Radial Force Control of IPMSM Considering Fundamental Magnetic Flux Distribution

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This paper proposes an optimal $d$-axis current to suppress the 2nd radial force, which is caused by the fundamental permanent flux. Under the no-load condition, the flux distribution is approximated in order to calculate the radial force. Considering the cyclic nature of 3-phase, the optimal $d$-axis current reference to suppress the 2nd radial force is derived. Simulations and experiments under both load and no-load conditions are performed to demonstrate the validity of the proposed optimal $d$-axis current reference.

Keywords: IPMSM, maxwell stress, radial force control, radial force, flux distribution, vibration suppression

1. Introduction

IPMSMs (Interior Permanent Magnet Synchronous Motors) are widely applied in many industrial applications. In these applications, it is essential to reduce the noise and vibration. Compared with other types of electrical machine, such as induction motor and switched reluctance motor, IPMSMs are relatively quiet. However, in applications, such as industrial servos, consumer products, and automotive drives, acoustic noise and vibration are important issues. Quietness enhances high commodity value of IPMSMs. Moreover the less weight IPMSMs become because of efficient vehicle\(^{(1)}\), the larger noise and vibration have been caused.

In IPMSMs, a strong electromagnetic force exists between the rotor magnets and the stator teeth, having components in both the tangential and radial directions. The tangential force results in torque ripple. The radial force which is stronger than the tangential force induces mechanical deformation and vibration of the stator. The causes of noise and vibration in PMSMs are investigated in (2). In (3) the relationship between skew and radial force vibration is examined. In (4) and (5), vibration caused by radial force is examined with structural analysis. In (6), with FEA (Finite Element Analysis) and experiments, a detailed study of the vibration is performed. Some relationships between vibration and structural characteristics are shown in (7) and (8). Some papers such as References (7) and (8) propose radial force reduction using structural changes. On the other hand, few methods to suppress vibration using current control are proposed. Reference (9) proposes radial force reduction with current control in condition of no-tooth effect. However this method is not versatile because the Reference (9) does not consider tooth effect and the relationship between current reference and radial force is obscure. Therefore, simpler methods which predict and reduce amplitude of radial force are required.

The radial force which is caused by the permanent magnet is mainly electrical 2nd order. This paper proposes an optimal $d$-axis current to suppress the 2nd radial force. The flux distribution on the no-load condition is approximated as a series of rectangle. Based on the approximate model, the radial forces on each tooth are calculated. Considering cyclic nature of 3-phase, the optimal $d$-axis current is derived. Second, $q$-axis influence is investigated with FEA. Finally, simulations and experiments under load and no-load conditions are performed to demonstrate the validity of the proposed optimal $d$-axis current reference.

2. Model and Maxwell Stress

2.1 dq Model of IPMSM

The voltage equation of IPMSM in $dq$-axis and the motor torque $T$ are represented by

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} L_{ds} + R & -\omega_c L_q \\ \omega_c L_{dq} & L_{qs} + R \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \omega_c \Psi_a \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$T = K_{mt} i_q + K_{rt} i_d i_q,$$

where $L_{ds}$, $L_{dq}$, $L_{qs}$, and $R$ are the inductances and resistance of the machine, $\omega_c$ is the angular speed, $\Psi_a$ is the airgap flux, $K_{mt}$ is the torque constant, and $K_{rt}$ is the torque constant of the rotor.
\[ \omega_m = \frac{1}{J_s + D_T} \omega_e = P \omega_m, \quad (3) \]

where \( v_d, q \) are the \( d \)-axis and \( q \)-axis voltages, \( i_d, q \) are the \( d \)-axis and \( q \)-axis currents, \( L_d, q \) are the \( d \)-axis and \( q \)-axis inductances, \( R \) is the stator winding resistance, \( \omega_e \) is the electric angular velocity, \( \psi_a \) is the back EMF constant, \( K_{mt} := P \psi_a, K_{mt} := P(L_d - L_q) \), \( P \) is the number of pole pairs, and \( J \) is the rotor inertia. In this paper, 2-phase/3-phase transform is absolute transformation.

2.2 The Electromagnetic Forces of IPMSM

\( \varphi_m \) refers to the stator position angle between the center of a U-phase tooth and a point where Maxwell stress is considered. Using the definition of Maxwell stress tensor, the radial and tangential electromagnetic forces are calculated as

\[ f_r(\varphi_m) = \frac{B_r^2(\varphi_m) - B_{\theta}^2(\varphi_m)}{2 \mu_0}, \quad (4) \]

\[ f_{\theta}(\varphi_m) = \frac{B_r(\varphi_m)B_{\theta}(\varphi_m)}{\mu_0}, \quad (5) \]

where \( B_r(\varphi_m) \) and \( B_{\theta}(\varphi_m) \) are the radial and tangential flux densities on \( \varphi_m \). In (4), \( B_r^2(\varphi_m) \) is much smaller than \( B_{\theta}^2(\varphi_m) \) and (4) can be approximated as

\[ f_r(\varphi_m) = \frac{B_{\theta}^2(\varphi_m)}{2 \mu_0}. \quad (6) \]

In this paper, the radial force \( F_{rU,vV,vW} \) acting on the surfaces of U, V and W-phase teeth are defined as

\[ F_{rU,vV,vW} = \int \int f_r(\varphi_m)dS, \quad (7) \]

where \( S \) is the internal surface area on a tooth.

