Modeling and Control of Radial Force due to Electromagnetic Force in IPMSMs

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ABSTRACT: In this paper, various methods to reduce noise and vibration of IPMSMs are introduced, especially focused on radial electromagnetic force fluctuation. Electrical 2nd and 6th order radial force is known to cause serious noise and vibration problem. Firstly the method to reduce radial force by structural designing is introduced. Secondly the modelling and control method for decreasing 2nd and 6th radial force vibration is shown. Finally, general overview is discussed to realize advanced motor designing and control technology.

KEY WORDS: Electromagnetic force, Radial force control, Electric Vehicle, Noise and vibration

Fig. 1 The magnetic attractive force in IPMSMs

1. Introduction

IPMSMs (Interior Permanent Magnet Synchronous Motors) are widely applied in many industrial applications. In these applications, IPMSMs face strong demands about the reduction of noise and vibration. In addition, the noise and vibration problems in the inside of cars remain to be one of the problems which should be improved. Furthermore, lower acoustic noise and vibration enhance the value of the product.

The magnetic attractive force which causes noise and vibration are produced by magnetic flux. Therefore it is important to grasp the flux distribution in IPMSMs. Fig. 1 shows the concept of magnetic attractive force. It can be seen that the magnetic attractive force has both the tangential and radial components in Fig. 1. Concentrated winding generally causes large fluctuation of tangential magnetic force which is called as torque ripple. Torque ripple triggers torsional resonant vibration and deteriorates control performance. On the other hand, the electromagnetic force fluctuation in radial direction is called as radial force and it induces elastic deformation when the frequency of radial force corresponds to natural frequency of the stator. Radial force is less acknowledged as a problem than torque ripple. However, in EV/HEV applications which are demanded very high performance, radial force comes to a head as the origin of noise and vibration. Fig. 2 shows typical radial force mode in IPMSMs.

In this paper, the cause of noise and vibration in radial direction due to electromagnetic force is classified into magnetostriction, rotor eccentricity, and rotating magnetic field. The methods to suppress each origin of noise and vibration by structural designing and current control are overviewed. In these origins, we focus on the vibration caused by rotation magnetic field, and suppression methods by structural designing and current control proposed by our group are
2. Fundamental concept of electromagnetic vibration

Vibration problem is generally formulated as:

\[
[F(\omega)] \times [H(\omega)] = [X(\omega)]
\]  

(1)

where \([F(\omega)]\) is radial force, \([H(\omega)]\) is frequency response function and \([X(\omega)]\) is motor vibration. It is notable that frequency response function is constructed by various modes which are linearly independent set. Therefore vibration reduction should be performed to each modes.

It is known that in IPMSMs electrical 2nd and 6th radial force usually cause serious noise and vibration. 2nd radial force is caused by fundamental magnetic flux and the amplitude is very large. This leads to serious vibration although the corresponding frequency response function which mode is pole pair order annular mode is small. The concept of 2nd radial force is shown in Fig. 3. On the other hand, the amplitude of 6th radial force is small compared with 2nd radial force. However, the frequency response function which corresponds to 6th radial force is very large because 6th radial force excites spatially 0th annular mode.

Fig. 4 shows the vibration mechanism of 16P24S motor (1). Fig. 4 also shows that in order to reduce motor vibrations there exist mainly two methods which are to improve the characteristics of frequency response function by structural designing and to reduce the fluctuation of radial force by current control. In (1) 3D map of the rotation speed, frequency, and vibration level is shown. As rotation speed increases, the frequency of the vibration also increases.

3. Overview of previous study

3.1. Modelling of flux distribution

It has been studied for a long time the method to make the modelling of flux distribution in IPMSMs (2). Z.Q.Zhu et al. generalize the modelling of flux distribution in previous researches (3)-(8). However, there are some faults in the studies that it is hard to apply this theory to actual application such as motor designing and current control designing. In (9) (10) and (11) this generalized modelling is extended to fractional order motor and modular winding motor (12) (13).

