Minimum Collision Avoidance Distance Control for Four-wheel-driven Electric Vehicles with Active Front and Rear Steerings

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Abstract—In this paper, an automatic collision avoidance method is studied, where it is formulated as an optimal control problem. Specifically, the problem is designed as minimization of the longitudinal distance to prevent collision. The optimal force inputs are obtained by the initial longitudinal and lateral velocities of the vehicle, as well as the lateral distance to avoid the obstacle. Then, the inputs are distributed to each tire force. By effectively using the tire-workload, collision avoidance performance is improved. Simulations and experiments are conducted to verify the effectiveness of the proposed approach.

I. INTRODUCTION

A key feature of electric vehicles (EVs) is that they are driven by motors. Therefore, from the viewpoint of vehicle stability control, EVs have advantages as follows [1].

1) The torque response of electric motors is 100–500 times faster than that of engines.
2) All wheels can be controlled independently by adopting small high-power in-wheel motors.
3) The output torque of an electric motor can be measured accurately from the motor current.

Based on these advantages, many traction control methods [2] and motion stabilization control methods [3] have been proposed. Moreover, it is desirable to make use of these characteristics for collision avoidance.

Many studies on collision avoidance have been proposed, such as methods based on potential field [4] or nonlinear programming problem [5]. In [5], Hattori et al. have formulated the obstacle avoidance problem as an optimal control problem with a free terminal time. It has been examined using a mass model to calculate optimal force to minimize collision avoidance distance. However, experimental results were not provided. In [6], collision avoidance using front and rear steering was achieved. However, vehicle motion is restricted due to the mechanical limit of the front and rear steering angle. One of the methods to improve the obstacle avoidance performance is having actuator redundancy. That is, not only the front and rear steering angle but the driving force difference moment should also be considered as the control inputs.

In this paper, the shortest avoidance trajectory is calculated using the method proposed in [5] under the assumption that the location of the stationary obstacle is known. The longitudinal and lateral forces for each tire are distributed to have the vehicle follow the trajectory given by optimal control. The method of tire-workload equalization [3] is employed as the proposed method using four wheel independent driving and active front and rear steerings. By effectively using the tire-workload, collision avoidance performance is improved. Simulations and Experiments are conducted to confirm the effectiveness of the proposed method from the viewpoint of avoidance distance.

II. VEHICLE MODELING

A. Experimental Vehicle

The experimental vehicle is shown in Fig. 1 and its specifications are shown in Table. 1. This vehicle is an electric vehicle “FPEV2-Kanon” produced by authors’ laboratory. Direct drive in-wheel motors are equipped inside each wheel, and the maximum front torque is ±500 Nm and the maximum rear torque is ±340 Nm. Moreover, active automatic front and rear steering systems are available, and the maximum front steering angle is ±0.35 rad and the maximum rear steering angle is ±0.15 rad.

B. Vehicle Model

An EV with four independent in-wheel motors and active front and rear steerings is modeled. Under the assumption that each front and rear steering angle defined as $\delta_i$ are small enough, equations of each longitudinal, lateral and yaw motion of the vehicle as shown in Fig. 2 are written as follows,

$$F_{xall} = F_{xf} + F_{xf} + F_{xfr} + F_{xrr}$$
$$F_{yall} = F_{yf} + F_{yf} + F_{yfr} + F_{yrr}$$
$$N_z = -\frac{d_f (F_{xf} - F_{xf}) - d_r (F_{xfr} - F_{xrr})}{2}$$
$$N_t = I_f (F_{yf} + F_{yfr}) - I_r (F_{yfr} + F_{yrr})$$
$$M_z = N_z + N_t$$
where $F_{x_{all}}$ is total longitudinal force of the vehicle, $F_{z_{ij}}$ are longitudinal force of each wheel, $F_{y_{all}}$ is total lateral force of the vehicle, $F_{y_{ij}}$ are lateral force of each wheel, $N_i$ is yaw-moment generated by longitudinal force of each wheel, $N_y$ is yaw-moment generated by lateral force of each wheel, $M_i$ is yaw-moment of the vehicle, $d_i$ are tread base and $l_i$ are wheel base. The subscript $i$ represents $f$ or $r$ ($f$ is front and $r$ is rear) and $j$ represents $f$ or $r$ ($l$ is left and $r$ is right)