3. Approximate Model of Flux Distribution

In this paper, the radial force is calculated using the proposed approximate flux distribution model. In this chapter, the radial flux distribution is derived. JMAG (electromagnetic field analysis software with the FEA) produced by JSOL Corporation is applied to this analysis. In FEA, it is supposed that ideal sinusoidal current flows.

3.1 Hypotheses for Approximation

This paper considers 12 poles 18 slots IPMSM. The winding pattern is concentrated winding.

The value of \( B_r(\varphi_m) \) is a function of \( i_d, i_q, \varphi_m \), and \( \theta_e \), where \( \theta_e \) is rotor angle. Moreover, it is assumed that the radial flux density \( B_r(\varphi_m) \) is expressed as

\[ B_r(\varphi_m) = B_{ri_d}(\varphi_m, i_d) + B_{ri_q}(\varphi_m, i_q) + B_{rm}(\varphi_m), \quad (8) \]

where \( B_{ri_d}(\varphi_m, i_d) \) and \( B_{ri_q}(\varphi_m, i_q) \) are the radial flux densities due to \( i_d \) and \( i_q \) on \( \varphi_m \). \( B_{rm}(\varphi_m) \) is the radial flux density which is generated by the permanent magnet on \( \varphi_m \). Firstly, in order to consider no-load condition, this paper neglects \( B_{ri_d}(\varphi_m, i_q) \). The effect of \( q \)-axis current is described at Section 4.4.

In this paper, the linearity between \( i_d \) and \( B_{ri_d}(\varphi_m) \) is assumed.

Fig. 1. Flux distribution \( B_{rm}(\varphi_m) \) by permanent magnet

3.2 Approximate Model of Flux Distribution by Permanent Magnet

The FEA result of the flux distribution of the permanent magnet at \( \theta_e = 0 \) deg. is shown in Fig. 1(b). As shown in Fig. 1(b), the flux distribution is nearly flat on the U-phase tooth, but unequal on the V-phase and W-phase teeth. Here, the flux distribution is approximated by a rectangle. The magnetic flux passes through the area \( \gamma S \) and no magnetic flux passes through the other area \( (1 - \gamma)S \). \( \gamma \) \((0 < \gamma \leq 1)\) is flux interlinkage area. It is expected that \( \gamma \) depends on the rotor structure, but this relationship is inevident. In this paper, \( \gamma \) is determined from FEA, such that \( \gamma = 1 \) on U-phase and \( \gamma = 0.5 \) on V-phase and W-phase.

The fluxes \( \phi_{mU,mV,mW} \) on a tooth surface are calculated as

\[ \phi_{mU} = \phi, \quad \phi_{mV,mW} = -\frac{1}{2} \phi, \quad (9) \]

where \( N \) is the number of commutating turns per a tooth and \( \phi := \sqrt{\frac{2}{3} \psi}. \) Here, \( \sqrt{\frac{2}{3}} \) is the coefficient to transform 2-phase into 3-phase.

The flux distribution of the permanent magnet is approximated as \( B_{rmj} := \frac{\phi_{mj}}{\gamma S} \), where \( B_{rmj} \) is the flux distribution of the \( j \)-phase teeth by the permanent magnet, \( \phi_{mj} \) is whole magnetic flux of the \( j \)-phase teeth, \( S_j \) is the interlinkage flux area on the \( j \)-phase teeth. Fig. 1(b) shows the approximate model of the flux distribution.

3.3 Approximate Model of Flux Distribution by \( d \)-axis Current

Under the assumption of Section 3.1, \( B_{ri}(\varphi_m) \) is calculated by the difference be-
4. Radial Force Approximation Using Flux Distribution

4.1 Radial Force on U-phase Tooth

Fig. 4(a) shows the flux distribution image on the U-phase tooth. The flux distribution $B_r(\varphi_m)$ on the U-phase tooth is calculated as follows:

$$B_r(\varphi_m) = \left( \frac{\phi}{S} + \frac{l_d i_d}{2S} \right)$$

By substituting (6) and (11) into (7), (12) is obtained.

$$F_{rU} = \int \int f_r(\varphi_m) dS$$

$$= B^2_r(\varphi_m) \cdot \frac{S}{2\mu_0}$$

$$= \frac{(\phi + l_d i_d)^2}{2\mu_0 S}$$

(12)

$F_{rU}, F_{rW} = \int \int f_r(\varphi_m) dS$

$$= \frac{B^2_r(\varphi_m)}{2\mu_0} \cdot \frac{\gamma S + B^2_r(\varphi_m)}{2\mu_0} \cdot (1 - \gamma) S$$

$$= \frac{(\phi + l_d i_d)^2 + (1-\gamma)^2 \phi^2}{8\mu_0 S}$$

(15)

