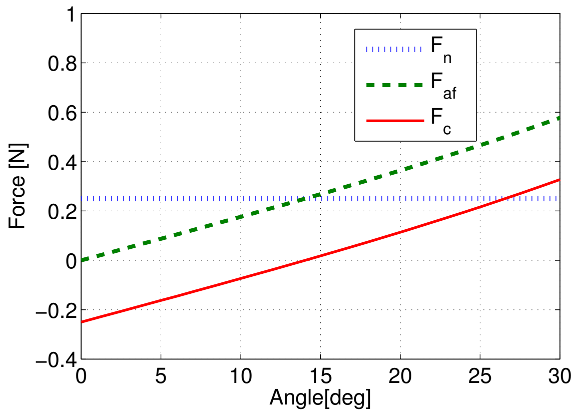


Fig. 4. Relationship between F_n and F_{af}

2) The relationship between F_{af} and F_n: In this case, (6) becomes

$$F_f + F_{af} + F_n - m_f g = m_f \dot{v_z}, \tag{25}$$

because F_c can be expressed as

$$F_c = F_n + F_{af} \tag{26}$$

In case of running at decelating ($a < 0$), F_{fx} is less than 0 and F_{af} is larger than 0. Therefore, when the robot runs at decelating, anti-dive force helps wheeled robot climb the step. However, in this case, F_n is less than 0 and prevented wheeled robot from climb the step.

F_{af} can be changed by changing ψ_f. The relashionship between F_n and F_{af} when the robot runs at decrelating ($a = -0.08$) is shown as Fig.4. As ψ_f becomes larger, F_{af} becomes larger and is larger than F_n.

Therefore, if suspensions are fit up as ψ_f is large, the effect of anti-dive force is larger than that of F_n, and it helps the wheeled robot climb the step.

V. Control System for Step Climbing[12]

From considerations in section III, increasing F_f at the moment of impact enables the front tire leave the ground. If μ_f or D is increased, F_f is increased as shown in (11). As is said in section III, D is increased at the moment of impact. Therefore, if μ_f can be controlled at the moment of impact, F_f is increased. In this section, the control system which is based on the slip ratio control method [12] is proposed.

A. Modeling of front and rear tire

The block diagram of front wheel in the z direction is shown as Fig.5, and the block diagram of rear wheel in the x direction is shown as Fig.6. The equations of the wheeled robot are expressed as

$$J_i \dot{\omega}_i = \tau_i - r F_i, \tag{27}$$

$$m_f \dot{v_f} = F_f - m_f g, \tag{28}$$

$$M \dot{v_x} = F_r - D, \tag{29}$$

$$v_{\omega_i} = r\omega_i, \tag{30}$$

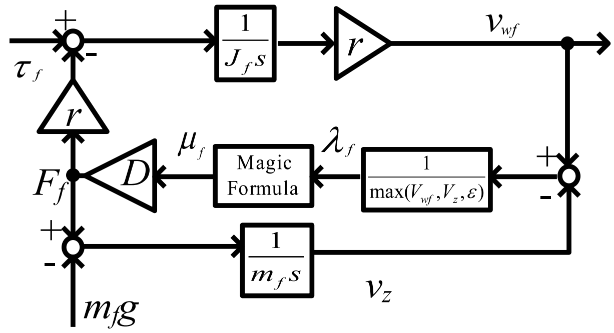


Fig. 5. Block diagram of front wheel in the z direction

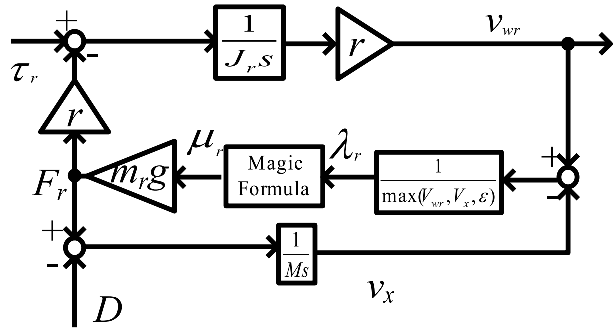


Fig. 6. Block diagram of rear wheel in the x direction

where ω_i [rad/s] is the wheel angular velocity, J_i [kg · m^2] is the wheel inertia, τ_i [N · m] is the motor torque, and $v_{\omega i}$ [m/s] is the wheel velocity. Also, i is indices for f/r (front/rear).

Also, the slip ratio of front and rear tire λ_i is expressed as eq(31),

$$\lambda_i = \frac{v_{\omega_i} - v_j}{\max(v_{\omega_i}, v_j, \epsilon)}, \tag{31}$$

where ϵ is a tiny value to prevent division by zero. j is indices for x/z. If i is f, j is z, and if i is r, j is x.