Moreover, provided that a vehicle can be modeled as a linear two-wheel vehicle, the relation between $F_{y_{ij}}$, cornering force $Y_i$, tire side slip angle $\alpha_i$, vehicle side slip angle $\beta$, yaw-rate $\gamma$ and steering angle $\delta_i$ are approximated as follows,

$$F_{y_{ij}} \approx Y_i = \frac{C_f}{1 + T_i \delta} \alpha_i = -C_f (\beta + \frac{l_f}{V} \gamma - \delta_f) \tag{6}$$

$$F_{y_{ij}} \approx Y_i = \frac{C_r}{1 + T_r \delta} \alpha_r = -C_r (\beta - \frac{l_r}{V} \gamma - \delta_r) \tag{7}$$

where $C_i$ are cornering stiffness. Generally, tire lateral force is known to reply of the primary delay for tire side slip angle and $T_i$ are time constant.

Vertical load of each wheel $F_{z_{ij}}$ are written as follows,

$$F_{z_{ij}} = \frac{1}{l_f} Mg - a_x M h_y + \rho_f a_y M h_y \frac{h_g}{l_f} \tag{8}$$

$$F_{z_{ij}} = \frac{1}{l_r} Mg - a_x M h_y + \rho_f a_y M h_y \frac{h_g}{l_r} \tag{9}$$

$$F_{z_{it}} = \frac{1}{l_f} Mg + a_x M h_y - \rho_f a_y M h_y \frac{h_g}{l_f} \tag{10}$$

$$F_{z_{rt}} = \frac{1}{l_r} Mg + a_x M h_y - \rho_f a_y M h_y \frac{h_g}{l_r} \tag{11}$$

where $\rho_i$ are roll stiffness distribution and $a_x, a_y$ are acceleration of longitudinal and lateral direction.

### C. Friction Circle

Moreover, the relation between $F_{x_{ij}}, F_{y_{ij}}$ and $F_{z_{ij}}$ has to satisfy the following equation in any case,

$$\sqrt{F_{x_{ij}}^2 + F_{y_{ij}}^2} \leq \mu F_{z_{ij}} \tag{12}$$

where $\mu$ is a coefficient of friction. This is called friction circle, which is shown in Fig. 3. Tire-workload of each wheel $\eta_{ij}$ which is the rate of resultant force in friction circle is defined from (12) as follows.

$$\eta_{ij} = \frac{\sqrt{F_{x_{ij}}^2 + F_{y_{ij}}^2}}{\mu F_{z_{ij}}} \approx \sqrt{\frac{F_{x_{ij}}^2 + F_{y_{ij}}^2}{\mu^2 F_{z_{ij}}^2}} \tag{13}$$

Many methods which decrease tire-workload of each wheel are proposed in order to avoid saturation of the tire forces [7], [8]. Our research group previously proposed a method for minimizing the square sum of the tire-workload of each wheel [3], and in this paper, this is used as the proposed method.

### III. TRAJECTORY GENERATION [5]

#### A. Problem setting

For trajectory generation of collision avoidance, the method Hattori et al. proposed in [5] is used. The collision avoidance problem is formulated as an optimal control problem with a free terminal time $t_e$. Assumed that it is possible to control the braking and steering of all tires independently without rotation, the dynamics of the vehicle can be represented by a mass model. The problem is solved as the trajectory control problem for a rigid body (Fig. 4). In [5], when initial velocity $v_{X0}, v_{Y0}$, lateral avoidance distance $Y_c$, vehicle mass $M$, vehicle maximum force $F_{max}$ are known, it is calculated that optimal control force $F_{opt}(t)$ which minimize collision avoidable distance $X_{ep}$. It is taken the vehicle starting position of collision avoidance at the origin and vehicle position at any time $t = X(t), Y(t)$ in the $XY$ plane. (14),(15) show initial and terminal condition and (16) shows the relationship of the friction circle in the mass point as constraint condition.

