Range Extension Autonomous Driving for Electric Vehicles
Based on Optimal Velocity Trajectory
Considering Road Gradient Information

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Electric Vehicles (EVs) have been considered as one of the solutions for environmental and energy problems. The mileage per charge of EVs, however, is shorter than that of Internal Combustion Engine Vehicles (ICEVs). In this paper, Range Extension Autonomous Driving (READ) system is proposed by taking the road gradient information into account, and the proposed READ system optimizes velocity trajectory to minimize the energy consumption in the case of autonomous driving. The main contribution of this study is the modeling energy consumption by considering road gradient information and motor loss. Moreover, both simulations and experiments are performed to demonstrate the effectiveness of the proposed method in terms of mileage per charge.

Keywords: electric vehicle, range extension autonomous driving, optimal control, road gradient information

1. Introduction

With the increasing concerns on environmental and energy problems, Electric Vehicles (EVs) have been intensively studied during the past years. EVs have many remarkable advantages over Internal Combustion Engine Vehicles (ICEVs), and three main advantages are listed as below.

(1) The response of torque by motor is much faster than that of engines (100 times).
(2) In-wheel motors enable independent driving force control.
(3) Motor torque can be measured precisely with motor current.

These advantages are demonstrated to be useful for motion control of EVs.

However, the short mileage per charge of EVs prevents them from wide spreading to a certain extent. In order to solve this problem, lots of methods were proposed. For example, a control system was proposed to reduce iron loss by regulating the flux density of motors. Some other research works considered optimization of driving force distribution in terms of motor efficiency. However, these studies did not consider minimization of the total energy consumption. Meanwhile, there is research on minimizing the amount of total energy consumption considering gradient resistance and velocity constraint. Yet the research does not take account of motor efficiency.

The authors’ research group has proposed Range Extension Control Systems (RECS) and Range Extension Autonomous Driving (READ) using purely motion control techniques, which does not change motor type and vehicle structures. The conventional methods are mainly focused on optimizing the velocity trajectory and the distribution of driving forces. READ can improve energy efficiency in autonomous driving systems for highway applications which have been intensively discussed recently. However, important factor such as road gradient resistance are not considered. READ considering road gradient information can be expected to reduce more energy consumption. In this study, road gradient information is assumed to be available from on-board devices such as Global Positioning System (GPS), and the velocity trajectory can be designed by formulating an optimization algorithm. The effectiveness of proposed method is verified by simulations and experiments.

2. Experimental Vehicle and Model

2.1 Experimental Vehicle

In this research, an original electric vehicle “FPEV2–Kanon” which the authors’ research group manufactured is used. The photo and the specification of the vehicle are shown in Fig. 1 and Tab. 1. This
vehicle has four outer-rotor type in-wheel motors. These motors are direct drive type. Therefore the reaction forces from road are directly transferred to the motor without backlash influence of the reduction gear. Tab. 1 shows the specification of vehicle. Tab. 2 shows the specification of the motors. Fig 2 shows efficiency maps of the front and the rear in-wheel motors. Fig. 3 illustrates energy flow diagram of power system on the vehicle. Lithium-ion battery is used as power source. The voltage of the main battery is 160 V. The voltage is boosted to 320 V by a chopper. In this paper, the chopper loss is neglected.

2.2 Vehicle Model In this section, a four wheel driven vehicle model is described. Using the model given in Fig. 4(a), the wheel dynamics is expressed as Eq.(1). From Fig. 4(b), the vehicle dynamics are expressed as Eq.(2) and (3)

\[ J_\omega \omega_j = T_j - rF_j, \]
\[ MV = F_{all} - \text{sgn}(V)F_{DR}(V, \theta) - M\text{g}\sin\theta, \]
\[ F_{DR}(V, \theta) = \mu_0 M\text{g}\cos\theta + b|V| + \frac{1}{2}\rho C_dAV^2, \]

where \( \omega_j \) is the wheel angular velocity, \( V \) is the vehicle velocity, \( T_j \) is the motor torque, \( F_j \) is the driving force of each wheel, \( F_{all} \) is the total driving force, \( M \) is the vehicle mass, \( r \) is the wheel radius, \( J_\omega \) is the wheel inertia, \( F_{DR} \) is the driving resistance, \( \mu_0 \) is rolling friction coefficient, \( \theta \) is road gradient, \( b \) is resistance vehicle velocity coefficient, \( \rho \) is air density, \( C_d \) is constant drag and \( A \) is frontal projected area. The subscript \( j \) represents \( f \) or \( r \), \( f \) stands for “front” and \( r \) represents “rear”.

