Maximum Efficiency Control of Wireless Power Transfer via Magnetic Resonant Coupling Considering Dynamics of DC–DC Converter for Moving Electric Vehicles

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Abstract—Wireless charging for moving electric vehicles could extend their cruising distance. Wireless power transfer via magnetic resonant coupling is suitable for this application. The transmitting efficiency can be maximized by using a DC–DC converter on the secondary side. The control system, however, must be designed properly to satisfy the response requirements depending on motion of the vehicle. Previous controllers were designed without considering the dynamics of the DC-DC converter for wireless power transfer via magnetic resonant coupling. This paper proposes the design method of secondary voltage control with a feedback controller using a novel DC–DC converter model based on the analysis of wireless power transfer system. Experiments show that the proposed model is effective and that the secondary voltage control improves not only the transmitting efficiency but also the charging power at any transmitting distance.

I. INTRODUCTION

Electric vehicles (EVs) have gathered attention as a solution for environmental problems. Their electric motors have the advantage of a faster response over internal combustion engines and EVs can achieve a high performance in motion control [1]. However, EVs need to be charged frequently due to their limited mileage per charge. Making a charging network for EVs will allow for the reduction in size of their energy storage systems, but it is important to keep such a network simple.

Wireless power transfer (WPT) can mitigate complicated charging operations by eliminating the use of wiring. In recent years, wireless charging for moving EVs have been receiving focus and are expected to extend the cruising distance of EVs [2]–[5]. In addition, an energy storage system of EVs could be reduced in size by connecting EVs to electrical infrastructure because the required power for driving an EV could be supplied by electricity infrastructure. When transmitting coils of the wireless charging system are buried underground, they should be installed at least several tens of centimeters deep, considering road maintenance. Therefore, long distance transmission is required.

WPT via magnetic resonant coupling is suitable for this application because of its highly efficient mid-range transmission and its robustness to misalignment [6], [7]. The transmitting efficiency can be maximized by using a DC-DC converter on the secondary side [8]–[13], because it is determined not only by the parameters of transmitting and receiving coils but also the load condition [14]. Previous controllers were designed without considering the dynamics of the DC-DC converter for WPT via magnetic resonant coupling. In the case of wireless charging for moving EVs, however, it is important that the control system design satisfies the response requirement depending on the motion of the vehicle.

This paper proposes the design method of a feedback controller for secondary voltage control using a novel DC-DC converter model, which is based on the analysis of WPT via magnetic resonant coupling instead of the DC circuit model of the inductive power transfer system [15], [16]. Experiments show the validity of the proposed model and the effectiveness of the maximum efficiency control by the secondary voltage control.
II. WIRELESS POWER TRANSFER VIA MAGNETIC RESONANT COUPLING

A. Input/output characteristics at resonance frequency

WPT via magnetic resonant coupling applies series or parallel resonance to a transmitter and a receiver. This paper uses a series-series (SS) circuit topology and its equivalent circuit is shown in Fig. 1. The transmitter and the receiver consist of the inductance $L_1$, $L_2$, the series–resonance capacitance $C_1$, $C_2$, and the internal resistance $R_1$, $R_2$ respectively. $v_1$, $i_1$, $v_2$, and $i_2$ stand for root-mean-square voltages and currents. $R_L$ is the load resistance and $L_m$ is the mutual inductance between $L_1$ and $L_2$.

If the power source angular frequency $\omega_0$ satisfies eq. (1), the voltage ratio $A_v$, and the transmitting efficiency $\eta$ are expressed as eq. (2) and (3) [17].

$$\omega_0 = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}$$  \hspace{1cm} (1)

$$A_v = \frac{v_2}{v_1} = \frac{R_1 R_2 + R_1 R_L + (\omega_0 L_m)^2}{(\omega_0 L_m)^2 R_L}$$ \hspace{1cm} (2)

$$\eta = \frac{(R_2 + R_L)(R_1 R_2 + R_1 R_L + (\omega_0 L_m)^2)}{(R_2 + R_L)(R_1 R_2 + R_1 R_L + (\omega_0 L_m)^2)}$$ \hspace{1cm} (3)