3.2. Magnetostriction

The noise and vibration generated by magnetostriction is studied in (14). It is known that magnetostrictive force is hard to solve completely. Magnetostriction also deteriorates iron loss. To reduce the influence of magnetostriction, it is preferred to use magnetic steel sheets which have excellent property on magnetostriction. (15) also shows the influence of magnetostriction.

3.3. Rotor eccentricity

Rotor Eccentricity also contribute to the noise and vibration caused by electromagnetic force. When rotor eccentricity exists, unbalanced radial force is generated. This force attracts rotor to one side and leads to noise and vibration. There are many previous researches such as (16)-(21).

3.4. Causes of rotating magnetic field

Even if the two causes above don’t exist, rotating magnetic field generates radial force ripple and it leads to noise and vibration. Radial force caused by rotating magnetic field is mainly electrical 2nd order. Considering the phase difference of U, V, and W-phase radial forces, it is known that 2nd order radial force has spatially Pth circular mode, in this sentence P denotes pole pairs. Until now, vibration reduction is mainly achieved by the structural designing based on Finite Element Analysis (FEA).

3.5. Reduction method

By structural designing

In (22) peak amplitude of radial force is reduced by cutting the edge of teeth. Peak amplitude of radial force can be also decreased by making holes in the rotor (23) (25). In (24) optimal stator design is studied with comparing some parameter of the motor topology. Skewing is well known to reduce cogging torque and torque ripple. Therefore it is...
natural to apply skewing method for reducing radial force 
(26) (27) (28) (29). In (1), to reduce 2nd order radial force, two structural designing methods to suppress motor vibration are proposed. One is to improve stator stiffness. Another method to suppress 2nd radial acceleration is to change pole slot combination. 2nd radial force excites mainly Pth spatial frequency. It means that the motor which has many pole pairs has low amplitude of transfer characteristic. The effect of changing pole slot combination is shown in (1) with experiments.

3.6. Reduction method by current control

On the other hand, few methods to suppress vibration using current control are proposed. (30) proposes radial force reduction with current control in condition of no-tooth effect. (31) and (32) propose harmonic radial force reduction method with harmonic current. These methods optimize current waveform through iterative calculation. (33) takes almost the same approach of (31) and lead the relationship between currents of U, V, and W-phase and radial force. Our group has already proposed current control methods (34) (35), and they are overviewed in this paper.

4. Modelling and Control Technology to reduce 2nd radial vibration

4.1. The Electromagnetic Forces of IPMSM

\( \phi_m \) refers to the stator position angle between the center of a U-phase tooth and a point where maxwell stress is considered. Maxwell stress is expressed as:

\[
fr(\phi_m) = \frac{B_r^2(\phi_m) - B_t^2(\phi_m)}{2\mu_0}, \quad fd(\phi_m) = \frac{B_s(\phi_m)B_t(\phi_m)}{\mu_0}
\]

where \( B_r(\phi_m) \) and \( B_t(\phi_m) \) are the radial and tangential maxwell stress on \( \phi_m \). In this paper, radial force \( F_{rU,V,W} \) on the surface of U, V and W-phase teeth is evaluated.

\[
F_{rU,V,W} = \int \int fr(\phi_m)dS
\]

where \( S \) is the surface area on a tooth facing on the air gap.

In this paper, radial force is calculated using approximate flux distribution. In this paper, JMAG (electromagnetic field analysis software with the FEA) produced by JSOL Corporation is applied to this analysis.