$$[X(0) X(0) Y(0) Y(0)]^T = [0 v_{X0} 0 0]^T \tag{14}$$

$$[Y(t_e) Y(t_e)]^T = [Y_c 0]^T \tag{15}$$

$$F_{X0}^2 + F_{Y0}^2 - F_{max}^2 \leq 0 \tag{16}$$

In [5], the following simultaneous equations are gained for the above problem. where $\nu_1, \nu_2$ are Lagrange constants.

$$\begin{cases} M \frac{d^2 X_{00}}{d t^2} - \frac{\nu_1}{\sqrt{2}} - \frac{\nu_2}{\sqrt{2}} = 0 \\ M \frac{d^2 Y_{00}}{d t^2} + \frac{\nu_1}{\sqrt{2}} - \frac{\nu_2}{\sqrt{2}} = 0 \end{cases} \tag{17}$$

$$M \frac{d^2 Y_{00}}{d t^2} + \frac{\nu_1}{\sqrt{2}} - \frac{\nu_2}{\sqrt{2}} = -\mu \sqrt{2} \frac{\nu_1}{\sqrt{2}} \frac{\nu_2}{\sqrt{2}} \frac{\nu_1}{\sqrt{2}} \frac{\nu_2}{\sqrt{2}} = 0$$

$$+ \frac{\nu_1}{\sqrt{2}} - \frac{\nu_2}{\sqrt{2}} = 0$$

$$Q = \ln \frac{-\nu_1}{\sqrt{2}} \frac{\nu_2}{\sqrt{2}} \frac{\nu_1}{\sqrt{2}} \frac{\nu_2}{\sqrt{2}} \frac{\nu_1}{\sqrt{2}} \frac{\nu_2}{\sqrt{2}}$$
When \( v_{X0}, v_{Y0}, Y_c, F_{max}, M \) are known, \( \nu_1, \nu_2 \) and \( t_e \) are calculated and \( F_{opt}(t) \) is gained from these variables.

\[
F_{opt}(t) = \left[F_{Xopt}(t) \ F_{Yopt}(t)\right]^T
\]

\[
= -F_{max} \frac{ \{ (1+\nu^2) t_e -3\nu_1 \nu_2 \} \sqrt{(\nu_1 t_e + \nu_2)^2 + t_e^2} }{2(1+\nu_1^2)^2} \tag{18}
\]

Because of \( F_{opt}(0) \neq 0 \), low-pass filter of 10 rad/s are inserted into the setpoint to satisfy \( F_{opt}(0) = 0 \) in order to have moderation in the control variables. In addition, the minimum avoidable distance \( X_{ep} \) by braking and steering is determined by the following equation.

\[
X_{ep} = -F_{max} \frac{ \{ (1+\nu^2) t_e -3\nu_1 \nu_2 \} \sqrt{(\nu_1 t_e + \nu_2)^2 + t_e^2} }{2(1+\nu_1^2)^2} + v_{X0} t_e - \frac{F_{max} 3\nu_1 \nu_2 \sqrt{\nu_2^2}}{2(1+\nu_1^2)^2} + \frac{F_{max} \nu_2^2 (2\nu_2^2 - 1) }{2(1+\nu_1^2)^2} Q \tag{19}
\]

In this paper, mass point tire-workload \( \hat{\eta} \) is introduced to express \( F_{max} = \hat{\eta} \mu M g \), \( \hat{\eta} \) is gravity acceleration. When \( \hat{\eta} \) is larger, \( X_{ep} \) becomes smaller. However, when \( \hat{\eta} \) is too large, there is a possibility that either tire-workload of each wheel are also too large. In addition, because the front and rear wheel steering angle and each wheel torque have limits, lateral and longitudinal force also are limited. Therefore, \( \hat{\eta} \) is to some extent small value.