B. Maximization of transmitting efficiency

The transmitting efficiency is determined by the angular frequency of the power source, the internal resistance of the coils, the mutual inductance between the two coils, and the load resistance. The angular frequency of the power source and the parameters of the coils can be designed before installation. However, the mutual inductance changes depending on the motion of the vehicle and the load resistance is determined by the state of charge and the output power of the vehicle. In order to maximize the transmitting efficiency, the load resistance should be controlled according to the change in the mutual inductance. The optimal value of the load resistance $R_{L_{\text{max}}}$ for maximizing the transmitting efficiency is expressed as follows [14]:

$$R_{L_{\text{max}}} = \sqrt{R_2 \left\{ \frac{(\omega_0 L_m)^2}{R_1} + R_2 \right\}}.$$ \hspace{1cm} (4)

Previous research has controlled the load resistance by using a DC–DC converter on the secondary side [8]–[10]. The system configuration is shown in Fig. 2. Ground facilities apply a DC source and an inverter to power source on the primary side. An EV consists of a receiver, a rectifier, a DC–DC converter, batteries, and a motor drive system. In this paper, the motor drive system is neglected as this is a fundamental study.

If the primary voltage is regulated, the load resistance can be determined by the secondary voltage control. Then the transmitting efficiency can be maximized by the secondary voltage, which is expressed as follows [12]:

$$v_2^* = \sqrt{\frac{R_2}{R_1}} \sqrt{R_1 R_2 + (\omega_0 L_m)^2} + \sqrt{R_1 R_2} v_1.$$ \hspace{1cm} (5)

This paper applies the secondary voltage control with a feedback controller to a maximum efficiency control and proposes the novel DC–DC converter model. Then, the model includes a characteristic of the secondary current in WPT via magnetic resonance coupling and the feedback controller is designed by the pole placement method.

III. MODELING OF DC–DC CONVERTER

A. Circuit configuration

Fig. 3 shows the circuit configuration of the DC–DC converter, where $E$ is battery voltage, $C$ is capacitance of the DC–link capacitor, $L$ is inductance of the reactor coil, $r$ is internal resistance of the reactor coil and batteries, $v_{dc}$ is voltage of the DC–link capacitor, and $i_L$ is an average value of the inflowing current to batteries. When the resonant frequency of WPT is much higher than the switching frequency of the DC–DC converter, $i_{dc}$ is defined as the average current flowing into the DC–link capacitor.

B. State space averaging method

The plant model of the DC–DC converter can be described by using the state space averaging method [18]. In this paper, the DC–DC converter is operated in a continuous conduction mode because switches of the DC–DC converter are alternately turned on and off. When $d(t)$ is defined as the duty cycle of the upper switch, the state space model of the DC–DC converter
is expressed as follows:

\[
\frac{d}{dt} x(t) = A(d(t))x(t) + B \begin{bmatrix} E \\ i_{dc}(t) \end{bmatrix} \tag{6}
\]

\[
v_{dc}(t) = c x(t) \tag{7}
\]

\[
x(t) := [i_L(t) \ v_{dc}(t)]^T
\]

\[
\begin{bmatrix} A \\ c \end{bmatrix} = \begin{bmatrix} -\frac{1}{C} & \frac{d(t)}{C} \\ -\frac{d(t)}{C} & 0 \end{bmatrix}
\]

\[
\begin{bmatrix} \Delta A \\ \Delta c \end{bmatrix} = \begin{bmatrix} -\frac{1}{C} & \frac{d(t)}{C} \\ -\frac{d(t)}{C} & 0 \end{bmatrix} \begin{bmatrix} \Delta v_{dc} \\ \Delta v_{dc} \end{bmatrix} \tag{9}
\]

\[
x(t) := X + \Delta x(t), \Delta x(t) := [\Delta i_L(t) \ \Delta v_{dc}(t)]^T
\]

\[
u(t) := U + \Delta u(t), \Delta u(t) := [\Delta d(t) \ \Delta i_{dc}(t)]^T
\]

\[
X := [I_L \ V_{dc}]^T, \ U := [D \ I_{dc}]^T
\]