4.2. Approximate Model of Flux Distribution by Permanent Magnet

FEA result of flux distribution of permanent magnet is shown in Fig. 6. As shown in Fig. 6, The flux distribution is nearly flat on U-phase teeth, but unequal on V and W-phase teeth. Here, the flux distribution is approximated by rectangle. The magnetic flux passes through the area \( \gamma S \) and no magnetic flux passes through the other area \( \gamma S \). where \( \gamma (0 < \gamma \leq 1) \) is flux interlinkage area. It is expected that \( \gamma \) depends on rotor structure, but this relationship is considered. Maxwell stress is expressed as

\[
\gamma = \frac{\psi_{mU,mV,mW}}{S} = \frac{1}{2}\psi
\]

where \( N \) is the number of turn/teeth, \( \psi := \sqrt{\frac{2}{\pi}} F_{rU,V,W} \). Here, coefficient \( \sqrt{\frac{2}{\pi}} \) is the coefficient to transform two-phase into three-phase.

The flux distribution of permanent magnet is approximated as \( B_{rmU,mV,mW} \), where \( B_{rm} \) is flux interlinkage of j-phase teeth by permanent magnet, \( \psi_{mU,mV,mW} \) is interlinkage flux of j-phase, \( S_j \) is interlinkage flux area on j-phase teeth. Fig. 1 shows the approximate model of flux distribution.

4.3. Approximate Model of Flux Distribution by d-axis Current

Based on lineal independency, \( B_{rU} \) is calculated by the difference between \( B_r \) as \( i_d = -1[A] \) and by \( B_{rm}(\phi_m) \). Fig. 7 shows the flux distribution of \( B_{rU}(\phi_m) \). The flux distribution on U-phase tooth is nearly flat. On the other hand, the flux distribution on V and W-phase teeth concentrate in side of U-phase. In this paper, for the sake of simplicity, flux distribution on V and W-phase teeth is assumed to be flat.

\[ \psi_{mU} = \frac{1}{2}l_d i_d, \quad \psi_{mV,mW} = \frac{1}{2}l_d i_d \]

where \( \phi_{U,V,W} \) is flux on U, V and W-phase teeth by d-axis current. Here, \( l_d = \sqrt{\frac{2}{\pi}} F_{rU} \).

4.4. Radial force approximation using flux distribution

The flux distribution \( B_r(\phi_m) \) and maxwell stress \( fr(\phi_m) \) on U-phase teeth are calculated as follows:

\[
B_r(\phi_m) = \frac{\psi}{S} + \frac{l_d i_d}{S}
\]

By substituting (2) and (6) into (3), (8) is obtained.
\[ F_r = \frac{B_r^2(\varphi_m)}{2\mu_0} \cdot \frac{(\psi + l_i d)^2}{2l_0 S} \] (7)

The flux distribution of PM is not flat on V and W phase teeth. Therefore, approximate radial force is derived in two areas. In the area

\[ B_r(\varphi_m) = \left( \frac{\psi}{2} + \frac{l_i d}{2S} \right), \quad B_r'(\varphi_m) = \frac{l_i d}{2S} \] (9)

\[ F_{rV}, F_{rW} = \frac{B_r^2(\varphi_m)}{2\mu_0}, \quad \gamma S + \frac{B_r^2(\varphi_m)}{2\mu_0}, \quad (1 - \gamma) S \]

(10)

\[ = \frac{(\psi + \frac{l_i d}{\mu_0 S})^2 + \frac{(1-\gamma)\psi^2}{l_0 S}}{8} \] (11)

4.5. Current Reference Method to Suppress 2nd Order Radial Force

In this chapter, d-axis current reference is derived to suppress 2nd order radial force, \( F_{rU} \), \( F_{rV} \), and \( F_{rW} \) refer to radial force of U, V and W-phase when rotor electrical angle is \( \theta_e \).

Generally, radial force is suppressed by reducing the difference between max. and min. radial force. When \( \theta_e \) is 0, \( \pi \text{[rad]} \), radial force on U-phase teeth is maximum and equals \( F_{rU}(0) \), which is obtained in (8). Radial force reaches the a minimum value when \( \theta_e = \frac{1}{3} \pi, \frac{2}{3} \pi \text{[rad]} \). But, at this point, approximation model is not accurate enough. Therefore, radial force at \( \theta_e = \frac{1}{3} \pi \text{[rad]} \) is used instead of \( \theta_e = \frac{1}{3} \pi\text{[rad]} \).