The relationship between μ_i and λ_i depends on the road condition, and is known to be Magic Formula [13]. F_f and F_r are expressed as (11) and (15).

B. Proposed control system for step climbing based on slip ratio control

The block diagram of the proposed control system is shown as Fig.7. From (31), the wheel angular velocity reference ω_i^* [rad/s] is calculated as

$$\omega_i^* = \frac{v_j}{r(1 - \lambda_i^*)}, \tag{32}$$

where λ_i^* is the slip ratio reference. Each controller is designed as PI law using pole placement method.

The front slip ratio reference is determined as (33), using $\hat{D}$ estimated by Reaction Force Observer (RFO) which is

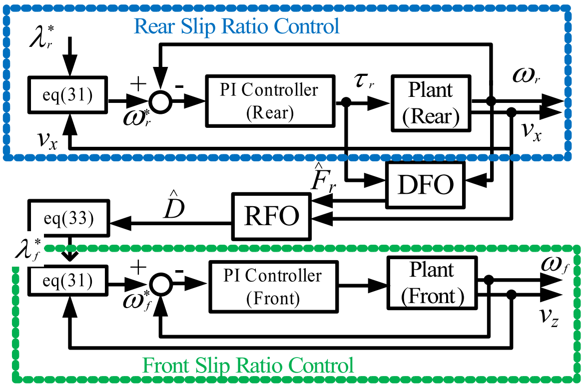


Fig. 7. Block diagram of front/rear slip ratio control

proposed in [12],

$$\lambda_f^* = \begin{cases} 0 & (\hat{D} \leqq D_{th}) \\ \lambda_{peak} & (\hat{D} > D_{th}) \end{cases}, \tag{33}$$

where D_{th} [N] is threshold determined at one's will. By controlling the front slip ratio and maximizing μ_f, F_f is increased because of (5).

VI. Simulation and Experiment of Step Climbing Using Wheeled Robot

In this section, effectiveness of the proposed method is verified by simulation and experiment.

A. Simulation

At first, effectiveness of the proposed method is verified by simulation.

Assumed that wheeled robot impacts the wall which height is infinite, the distance from the front tire to the ground z is verified. The ways of running are describes as:

- Conv:The robot runs at a constant speed (0.15 m/s) without suspension ($\psi_f = 0$),
- Prop1:The robot runs at accelerating ($a = 0.08$) without suspension ($\psi_f = 0$),
- Prop2:The robot runs at decelatin ($a = -0.08$) with suspension ($\psi_f = 20$).

v_x when the robot contacts the step is same (0.15 m/s) in all cases. Assumed the experimental wheeled robot, simulation model is established using the parameters in Table.I. In this simulations, m_f and m_r are same. Also, μ_{f0} and μ_{r0} are 0.5. The gain of controller is determined by pole placement method. And the pole of feedback control of front tire is -30 rad/s, and that of rear tire is the same. The front slip ratio reference is calculated by (33), which D_{th} is 10 N. The results are shown as Fig.8. Although D and v_x at the moment of impact is same in all cases, z in Prop1 is larger than that in Conv, and z in Prop2 is larger than that in Prop1. It is said that the way of step climbing using load change by changing speed and anti-dive force is effective.

B. Experiment

In this section, effectiveness of the proposed method is verified by experiments using experimental wheeled robot.

the distance from the front tire to the ground z is verified. The ways of running are describes as:

- Conv:The robot runs at a constant speed (0.15 m/s) without suspension ($\psi_f = 0$),
- Prop1:The robot runs at accelerating ($a = 0.08$) without suspension ($\psi_f = 0$),
- Prop2:The robot runs at decelatin ($a = -0.08$) with suspension ($\psi_f = 20$).

v_x when the robot contacts the step is same (0.15 m/s) in all cases.

Experimental results is shown as Fig.9. Although D and v_x at the moment of impact is same in all cases, z in Prop1 is larger than that in Conv, and z in Prop2 is larger than that in Prop1. It is said that the way of step climbing using load change by changing speed and anti-dive force is effective.

Moreover, how long it takes to climb the step is measured (Fig.9(c)). The most quick way is case 3. It is said that, from the point of rapidity, the way of climbing step using anti-dive force (Prop2) is much better than others.

VII. Conclusion

In this paper, a control method for the wheeled robot whose front and rear load are same and which does not use actuators for changing front and rear mass to climb the step is proposed, and whether it is effective if robot runs at changing speed is verified. We examine whether or not the robot can climb the step by controlling front tire at the moment it impacts the step[12]. However, impact force arising at the moment of impact must be increased in order to climb the step, and it makes wheeled robot easier to destroy. Therefore, we verify whether the variable loads by changing speed and anti-dive force are effective for step-climbing of wheeled robot. In conclusion, from safety and increasing the distance of step climbing, the way for wheeled robot with suspensions to contact the step at decelerating speed is better by simulations and experiments.