Then, the equilibrium point satisfies following equations.

\[
I_L = \frac{I_{dc}}{D} \tag{10}
\]

\[
V_{dc} = \frac{E D - r I_{dc}}{D^2} \tag{11}
\]

C. Secondary current on wireless power transfer

Fig. 4 shows the WPT system for maximum efficiency control using the DC-DC converter. In this paper, a rectangular voltage is used as a power source on the primary side. When the SS circuit topology is applied to the WPT system, the transfer function from the primary voltage to the secondary current has a high gain at the resonant frequency in comparison with other frequencies [19]. Therefore, the waveform of the secondary current \( i_2 \) becomes a nearly sinusoidal wave with the resonant frequency regardless of the waveform of the primary voltage. Additionally, if the variation of the DC-link voltage can be ignored, the waveform of the secondary voltage \( v_2 \) becomes a rectangular wave, which has the same amplitude as the DC-link voltage \( v_{dc} \) and the resonant frequency due to the conduction of the diodes. This paper focuses on fundamental waves with the resonant frequency and analyzes the equivalent circuit of the WPT system at the resonant frequency.

If the power factor of the fundamental waves on the rectifier is equal to 1 and losses of the rectifier can be ignored, the whole of the rectifier and load are equated with a pure electrical resistance [10]. An equivalent resistance \( R_L \) is expressed as follows:

\[
R_L = \frac{v_{20}}{i_2}, \tag{12}
\]

where \( v_{20} \) is defined as the root–mean–square voltage of the fundamental secondary voltage. On the other hand, the voltage ratio of the fundamental secondary voltage \( v_{20} \) to fundamental primary voltage \( v_{10} \) is described as follows:

\[
A_v = \frac{v_{20}}{v_{10}} \tag{13}
\]

From eq. (2), (12), and (13), the root–mean–square value of the secondary current \( i_2 \) is expressed as follows:

\[
i_2 = \frac{\omega_0 L_m v_{10} - R_1 v_{20}}{R_1 R_2 + (\omega_0 L_m)^2}, \tag{14}
\]

where \( v_{10} \) and \( v_{20} \) can be expressed by the Fourier series expansion of the primary voltage \( v_1 \) and the secondary voltage \( v_2 \). As a result, \( i_2 \) can be described as follows:

\[
i_2 = \frac{2 \sqrt{2}}{\pi} \frac{\omega_0 L_m v_{10} - R_1 v_{20}}{R_1 R_2 + (\omega_0 L_m)^2}. \tag{15}
\]

Then, the average current flowing into the DC–link capacitor \( i_{dc} \) is expressed as follows:

\[
i_{dc} = \frac{8 \omega_0 L_m v_{10} - R_1 v_{dc}}{\pi^2 R_1 R_2 + (\omega_0 L_m)^2}, \tag{16}
\]

where \( v_{dc} \) is equal to \( v_{2} \). When Eq. (16) is linearized around an equilibrium point, the microscopic fluctuation in \( i_{dc} \) is expressed as follows:

\[
\Delta i_{dc} = \frac{8 \omega_0 L_m}{\pi^2 R_1 R_2 + (\omega_0 L_m)^2} \Delta v_{dc}. \tag{17}
\]

Applying eq. (17) to eq. (8), the linearized model of the DC–DC converter is expressed as follows:

\[
\frac{d}{dt} \Delta x(t) = \Delta A \Delta x(t) + \Delta B \Delta u(t) \tag{18}
\]

\[
\Delta v_{dc}(t) = \Delta c \Delta x(t) \tag{19}
\]

\[
\begin{bmatrix} \Delta A \\ \Delta c \end{bmatrix} = \begin{bmatrix} -\frac{1}{C} & \frac{D}{C} \\ -\frac{D}{C} & 0 \end{bmatrix}
\]

\[
x(t) := X + \Delta x(t), \Delta x(t) := [\Delta i_L(t) \ \Delta v_{dc}(t)]^T
\]