If three-phase equilibrium is correct in IPMSM, \( F_{rU}(\frac{2}{3} \pi) \) equals \( F_{rV}(0) \). \( F_{rU}(0) \) is approximated in (11). Moreover, by cyclic nature, \( F_{rU}(\theta_e) \) has the equal values at electrical angle \( \theta_e = \frac{1}{3} \pi, \frac{2}{3} \pi, \frac{4}{3} \pi, \frac{5}{3} \pi \text{[rad]} \). So, if following equation is true, it is predicted that 2nd radial force is suppressed largely.

\[ F_{rU}(0) = F_{rU} \left( \frac{2}{3} \pi \right) \] (12)

From (10), (13), (14), the d-axis current which achieves minimum 2nd order radial vibration is represented by

\[ i_d = \left( -1 \pm \sqrt{\frac{1 - \gamma}{3\gamma}} \right) \frac{\psi}{l_i} \] (13)

By taking into consideration of \( 0 < \gamma \leq 1 \), square root is real number. Here, plus sign in (13) is selected in order to use minimum current amplitude. Using Table. 1, d-axis current reference is calculated by

\[ i_d = -29.1[A] \] (14)

4.6. Experiment

It is difficult to measure radial force directly. In this paper, radial acceleration outside the stator is evaluated in stead of radial force. The velocity is controlled by load motor. Drive motor controls current and radial acceleration on the stator of the drive motor is measured by accelerometer.

In this paper, the negative d-axis current reference is limited within -20 A because of the motor rating. Experimental results with \( n = 1000, 2000 \text{ rpm} \) are shown in Fig. 8(a), and 8(b) respectively. Here, the frequency of \( x \)-axis is normalized by electrical angle frequency. Fig. 8(a), and 8(b) shows d-axis current suppress 2nd order acceleration effectively, but deteriorate 6th order acceleration. 2nd order spectrums extracted in run-up experiment are shown in Fig. 8(c). 2nd order acceleration is suppressed at all rotation speed.

5. Modelling and Control Technology to reduce 6th radial vibration

5.1. The relationship between current, flux linkage and radial force

Flux passing through a U-phase tooth \( \phi_u(t) \) is expressed with flux linkage in U-phase \( \psi_u(t) \) as

\[ \phi_u(t) = \frac{\psi_u(t)}{N} \] (15)

where, \( N \) is turn number per one phase, and all coils are connected as series. We assume the tangential flux distribution \( B_\theta(t) \) is small, and all flux linkage is generated by the radial flux distribution \( B_r(t) \). Radial force on a U-phase tooth \( f_u(t) \) is calculated based on Maxwell stress. These assumptions are expressed as:

\[ f_u(t) = \int B_r(t) dS, \quad f_u(t) = \int \frac{B_r(t)^2}{2\mu_0} dS. \] (16)

where \( S \) is a tooth area facing air region. This paper has the assumption that flux is distributed equally over the tooth area \( S \). With this assumption, (16) are rewritten as (17).

\[ \phi_u(t) = B_r(t) S, \quad f_u(t) = \frac{\psi_u^2(t)}{2\mu_0 S} \] (17)

In this paper, we consider constant speed condition and radial force is regarded as a function for electrical angle \( \theta \). Substituting (15) into (17), radial force \( f_u(t) \) is expressed as a function of \( \psi_u(\theta) \).

\[ f_u(\theta) = \frac{\psi_u^2(\theta)}{2\mu_0 S N^2} = A\psi_u^2(\theta), \quad A := \frac{1}{2\mu_0 S N^2} \] (18)

(18) is used as the approximation of radial force in this paper.