In this paper, we investigate only the distance which the robot surfaces from the ground. We will discuss whether or not the robot can climb the real step in the future work.

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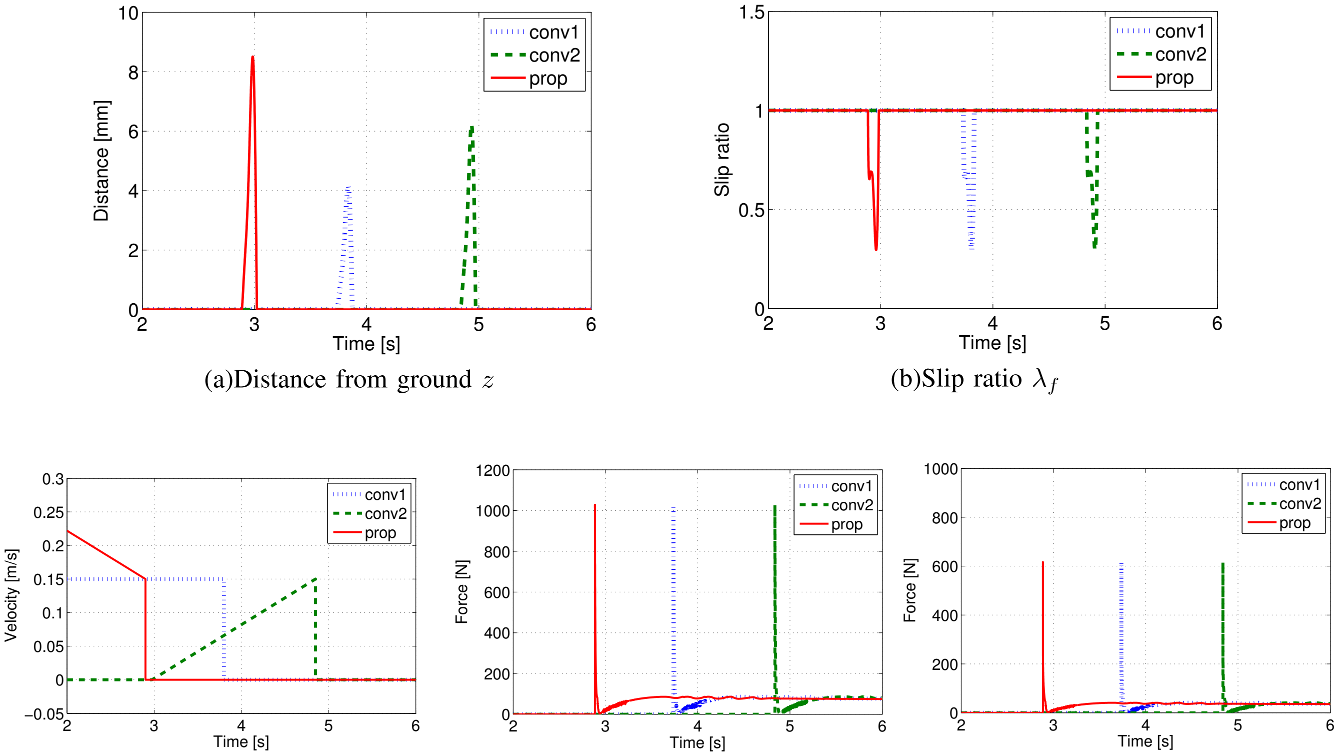
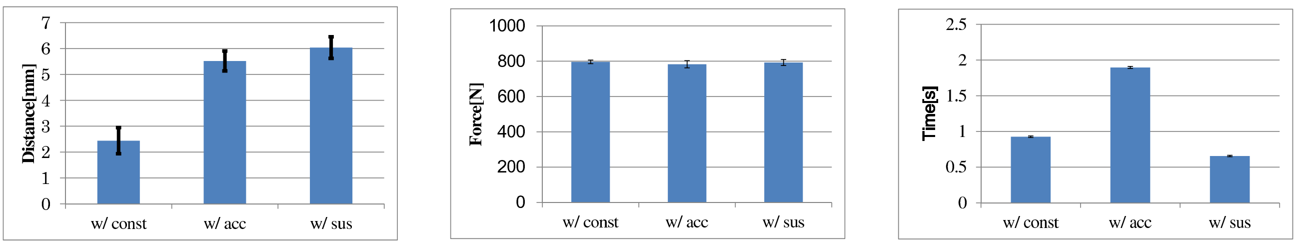


(a)Distance from ground z
(b)Slip ratio λ_f
(c)Velocity v_x
(d)Reaction force D
(e)Driving force F_f

Fig. 8. Simulation results)



(a)Distane z
(b)Reaction force D
(c)Time

Fig. 9. Experimental results

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