The slip ratio \( \lambda_j \) is defined as

\[ \lambda_j = \frac{V_{\omega_j} - V}{\max(V_{\omega_f}, V_{\omega_r})} \]

where \( V_{\omega_j} = r\omega_j \) is the wheel speed and \( \epsilon \) is a small constant to avoid zero division. The slip ratio \( \lambda_j \) is known to be related with the coefficient of friction \( \mu_j \), as shown in Fig. 5. In region \(|\lambda_j| \ll 1\), \( \mu_j \) is nearly proportional to \( \lambda_j \). Then, for longitudinal acceleration cases,

\[ F_j = \mu_j N_j \approx D_j N_j \lambda_j, \]

where \( D_j \) is the normalized driving stiffness.

In this paper, \( F_j \) and \( F_r \) are distributed \( F_{all} \) equally.


\[ F_j = \frac{1}{4} F_{all}, \]  
(6)

The normal forces of each wheel during the longitudinal acceleration process are calculated as follows:

\[ N_f(V, \theta) = \frac{1}{2} l_f M g \cos \theta - \frac{h_f}{l} M V^2, \]  
(7)

\[ N_r(V, \theta) = \frac{1}{2} l_r M g \cos \theta + \frac{h_r}{l} M V^2, \]  
(8)

where \( N_f \) and \( N_r \) are respectively the front and rear normal forces, \( l_f \) and \( l_r \) are respectively the distances from the center of gravity to the front and rear axles, \( l \) is the wheelbase, and \( h \) is the height of the center of gravity. The acceleration direction is defined as positive when the vehicle is accelerating.

### 2.3 Power Flow Model

The inverter input power \( P_{in} \) in Fig. 3 considering the slip ratio and motor loss is expressed as

\[ P_{in} = P_{out} + P_c + P_r, \]  
(9)

where \( P_{out} \) is the sum of the mechanical output of each motor, \( P_c \) is the sum of the copper loss of each motor, and \( P_r \) is the sum of the iron loss of each motor, and neglecting the inverter loss and mechanical loss.

When each wheel angular acceleration is small, torque \( T_j \) is proportional to driving force. \( T_j \) is expressed as

\[ T_j \approx r F_j, \]  
(10)

When the slip ratio \( \lambda_j \) is small enough, \( \omega_j \) is expressed as

\[ \omega_j = \frac{V}{\tau(1-\lambda_j)} \approx \frac{V}{\tau}(1+\lambda_j), \]  
(11)

By substituting Eq.(5) for Eq.(6) and Eq.(11), \( \lambda_j \) is expressed as

\[ \lambda_j = \frac{F_j}{D_j N_f(V, \theta)} = \frac{F_{all}}{4D_j N_f(V, \theta)}, \]  
(12)

Approximate \( P_{out} \), \( P_c \) and \( P_r \) are expressed as

\[ P_{out} = 2 \sum_{j=f,r} \omega_j T_j, \]

\[ = V F_{all} \sum_{j=f,r} \left( 1 + \frac{F_{all}}{4D_j N_f(V, \theta)} \right), \]

\[ P_c = \frac{r^2}{2} F_{all}^2 \sum_{j=f,r} \frac{R_j}{4K_{ij}^2}, \]

\[ P_r = \frac{r^2}{2} V^2 \sum_{j=f,r} \frac{P_{ej}^2}{R_{ej}} \left( \frac{L_{ej} F_{all}^2}{4K_{ij}} \right)^2 + \Psi_j^2, \]

where \( R_j \) is the armature winding resistance of the motor, \( K_{ij} \) is the torque coefficient of the motor, \( P_{ej} \) is the number of pole pairs, \( L_{ej} \) is the q-axis inductance and \( \Psi_j \) is the interlinkage magnetic flux.