5.2. Assumption on flux linkage

It is also assumed that flux linkage caused by permanent magnet \( \psi_{um}(\theta) \) and flux linkage caused by current \( \psi_u(\theta) \) satisfy linear independency.

\[ \psi_u(\theta) = \psi_{um}(\theta) + \psi_{u}(\theta) \] (19)

This paper considers 12 poles 18 slots IPMSM. The winding pattern is concentrated winding. To consider 6th radial force, \( \psi_{um}(\theta) \) is defined as:
ψ_{5,7} \text{ shown in Table 1.}

The parameter of IPMSM is as:

\begin{align*}
\psi_{\alpha} &= \psi_{1m} \cos \theta + \psi_{\alpha m} \cos 5\theta + \psi_{\gamma m} \cos 7\theta 
\end{align*} \tag{20}

5th and 7th flux linkage \( \psi_{\alpha m} \) and \( \psi_{\gamma m} \) have negative value when they have opposite phase against fundamental flux linkage. On ground of the symmetry, flux linkage on U-phase tooth is considered. The parameter of IPMSM is shown in Table 1.

\( q \)-axis current reference \( i_d, i_q \) is defined as:

\begin{align*}
i_d &= I_{d0} + i_{d6}, \quad i_{d6} = I_{d6} \cos(6\theta - \theta_{d6}) 
\end{align*} \tag{21}

\begin{align*}
i_q &= I_{q0} + i_{q6}, \quad i_{q6} = I_{q6} \cos(6\theta - \theta_{q6}). 
\end{align*} \tag{22}

5.3. The Influence of d-axis Harmonic Current

Flux linkage on U-phase caused by d-axis harmonic current \( \psi_{wu}(\theta) \) is shown as:

\begin{align*}
\begin{bmatrix} 
\psi_{wu}(\theta) \\
\psi_{wu}(\theta) \\
\psi_{wu}(\theta)
\end{bmatrix} &= C_{wu}^{uvw} \begin{bmatrix} L_d I_{d0} + L_d I_{d6} \cos(6\theta - \theta_{d6}) \\
L_q I_{q0}
\end{bmatrix} \\
&= \sqrt{\frac{2}{3}} L_d I_{d0} \begin{bmatrix} \cos \theta \\
\cos(\theta - \frac{2}{3}\pi) \\
\cos(\theta - \frac{4}{3}\pi)
\end{bmatrix} - \sqrt{\frac{2}{3}} L_q I_{q0} \begin{bmatrix} \sin \theta \\
\sin(\theta - \frac{2}{3}\pi) \\
\sin(\theta - \frac{4}{3}\pi)
\end{bmatrix} \\
&+ \frac{\sqrt{1}}{6} L_d I_{d6} \begin{bmatrix} \cos(5\theta - \theta_{d6}) + \cos(7\theta - \theta_{d6}) \\
\cos(5\theta - \theta_{d6} + \frac{2}{3}\pi) + \cos(7\theta - \theta_{d6} - \frac{2}{3}\pi) \\
\cos(5\theta - \theta_{d6} + \frac{4}{3}\pi) + \cos(7\theta - \theta_{d6} - \frac{4}{3}\pi)
\end{bmatrix} 
\end{align*} \tag{23}

Substituting (24) (20) and (19) into (18) all radial force is calculated. It is presumable 6th radial force caused by d-axis harmonic current is considered as a function of drive condition \( I_{d0}, I_{q0} \) and d-axis harmonic current \( i_{d6} \). Therefore, 6th radial force caused by d-axis harmonic current \( f_{r6d}(I_{d0}, I_{q0}, i_{d6}) \) is extracted in (18) as:

\begin{align*}
f_{r6d}(I_{d0}, I_{q0}, i_{d6}) &= K_{dr}(I_{d0}, I_{q0})i_{d6} 
\end{align*} \tag{25}

\begin{align*}
K_{dr}(I_{d0}, I_{q0}) := \frac{A}{3} (\Psi_{1m} + L_d I_{d0} L_q) 
\end{align*} \tag{26}

where, \( \Psi_{1m} := \sqrt{2}\psi_{1m} \). In this paper, 2-phase/3-phase transform is absolute transformation. In Eq. (26), it is known that \( f_{r6d} \) are not affected by \( I_{q0} \). To verify the accuracy of (26), FEA is performed on the condition that \( I_{d0} = 1 \) A and \( \theta_{d6} = 0 \) deg. The FEA result is shown in Fig.9(a) and 9(b). Although a lot of assumptions have been made to lead (26), we can see that 6th radial force model (26) differs very little from FEA result.