IV. CONTROL SYSTEM

\( \gamma^* \) = 0 and \( F_{opt}(t) \) calculated in section III are inputted to the control system shown in Fig. 4, and are distributed to longitudinal and lateral force of each tire to realize collision avoidance. In this paper, vehicle side slip angle is estimated from a side slip angle observer [9], yaw-rate is measured from the gyro sensor. In the following discussions, the controller are the same, but the longitudinal and lateral force distribution methods are different.

A. Only Lateral Force(conv.1)

Steering avoidance by only lateral force is performed without generating any longitudinal force as conventional method1(conv.1). The command value of the lateral position, the lateral velocity and the lateral force equation is obtained from the fifth-order polynomial to satisfy (14) and (15).

\[
Y(t) = \frac{Y_c}{t_e} t^3 - 15 \frac{Y_c}{t_e^2} t^4 + 6 \frac{Y_c}{t_e^3} t^5 \tag{20}
\]

\[
F_{y_{max}} \leq \frac{Y_{y_{max}}}{\frac{10}{3} \hat{\eta} \mu g} \tag{21}
\]

\[
t_e = \sqrt{\frac{10Y_c}{\frac{3}{3} \hat{\eta} \mu g}} \tag{22}
\]

Terminal time \( t_e \) varies by the mass point tire-workload \( \hat{\eta} \).

In addition, the lateral force distribution law based on the two-wheel vehicle model in (2) and (4) as follows.

\[
\begin{bmatrix}
F_{yf}
F_{yr}
\end{bmatrix}
= \begin{bmatrix} 2 & 2 \\ 2t_f & -2t_r \end{bmatrix}^{-1}
\begin{bmatrix} F_{yll} \\ N_l \end{bmatrix} \tag{23}
\]

\( N_l = 0 \) are inputted as the reference.

\[
\begin{align*}
F_{opt}(t) = \left[F_{Xopt}(t) \ F_{Yopt}(t)\right]^T \\
= -F_{max} \frac{ \{ (1+\nu^2) t_e -3\nu_1 \nu_2 \} \sqrt{(\nu_1 t_e + \nu_2)^2 + t_e^2} }{2(1+\nu_1^2)^2} \\
+ v_{X0} t_e - \frac{F_{max} 3\nu_1 \nu_2 \sqrt{\nu_2^2}}{2(1+\nu_1^2)^2} + \frac{F_{max} \nu_2^2 (2\nu_2^2 - 1) }{2(1+\nu_1^2)^2} Q
\end{align*}
\]

B. Equal allocation method(conv.2)

Steering and braking avoidance is performed by distributing equal driving force to each the left front and rear wheel and the right front and rear wheel as conventional method2(conv.2). (24) shows driving distribution method from (1) and (3).

\[
\begin{bmatrix} F_{zlf} \\ F_{zrf} \\ F_{zfr} \\ F_{zrr} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2t_f & -2t_r \end{bmatrix}^{-1}
\begin{bmatrix} F_{zall} \\ N_z \end{bmatrix} \tag{24}
\]

In (23), \( N_l = -N_z \) is inputted as the reference to satisfy \( M_z = 0 \).

C. Equalization of Tire-workload for each wheel[3](prop)

Steering and braking avoidance is performed by tire-workload equalization method with active front and rear steering and four wheel independent driving as proposed method(prop.). In the case of the vehicle which can be regarded as a linear two-wheel vehicle, (1)~(5) can be approximated as follows,

\[
\begin{bmatrix} F_{xall} \\ F_{yall} \\ M_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2t_f & -2t_r & -2t_f & -2t_r & -2t_f & -2t_r \end{bmatrix}
\begin{bmatrix} F_{yf} \\ F_{yr} \\ F_{zlf} \\ F_{zrf} \\ F_{zfr} \\ F_{zrr} \end{bmatrix} \tag{25}
\]

where the left-hand vector is defined as \( b \), the right-hand side coefficient matrix as \( A \), the vector of lateral and longitudinal force of all wheels as \( u \).