\[
u(t) := U + \Delta u(t), \Delta u(t) := [\Delta d(t) \ \Delta i_{dc}(t)]^T
\]

\[
X := [I_L \ V_{dc}]^T, \ U := D
\]
Therefore, the transfer function from $\Delta d(s)$ to $\Delta v_{dc}(s)$ is described as follows:

$$\Delta P_v(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}.$$  

(20)

$$a_1 = \frac{r}{L} + \frac{8}{\pi^2} \frac{1}{C} \{R_1 R_2 + (\omega_n L_m)^2\}$$

$$a_0 = \frac{1}{LC} \left\{ D^2 + \frac{8}{\pi^2} \frac{r R_1}{R_1 R_2 + (\omega_n L_m)^2} \right\}$$

$$b_1 = \frac{I_L}{C}, \quad b_0 = \frac{r I_L - D V_{dc}}{LC}$$

IV. CONTROL SYSTEM DESIGN

A. Feedback controller for secondary voltage control

A feedback controller for the secondary voltage control is designed by eq. (20). To apply the pole placement method, we use a PID controller $C_{PID}(s)$, which is expressed as follows:

$$C_{PID}(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{\tau_D s + 1}.$$  

(21)

A discretized controller of $C_{PID}(s)$ is obtained by Tustin transform, which is defined as follows:

$$s = \frac{2(z - 1)}{T (z + 1)},$$  

(22)

where $T$ is a control period and equates the switching frequency of the DC–DC converter in this paper.

The secondary voltage control is implemented by using the discretized controller $C_{PID}(z)$ and its block diagram is shown in Fig. 5. Then an equilibrium point has to be defined adequately because the plant model is a small signal model around the equilibrium point.

B. Defining the equilibrium point

$V_{dc}$ is determined as a reference of the secondary voltage which maximizes the transmitting efficiency. When the waveform of the secondary voltage can be equated with a rectangular wave, $V_{dc}$ can be obtained by eq. (5) and $I_{dc}$ can be calculated by eq. (16). $D$ and $I_L$ are obtained by eq. (10) and (11). Therefore, the equilibrium point is expressed as follows:

$$V_{dc} = \sqrt{\frac{R_2}{R_1} \sqrt{R_1 R_2 + (\omega_n L_m)^2} + \sqrt{R_1 R_2} v_1},$$  

(23)

$$I_{dc} = \frac{8 \omega_n L_m v_1 - R_1 V_{dc}}{\pi^2 R_1 R_2 + (\omega_n L_m)^2},$$  

(24)

$$D = \frac{E + \sqrt{E^2 - 4 V_{dc} I_{dc}}}{2 V_{dc}},$$  

(25)

$$I_L = \frac{I_{dc}}{D}.$$  

(26)

V. EXPERIMENT

A. Experimental setup

The experimental equipment is shown in Fig. 6. The characteristics of the transmitter and the receiver are expressed in Table 1 and the specification of the DC–DC converter is shown in Table 2. The transmitting distances were gradually increased in 50 mm increments from 200 mm to 450 mm during the experiment. The mutual inductance between the transmitter and the receiver for each transmitting distance is indicated by Table 3. These values were measured by a LCR meter (NF Corporation ZM2371).

The primary voltage $v_1$ was generated by a function generator (Tektronix AFG3021B) and a high speed bipolar amplifier (NF Corporation HSA4014). The voltage amplitude
TABLE I. CHARACTERISTICS OF TRANSMITTER AND RECEIVER.