The electrical angular velocity of the motor \( \omega_{ej} \) and the equivalent iron loss resistance \( R_{ej} \) are expressed as

\[ \omega_{ej} = \frac{P_{ej} V}{r}, \]

\[ \frac{1}{R_{ej}} = \frac{1}{R_{ej}} + \frac{1}{|\omega_{ej}|}, \]
(17)

where the first and second terms on the right-hand side represent the eddy current loss and hysteresis loss respectively.

Road gradient can be estimated from the distance traveled on condition that grade map data is stored in advance. Therefore, the road gradient function \( \theta \) can be described by distance traveled \( X \). Then the inverter input power is expressed by \( V, X \) and \( F_{all} \) as

\[ \theta = \theta(X), \]
(18)

\[ P_{in}(V, X, F_{all}) = P_{out}(V, X, F_{all}) + P_c(F_{all}) + P_r(V, F_{all}). \]
(19)

### 3. Optimization of Velocity Trajectory Considering Road Gradient Information

#### 3.1 The Evaluation Function and the Constraint Condition

In this section, on the assumption that the EVs are autonomous driving, we propose READ which calculates optimal velocity trajectory which minimizes the total amount of energy consumption from initial time \( t_0 \) to final time \( t_f \). EVs can regenerate kinematic energy and potential energy. Therefore, minimization of total energy consumption is equal to maximization of regenerative energy. The evaluation function and the constraint conditions are described as

\[ \min_{x} W_{in} = \int_{t_0}^{t_f} P_{in}(x(t), u(t)) dt, \]  
(20)

s.t. \( x(t) = f(x(t), u(t)) \),  
(21)

\( \chi(x(t_0)) = x(t_0) - x_0 = 0 \),  
(22)

\( \psi(x(t_f)) = x(t_f) - x_f = 0 \).  
(23)

where

\[ x(t) = [V(t), X(t)]^T, u(t) = F_{all}(t) \]  
(24)

\[ f(x(t), u(t)) = f(V(t), X(t), F_{all}(t)) \]  
(25)

\[ = \left[ \frac{1}{M} [F_{all}^2 - \sin(V(t))] F_{DR}(V(t), \theta(X)) - M g \sin \theta(X) ] \right]/V(t), \]
(26)

where \( X(t) \) is the distance traveled and \( x_0 \) is the initial condition of velocity and distance traveled. \( x_f \) is the final condition of velocity and distance traveled. Penalty function \( P(x(t_f)) \) which turns the constrained problem into the unconstrained problem on final condition is described as

\[ P(x(t_f)) = \frac{1}{2} \sigma ||\psi(x(t_f))||^2, \]  
(26)

where \( \sigma \) is the penalty parameter. Concomitant variable \( \nu(t) \) is used to turn the constrained problem into an unconstrained problem using final condition. Then the Hamiltonian function \( H \) is described as

\[ H(x, u, \nu) = P_{in}(x, u) + \nu^T f(x, u). \]  
(27)

Therefore, the evaluation function \( J \) without constraints is defined as
4. Simulation

In this section, a simulation on the three trajectories are conducted.

4.1 Simulation Conditions

Fig. 6(a) shows the test road at National Traffic Safety and Environment Laboratory. Fig. 6(b) illustrates the road gradient profile of the test road. The simulation conditions are given in Tab. 1–4. The initial condition is given as $t_0 = 0.00 \text{ s}$, $V_0 = 30.0 \text{ km/h}$, $X_0 = 0.00 \text{ m}$. The final condition is given as $t_f = 24.7 \text{ s}$, $V_f = 0.00 \text{ km/h}$, $X_f = 103 \text{ m}$.