On the condition that \( \mu \) of all wheels are equal, the performance index \( J \), weighted least squares solution \( u_{opt} \) and weighting matrix \( W \) are written as follows,

\[
J = \sum_{i=f,r j=l,r} (\mu \eta_{ij})^2 u^T W u \tag{26}
\]

\[
u_{opt} = W^{-1} A^T (AW^{-1} A^T)^{-1} b \tag{27}
\]

\[
W = \text{diag} \left( \frac{1}{r_f}, \frac{1}{r_f}, \frac{1}{r_f}, \frac{1}{r_r}, \frac{1}{r_r}, \frac{1}{r_r}, \frac{1}{r_r}, \frac{1}{r_r} \right) \tag{28}
\]

V. SIMULATION AND EXPERIMENT

In simulation and experiment, \( X_{ep} \) are compared to satisfy \( \eta_{ijmax} = 0.20, 0.25, 0.30 \) by adjusting \( \hat{\eta} \) \( F_{max} \).
A. Condition

The roll stiffness distribution are assumed as $\rho_i = 0.5$, the friction coefficient as $\mu = 0.8$ and the front and rear cornering stiffness as $C_f = 11220, C_f = 23600$. The proportional gain in the yaw-rate feedback controller is decided in order that the pole of the system of the plant between input $N_{in}$ and output $\gamma$ became -10 rad/s. The proportional and integral gain of lateral force controller which composed of feedforward and feedback controller were also decided in order that the pole of the system became -10 rad/s in the case of $T_i = 0.1585$. Avoidance is done by the route shown in Fig. 4. $v_{X0} = 30$ km/h, $v_{Y0} = 0$ km/h, $Y_c = 1$ m are set.

B. Simulation

Fig. 8, 9, 10 show simulation results to satisfy $\eta_{ijmax} = 0.25$ by adjusting $\hat{\eta}_i$. According to Fig. 8, 9, 10 (b) (c) (d), it is confirmed that the front and rear steering angle, lateral and longitudinal force are within the mechanical limit. According to Fig. 8, 9, 10, it is confirmed that $M_z$ is suppressed in the about range from -50 Nm to 50 Nm. According to Fig. 8, 9, 10 (f), it is confirmed that yaw-rate 0 control is achieved. They are in the region where the lateral and longitudinal force can freely occur and satisfy the condition of no yaw motion. According to Fig. 8, 9, 10 (h), it is confirmed that $\eta_{ijmax}$ is controlled to 0.25 and more equalized by prop.

C. Experiment

$F_{zall}, F_{xall} = 0$ are inputted after the avoidance has been completed and other conditions were the same as simulations. Fig. 11, 12, 13 show experiment results to satisfy $\eta_{ijmax} = 0.25$ by adjusting $\hat{\eta}_i$. According to Fig. 11, 12, 13 (c), front and rear lateral forces are not consistent with each command value and this maybe be caused by different poles in the lateral force PI controller. According to Fig. 11, 12, 13 (d) (f), it is confirmed that driving force and yaw-rate are controlled.

The shortest avoidance distance results in the case of each $\eta_{ijmax}$ are shown in Fig. 7. Fig. 6(a) is the data obtained by adjusting $\hat{\eta}_i$ to constrain $\eta_{ijmax}$ in each method. Fig. 6(b) is the experimental data obtained from using $\hat{\eta}$ adjusted to satisfy $\eta_{ijmax} = 0.20, 0.25, 0.30$ gained by Fig. 6(a) and there are 10 samples in each method. It is confirmed that the experimental results are gained similar to simulation results.