5.4. The Influence of q-axis Harmonic Current

In a similar way, flux linkage generated by current with q-axis harmonic current, \( \psi_{wq}(\theta) \) is calculated as:

\begin{align*}
\begin{bmatrix} 
\psi_{wq}(\theta) \\
\psi_{wq}(\theta) \\
\psi_{wq}(\theta)
\end{bmatrix} &= C_{wq}^{uvw} \begin{bmatrix} L_d I_{d0} \\
L_q I_{q0} + L_q I_{q6} \cos(6\theta - \theta_{q6})
\end{bmatrix} \\
&= \sqrt{\frac{2}{3}} L_d I_{d0} \begin{bmatrix} \cos \theta \\
\cos(\theta - \frac{2}{3}\pi) \\
\cos(\theta - \frac{4}{3}\pi)
\end{bmatrix} - \sqrt{\frac{2}{3}} L_q I_{q0} \begin{bmatrix} \sin \theta \\
\sin(\theta - \frac{2}{3}\pi) \\
\sin(\theta - \frac{4}{3}\pi)
\end{bmatrix} \\
&+ \frac{\sqrt{1}}{6} L_d I_{d6} \begin{bmatrix} \cos(5\theta - \theta_{d6}) - \cos(7\theta - \theta_{d6}) \\
\cos(5\theta - \theta_{d6} + \frac{2}{3}\pi) - \cos(7\theta - \theta_{d6} - \frac{2}{3}\pi) \\
\cos(5\theta - \theta_{d6} + \frac{4}{3}\pi) - \cos(7\theta - \theta_{d6} - \frac{4}{3}\pi)
\end{bmatrix} 
\end{align*} \tag{27}

Substituting (28) (20) and (19) into (18), the 6th radial force generated by q-axis harmonic current \( f_{r6q}(I_{d0}, I_{q0}, i_{q6}) \) is extracted as:

\begin{align*}
f_{r6q}(I_{d0}, I_{q0}, i_{q6}) &= K_{qr}(I_{d0}, I_{q0})i_{q6}, \quad K_{qr}(I_{d0}, I_{q0}) := \frac{A}{3} L_q^2 I_{q0} 
\end{align*} \tag{29}

Fig.9(c) shows the FEA results which are performed on the condition that \( I_{q6} = 1 \) A, \( \theta_{q6} = 0 \) A, and \( I_{d0} = 0 \) A. Fig.9(c) shows that the Eq. (29) can predict 6th radial force caused by q-axis harmonic current well.
5.6. Experimental Results

In experiment, radial acceleration outside the stator is evaluated instead of radial force. The velocity is controlled at 800rpm by load motor. Current controller is designed as PI feedback controller and feedforward controller of Perfect Tracking Controller \(^{(36)}\). Optimal \(d\)-axis harmonic current reference is recalculated through experiment.

Experimental result is shown in Fig. 11. Large 6th radial vibration is observed in w/o control spectrum. Injecting optimal \(d\)-axis harmonic current, 6th radial vibration is suppressed largely.

6. Conclusion

In this paper, the origin of noise and vibration in IPMSMs are classified and the methods to reduce noise and vibration are overviewed. The quietness is one of the key technologies in future electric vehicles. Magnetostriiction needs to be studied more and the reduction method of noise and vibration caused by magnetostriiction is desired by the improvement of magnetic steel sheet and analysis technology.

Uniformed modelling of radial force which shows structural designing and controller designing will be proposed in our future works.

References


Fig. 11 6th radial force control (Experimental Results)