4.2 Control System

Vehicle velocity control system is designed to control the EVs velocity automatically. Fig. 7 shows the system which is composed of a feedforward controller and feedback controller. Front and rear torque reference $T^*_j$ is given as

$$T^*_j = rF^*_j + \frac{J_0}{r}(1 + \lambda)$$

(34)

The second term of right hand side compensates inertia of the wheels. Vehicle velocity controller $C_P(s)$ is a PI controller, and it is designed by the pole placement method. The plant of vehicle velocity is expressed as

$$\frac{V}{F_{all}} = \frac{1}{M\dot{V}}$$

(35)

In the simulation and experiment, the poles of vehicle velocity controller are set to $-5 \text{ rad/s}$.

4.3 Simulation Results

Simulation results are shown in Fig. 8. Fig. 8(d)–8(f) show the dominant loss of power flow model. The dominant losses are expressed as

$$P_M = \frac{d}{dt}\left(\frac{1}{2}MV^2\right) = MV\dot{V}$$

(36)

$$P_{DR} = F_{DR}V$$

(37)

where $P_M$ is the power stored as kinetic energy of the vehicle mass, $P_{DR}$ is the loss caused by the driving resistance.

When vehicle reduces in the speed at a constant deceleration on the downward slope as conventional trajectory 1, motors convert potential energy and kinematic energy to electric energy. Then motors must generate larger total braking force than road gradient resistance. Therefore copper loss becomes large and regenerative energy becomes small.

Conventional trajectory 2 is calculated not considering road gradient information. When the vehicle runs on the flat area from 0.00 to 23.0 m, vehicle reduces speed more quickly than that of conventional trajectory 1 to prevent the vehicle from losing kinematic energy from driving resistance. Then regenerative energy of conventional trajectory 2 is larger than that of conventional trajectory 1. When vehicle runs on the
downward slope from 23.0 to 60.0 m, deceleration of conventional trajectory 2 is larger than that of conventional trajectory 1. Then copper loss of conventional trajectory 2 is slightly bit larger than that of conventional trajectory 1.

Proposed trajectory 3 is calculated considering gradient information. On the flat area, the vehicle reduces in speed at an optimal deceleration to prevent the vehicle from losing kinetic energy by driving resistance. On the other hand, copper loss is larger than that of conventional trajectories. On the downward slope, vehicle speeds up at an optimal acceleration to prevent the vehicle from losing potential energy by the copper loss. Then kinetic energy becomes large to regenerate potential energy. Inverter input power of trajectory 3 is larger than that of conventional both conventional trajectory 1 and 2 to suppress copper loss on the flat area.

Fig. 8(c) shows regenerative energy on conventional trajectory 1, 2 and proposed trajectory 3. Regenerative energy of trajectory 1, 2, and 3 are 58.5 kWs, 58.2 kWs, and 66.2 kWs, respectively. Regenerative energy of proposed trajectory improved about 13.0 % compared with that of conventional trajectory 1 and 2. Therefore, the algorithm to maximize regenerative energy should be designed in consideration of two cases: 1) Reduce regenerative braking if it increases the copper loss; 2) Increase regenerative braking in the case it has little influence on the copper loss, i.e., hard braking can reduce driving resistance loss.

5. Experiment

Experiments were conducted on the test road shown in Fig. 6(a) under the same condition as simulation. Vehicle velocity \( V \), inverter input power \( P_{in} \), mechanical output \( P_{out} \), total loss \( P_L \) including motor copper loss, motor iron loss and inverter loss and driving resistance loss \( P_{DR} \) are calculated as

\[
V = \frac{r}{4} \sum_{j=f,r} \sum_{i=1} \omega_{ij}, \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots 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6. Conclusions and Future Works

This paper proposed a READ system that can optimally generate vehicle velocity to reduce energy consumption considering road gradient information. In the experiments, it is demonstrated that the proposed method increases regenerative energy by 14 % in comparison with the conventional
READ systems. However, it should be noted that the copper loss was accumulated due to the incorrect information of traveling distance, and to reduce copper loss, it is desirable to include algorithms such as Kalman filter for vehicle velocity or traveling distance estimation.

The future works include: 1) Introduce optimal driving-braking force distribution ratio to the proposed system, 2) Consider constraints on speed and acceleration on general roads and highways.

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