In this paper, deviation occurs in avoidance trajectory as shown in Fig. 7, because the longitudinal and lateral forces are controlled instead of the position. (For $Y_c = 1$ m, the distances of conv.1, conv.2 and prop are 0.97 m, 0.91 m, and 0.94 m, respectively). It should be noted that, because the distance $Y_c$ of prop only slightly differs from that of conv.1 and conv.2, the $X_{cp}$ are compared by assuming that the three methods have the same $Y_c$

Table II shows the experiment results of the avoidable distance in the case of each $\eta_{ijmax}$. In the case of $\eta_{ijmax} = 0.25$, the distances are 16.48 m in conv.1, 13.62 m in conv.2 and 13.27 m in prop. $X_{cp}$ are 19.48 % reduction with respect to conv.1 and 2.57 % reduction with respect to conv.2 by using prop. The effect of prop to conv.2 is small. However, further effect of the prop to conv.2 is expected to adjust each $Y_c$ to equalize real lateral movement distance because conv.2 is poor tracking performance to the command trajectory. Clearly, the experimental results are similarly to the simulation ones.

VI. Conclusion

In this paper, a method of tire-workload equalization for each wheel in emergency avoidance for electric vehicle with four wheel independent driving and active front and rear steerings was studied. The shortest avoidance trajectory was calculated using the method proposed by Hattori et al. and the longitudinal and lateral forces are distributed properly in order to have the vehicle follow the trajectory. The effectiveness was verified by simulations and experiments. It was shown that a significant reduction of the longitudinal running distance with respect to the conv.1 and the advantage of tracking performance to the command trajectory with respect to the conv.2.

For future works, collision avoidance for a moving object will be considered.

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REFERENCES


(a) Optimal force. (b) Steering angle. (c) Lateral force. (d) Driving force.

(e) Moment. (f) Yaw-rate. (g) Slip angle. (h) Tire workload.

Fig. 8. Simulation results (conv.1, \( \eta_{j_{max}} = 0.25 \)).

(a) Optimal force. (b) Steering angle. (c) Lateral force. (d) Driving force.

(e) Moment. (f) Yaw-rate. (g) Slip angle. (h) Tire workload.

Fig. 9. Simulation results (conv.2, \( \eta_{j_{max}} = 0.25 \)).

(a) Optimal force. (b) Steering angle. (c) Lateral force. (d) Driving force.

(e) Moment. (f) Yaw-rate. (g) Slip angle. (h) Tire workload.

Fig. 10. Simulation results (prop, \( \eta_{j_{max}} = 0.25 \)).

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TABLE II
EXPERIMENT RESULTS(AVOIDANCE DISTANCE FOR EACH $\eta_{ijmax}$).

<table>
<thead>
<tr>
<th>$\eta_{ijmax}$</th>
<th>conv.1 [m]</th>
<th>conv.2 [m]</th>
<th>prop [m]</th>
<th>v.s. conv.1 [%]</th>
<th>v.s. conv.2 [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>17.71 ($\eta = 0.174$)</td>
<td>14.81 ($\eta = 0.168$)</td>
<td>14.72 ($\eta = 0.170$)</td>
<td>16.89</td>
<td>0.61</td>
</tr>
<tr>
<td>0.25</td>
<td>16.48 ($\eta = 0.210$)</td>
<td>15.62 ($\eta = 0.202$)</td>
<td>15.27 ($\eta = 0.209$)</td>
<td>19.48</td>
<td>2.57</td>
</tr>
<tr>
<td>0.30</td>
<td>15.29 ($\eta = 0.243$)</td>
<td>12.77 ($\eta = 0.234$)</td>
<td>12.90 ($\eta = 0.246$)</td>
<td>18.25</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Fig. 11. Experiment results(conv.1, $\eta_{ijmax} = 0.25$).

Fig. 12. Experiment results(conv.2, $\eta_{ijmax} = 0.25$).

Fig. 13. Experiment results(prop, $\eta_{ijmax} = 0.25$).