<table>
<thead>
<tr>
<th></th>
<th>Primary coil</th>
<th>Secondary coil</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance $R_1$, $R_2$</td>
<td>1.28 Ω</td>
<td>1.24 Ω</td>
</tr>
<tr>
<td>Inductance $L_1$, $L_2$</td>
<td>638 μH</td>
<td>642 μH</td>
</tr>
<tr>
<td>Capacitance $C_1$, $C_2$</td>
<td>3990 pF</td>
<td>3990 pF</td>
</tr>
<tr>
<td>Resonant frequency $f_1$, $f_2$</td>
<td>99.8 kHz</td>
<td>99.4 kHz</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>448 mm</td>
<td></td>
</tr>
<tr>
<td>Number of turns</td>
<td>56 turns</td>
<td></td>
</tr>
</tbody>
</table>

TABLE II. SPECIFICATION OF DC–DC CONVERTER.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery voltage $E$</td>
<td>12 V</td>
</tr>
<tr>
<td>Internal resistance $r$</td>
<td>800 mΩ</td>
</tr>
<tr>
<td>Inductance $L$</td>
<td>511 μH</td>
</tr>
<tr>
<td>Capacitance $C$</td>
<td>3400 μF</td>
</tr>
<tr>
<td>Carrier frequency $f_c$</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

TABLE III. MUTUAL INDUCTANCE BETWEEN TRANSMITTER AND RECEIVER IN EACH TRANSMITTING DISTANCE.

<table>
<thead>
<tr>
<th>Transmitting distance [mm]</th>
<th>Mutual inductance $L_{m}$ [μH]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>86.0</td>
</tr>
<tr>
<td>250</td>
<td>59.2</td>
</tr>
<tr>
<td>300</td>
<td>42.2</td>
</tr>
<tr>
<td>350</td>
<td>30.5</td>
</tr>
<tr>
<td>400</td>
<td>23.2</td>
</tr>
<tr>
<td>450</td>
<td>18.0</td>
</tr>
</tbody>
</table>

of $v_1$ was tuned to 20 V and the frequency of $v_1$ was set to 99.0 kHz. To equate the flowing current into the DC–link capacitor with the average value, the switching frequency was set to 10 kHz, which is much lower than the resonant frequency of WPT. The feedback controller was designed to place closed loop poles at -1000 rad/s (multiple root).

B. Secondary current of wireless power transfer

The secondary current of WPT was measured to verify the validity of eq. (15) in each transmitting distance. Then, the secondary voltage was regulated to the amplitude of 18 V by using the DC–DC converter.

Fig. 7 shows a comparison between the value calculated by eq. (5) and the measured value. The results show that eq. (5) is acceptable, as these values closely matched. However, the error of the secondary current was increased according to the increase of the transmitting distance. This is because the waveform of the secondary voltage could not be equated with a rectangular wave due to the variation of the DC–link voltage. Using a DC–link capacitor which has a large capacitance and using higher frequencies are effective for the problem.

C. Transmitting efficiency

The effectiveness of the secondary voltage control for maximizing the transmitting efficiency is verified by experiments. The secondary voltage was controlled to satisfy eq. (5) by the secondary voltage control. Without control, the duty cycle $d$ was set to 1, therefore conducting the upper switch of the DC–DC converter at all times.

The experimental result of the transmitting efficiency is shown in Fig. 8. At long distance transmission, the secondary voltage control can increase the transmitting efficiency. When the transmitting distance was short, however, high transmitting efficiency was achieved regardless of the secondary voltage control. In the case of wireless charging for moving EVs, the secondary voltage control is effective because long distance transmission is required.
D. Charging power

The experimental result of the charging power is shown in Fig. 10. At any transmitting distance, the charging power with the maximum efficiency control was increased. Therefore the maximum efficiency control can achieve not only highly efficient transmission but also high power charging. This result indicates that the secondary voltage control for maximizing the transmitting efficiency is suitable for EV applications regardless of the transmitting distance.

VI. CONCLUSION

This paper proposed a novel DC-DC converter model based on the analysis of WPT and designed a feedback controller for the maximum efficiency control of WPT. Experimental results demonstrated that the analysis of the secondary current of WPT is acceptable, and that the maximum efficiency control through secondary voltage control is suitable for EV applications at any transmitting distance.

In future works, the response characteristics of the secondary voltage control and its stability will be discussed. To implement a wireless charging system for EVs in motion, the maximum efficiency control according to the change in the mutual inductance will be designed and the motor drive system will be considered.

REFERENCES