4.2 Radial Forces on V and W-phase Teeth

The flux distribution of the permanent magnet is not flat on the V-phase and W-phase teeth. Therefore, approximate model is derived in two areas. Fig. 4(b) shows the flux distribution image on the V-phase and the W-phase teeth. The flux distribution $B_r(\varphi_m)$, $B'_r(\varphi_m)$ and the radial force $F_{rV}, F_{rW}$ on the V-phase and the W-phase teeth at $\theta_e = 0$ Deg. are calculated as

$$B_r(\varphi_m) = \left( \frac{\phi}{2S} + \frac{l_d i_d}{2S} \right)$$

(13)

$$B'_r(\varphi_m) = \frac{l_d i_d}{2S}$$

(14)

4.3 Current Reference Method to Suppress Radial Force

In this section, $d$-axis current reference is derived to suppress the 2nd order radial force. $F_{rU}(\theta_e)$, $F_{rV}(\theta_e)$, and $F_{rW}(\theta_e)$ refer to the radial forces of U-phase, V-phase and W-phase teeth when rotor electrical angle is $\theta_e$. When $\theta_e$ is 0 or $\pi$ rad., the radial force on U-phase tooth is maximum and equal to $F_{rU}(0)$, which is obtained in (12). The radial force on U-phase tooth is minimum when $\theta_e$ is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ rad. At this point, it is difficult to approximate radial force because the flux which does not interlink exists. Therefore, the radial force $F_{rU}(\frac{\pi}{2})$ is used instead of $F_{rU}(\frac{\pi}{2})$. When
three-phase equilibrium is correct in IPMSM, \( F_{rU}(\frac{2}{3}\pi) \) equals \( F_{rV}(0) \). In Fig. 5, the physical relationships between teeth and magnet when \( \theta_e \) changes are shown. \( F_{rV}(0) \) is approximated in (15). Moreover, by cyclic nature, \( F_{rU}(\theta_e) \) has the equal values at electrical angle \( \theta_e = \frac{1}{6}\pi, \frac{2}{6}\pi, \frac{3}{6}\pi, \frac{5}{6}\pi[\text{rad}] \). Therefore, when the following equation is true, it is predicted that 2nd radial force is suppressed largely.

\[
F_{rU}(0) = F_{rU}\left(\frac{2}{3}\pi\right)
\]  

(16)

4.4 The Influence of Flux Distribution by \( q \)-axis Current

In this section, the influence of flux distribution attributed by \( q \)-axis current is considered. The FEA result of the flux distribution by \( q \)-axis current is shown in Fig. 6. Fig. 6 shows that the flux distribution on U-phase tooth is 0. The \( dq/3 \)-phase transform matrix \( C_{UVW}^{dq} \) at \( \theta_e = 0 \) is given by

\[
C_{UVW}^{dq} = \begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & 0 \\
-\frac{1}{3} & 0 & \frac{\sqrt{3}}{3} \\
-\frac{1}{3} & -\frac{\sqrt{3}}{3} & 0
\end{bmatrix}.
\]  

(19)

This means the flux by \( q \)-axis current passes only through the V-phase and the W-phase teeth. In Fig. 6, the magnetic flux and the \( q \)-axis flux affect each other, and this leads to asymmetry of the flux distribution on the V-phase and W-phase teeth.

4.5 Verification of Radial Force Estimate Equation

In this section, the accuracy of the op-
Experimental results at 1500 rpm ($f_e = 150$ Hz) are shown in Fig. 9. Here, horizontal axis is the frequency normalized by electric angle frequency $f_e$. In this paper, the negative $d$-axis current reference is limited within -25 A because of the motor rating. Fig. 9(a) and 9(b) show the 2nd order acceleration on load condition is slightly larger than that on no-load condition. This result corresponds to FEA result as shown in Fig. 8(d). 2nd order acceleration on load condition is less reductive by $d$-axis current than that on no-load condition in Fig. 9(g) and 9(h). In no-load and load conditions, the radial forces of 2nd order are largely suppressed by the $d$-axis current.

Fig 10 shows the change of amplitude of 2nd acceleration. In each condition, 2nd accelerations are largely decreased. Amplitude of 2nd acceleration on load condition is larger than that on no-load condition.

6. Conclusion

In this paper, an optimal $d$-axis current reference which reduces 2nd radial force is proposed. The relationship between $d$-axis current and 2nd radial force is examined. The approximate model of the 2nd radial force predicts and controls the amplitude of 2nd radial acceleration. Experiments are performed to demonstrate the validity of this modeling. In automotive applications, IPMSMs use wide range of rotation speed, and the noise and vibration cause serious problem when the frequency of 2nd radial force corresponds to the frequency of res-
The amplitude of high order radial force is smaller than that of 2nd radial force. However, high order radial force, specially 6th order, can easily transmit stator vibration and generate large acceleration when rotating speed equals stator natural frequency. In our future work, high order radial force modeling and control method will be realized.

References
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